HYPERNUCLEAR SPECTROSCOPY IN THE UNITARY SYMMETRY MODEL

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The unitary symmetry model[1,2] based on the group SU(3) or SU(3)/Z(3) has been proved to be successful in classifying the existing hadrons (= strongly interacting particles). The nuclei, being bound states of the nucleons, have to belong to unitary multiplets accordingly. None of them is a unitary singlet, since they have nonvanishing hypercharge Y: $Y=Z+N=A$, where Z, N, and A are the numbers of protons, neutrons, and nucleons inside nuclei, respectively.

Therefore, to each isobaric multiplet of nuclei or of their excited states corresponds a unitary multiplet of hypernuclei which have the same spin and parity and suitable isobaric spin and hypercharge spectra. Hyperfragments are fortunate examples of such particles; they are stable with respect to the strong interactions. However, it is clear that there is an enormous number of the other hypernuclei and their excited states, though some of them may be unstable. Here we discuss only a few cases of the light nuclei as illustrations.

We assume that nuclei are the members of the unitary multiplets which have the highest hypercharge. The assumption is compatible with the fact that neither baryons nor hypernuclei with positive strangeness have been found in nature. We shall see that an irreducible representations to which nuclei belong can be uniquely determined by this assumption.

Let $D(\lambda_1,\lambda_2)$ be an irreducible representation of the group SU(3), which is characterized by two non-negative integers λ_1 and λ_2. The subquantum numbers, isobaric spin I and hypercharge Y, of submultiplets in the representation $D(\lambda_1,\lambda_2)$ are given by[3]

$$I=\tfrac{1}{2}(\lambda_1'-\lambda_2')+\tfrac{1}{2}\lambda_2 \tag{1}$$

and

$$Y=\lambda_1'+\lambda_2'-\tfrac{1}{3}(2\lambda_1+\lambda_2), \tag{2}$$

where λ_1' and λ_2' are integers such that

$$\lambda_1 \geqslant \lambda_1' \geqslant 0 \text{ and } \lambda_2 \geqslant \lambda_2' \geqslant 0. \tag{3}$$

Since the maximum value of Y is reached for $\lambda_1'=\lambda_1$ and $\lambda_2'=\lambda_2$, the nuclei with quantum numbers I and $Y=A$ have to belong to the representation $D(\lambda_1,\lambda_2)$ characterized by

$$\lambda_1=2I \tag{4}$$

and

$$\lambda_2=\tfrac{3}{2}A-I, \tag{5}$$

according to the assumption. The dimension of the representation, i.e., the number of the particles of the unitary multiplet, is[4]

$$N(\lambda_1,\lambda_2)=\tfrac{1}{2}(\lambda_1+1)(\lambda_2+1)(\lambda_1+\lambda_2+2). \tag{6}$$

Using Eqs. (4), (5), and (6) and experimental data on nuclei[5,6] and hyperfragments,[7] we give in Table I a list of unitary multiplets which involve the ground states of the nuclei with $Z\leqslant 7$.

We summarize various features of our prediction:

(a) All the observed hyperfragments are located in the listed unitary multiplets (column 5).

(b) Spin and parity of the hyperfragments are determined by those of the nuclei in the same multiplets (column 6).

(c) The predicted spins for ${}_\Lambda H^3$ and ${}_\Lambda H^4$ are compatible with current experimental data.[7,8]

(d) The spin and parity of ${}_\Lambda He^5$, ${}_\Lambda He^7$, ${}_\Lambda Be^7$, ${}_\Lambda Be^9$, and ${}_\Lambda B^{11}$ are all $\tfrac{3}{2}^-$, contrary to an expectation[7] that they would have spin $\tfrac{1}{2}$. The latter has been conjectured from the argument that for the cases where the core nuclei have spin zero, the extra lambda particle would be in the lowest orbital state (s state) independent of the property of the residual core nuclei. This turns out not to be the case in our model, since the lambda particle behaves like a nucleon in the spirit of the unitary-symmetry model, so that it is in a $p_{3/2}$ state for the above-mentioned hyperfragments.

(e) The spin of ${}_\Lambda Li^8$ has been suggested to be unity,[7,9,10] though this seems not to be conclusive, while our prediction is 0^+. This may offer a crucial test of the theory,[11] together with the measurement of spin and parity of the other hyperfragments.

(f) ${}_\Lambda B^9$ has not yet been reported to exist. According to our prediction, it is expected to be stable with respect to the strong interactions; $B^8+\Lambda=8587.3$ MeV $>$ Li^9. On the other hand, ${}_\Lambda Be^{9*}$, the other member of the isobaric triplet, is expected to be unstable; e.g., a reaction

$${}_\Lambda Be^{9*}\to {}_\Lambda He^5+He^4, \text{ etc.,}$$

would be possible.

Table I. Unitary multiplets of the hypernuclei.

Representation $D(\lambda_1, \lambda_2)$	Dimension $N(\lambda_1, \lambda_2)$	Isobaric multiplets contained[a] (I, Y)	Nuclei[b]	I, I_z	Mass (MeV)	Hyperfragments	I, I_z	Mass[c] (MeV)	Spin, parity[b] J^P
$D(0,3)$	10	$\underline{(0,2)}, (\frac{1}{2},1)(1,0), (\frac{3}{2},-1)$	H^2	$0,0$	1875.5	$_\Lambda n^2$	$\frac{1}{2},-\frac{1}{2}$	>2055.0	1^+
						$_\Lambda H^2$	$\frac{1}{2},\frac{1}{2}$	>2053.7	
$D(1,4)$	35	$\underline{(\frac{1}{2},3)}, (1,2), \cdots, (\frac{5}{2},-1)$	H^3	$\frac{1}{2},-\frac{1}{2}$	2808.8	$_\Lambda H^3$	$0,0$	2990.7 ± 0.2	$\frac{1}{2}^{(+)}$
		$\underline{(0,2)}, (\frac{1}{2},1), \cdots, (2,-2)$	He^3	$\frac{1}{2},\frac{1}{2}$	2808.3				
$D(0,6)$	28	$\underline{(0,4)}, \underline{(\frac{1}{2},3)}, \cdots, (3,-2)$	He^4	$0,0$	3727.1	$_\Lambda H^4$	$\frac{1}{2},-\frac{1}{2}$	3922.2 ± 0.1	0^+
						$_\Lambda He^4$	$\frac{1}{2},\frac{1}{2}$	3921.3 ± 0.3	
$D(1,7)$	80	$\underline{(\frac{1}{2},5)}, (1,4), \cdots, (4,-2)$	He^5	$\frac{1}{2},-\frac{1}{2}$	4667.6	$_\Lambda He^5$	$0,0$	4839.5 ± 0.1	$\frac{3}{2}^-$
		$\underline{(0,4)}, (\frac{1}{2},3), \cdots, (\frac{7}{2},-3)$	Li^5	$\frac{1}{2},\frac{1}{2}$	4667.2				
$D(0,9)$	55	$\underline{(0,6)}, \underline{(\frac{1}{2},5)}, \cdots, (\frac{9}{2},-3)$	Li^6	$0,0$	5601.2	$_\Lambda He^6$	$\frac{1}{2},-\frac{1}{2}$	···	1^+
						$_\Lambda Li^6$	$\frac{1}{2},\frac{1}{2}$	···	
$D(1,10)$	143	$\underline{(\frac{1}{2},7)}, \underline{(1,6)}, \cdots, (11/2,-3)$	Li^7	$\frac{1}{2},-\frac{1}{2}$	6533.4	$_\Lambda Li^7$	$0,0$	6711.1 ± 0.2	$\frac{3}{2}^-$
						$_\Lambda He^7$	$1,-1$	6716.8 ± 0.9	
		$\underline{(0,6)}(\frac{1}{2},5), \cdots, (5,-4)$	Be^7	$\frac{1}{2},\frac{1}{2}$	6533.8	$_\Lambda Li^{7*}$	$1,0$	···	
						$_\Lambda Be^7$	$1,1$	6716.2 ± 0.5	
$D(0,12)$	91	$\underline{(0,8)}, \underline{(\frac{1}{2},7)}, \cdots, (6,-4)$	Be^8	$0,0$	7454.4	$_\Lambda Li^8$	$\frac{1}{2},-\frac{1}{2}$	7642.4 ± 0.2	0^+
						$_\Lambda Be^8$	$\frac{1}{2},\frac{1}{2}$	7642.9 ± 0.3	
$D(1,13)$	224	$\underline{(\frac{1}{2},9)}, \underline{(1,8)}, \cdots, (7,-4)$	Be^9	$\frac{1}{2},-\frac{1}{2}$	8392.2	$_\Lambda Be^9$	$0,0$	8563.3 ± 0.2	$\frac{3}{2}^-$
						$_\Lambda Li^9$	$1,-1$	8577.5 ± 0.3	
		$\underline{(0,8)}, (\frac{1}{2},7), \cdots, (13/2,-5)$	B^9	$\frac{1}{2},\frac{1}{2}$	8392.8	$_\Lambda Be^{9*}$	$1,0$	···	
						$_\Lambda B^9$	$1,1$	···	
$D(0,15)$	136	$\underline{(0,10)}, \underline{(\frac{1}{2},9)}, \cdots, (15/2,-5)$	B^{10}	$0,0$	9323.9	$_\Lambda Be^{10}$	$\frac{1}{2},-\frac{1}{2}$	9499.3 ± 0.6	3^+
						$_\Lambda B^{10}$	$\frac{1}{2},\frac{1}{2}$	9498.2 ± 1.0	
$D(1,16)$	323	$\underline{(\frac{1}{2},11)}, (1,10), \cdots, (17/2,-5)$	B^{11}	$\frac{1}{2},-\frac{1}{2}$	10 251.9	$_\Lambda B^{11}$	$0,0$	10 429.4 ± 0.6	$\frac{3}{2}^-$
		$\underline{(0,10)}, (\frac{1}{2},9), \cdots, (8,-6)$	C^{11}	$\frac{1}{2},\frac{1}{2}$	10 253.4				
$D(0,18)$	190	$\underline{(0,12)}, \underline{(\frac{1}{2},11)}, \cdots, (9,-6)$	C^{12}	$0,0$	11 174.2	$_\Lambda B^{12}$	$\frac{1}{2},-\frac{1}{2}$	11 357.0 ± 0.3	0^+
						$_\Lambda C^{12}$	$\frac{1}{2},\frac{1}{2}$	11 356.0[d]	
$D(1,19)$	440	$\underline{(\frac{1}{2},13)}, (1,12), \cdots, (10,-6)$	C^{13}	$\frac{1}{2},-\frac{1}{2}$	12 109.7	$_\Lambda C^{13}$	$0,0$	12 278.8 ± 0.5	$\frac{1}{2}^-$
		$\underline{(0,12)}, (\frac{1}{2},11), \cdots, (19/2,-7)$	N^{13}	$\frac{1}{2},\frac{1}{2}$	12 111.4				
$D(0,21)$	253	$\underline{(0,14)}, \underline{(\frac{1}{2},13)}, \cdots, (21/2,-7)$	N^{14}	$0,0$	13 040.4	$_\Lambda C^{14}$	$\frac{1}{2},-\frac{1}{2}$	13 212.0 ± 0.7	1^+
						$_\Lambda N^{14}$	$\frac{1}{2},\frac{1}{2}$	13 215.2 ± 0.5	

[a]The isobaric multiplets (I, Y) which are underlined are nuclei and hyperfragments which are listed in the next columns. For multiplets of the same row there holds the relation $I + \frac{1}{2}Y = \text{const}$.

[b]Use is made of the tables in references 5 and 6.

[c]Calculated from the table in reference 7.

[d]Calculated from the data of A. Z. M. Ismail, I. R. Kenyon, A. W. Key, S. Lokanathan, and Y. Prakash, Phys. Letters 1, 199 (1962).

(g) $_\Lambda n^2$ and $_\Lambda H^2$ are likely to be unstable,[12] while the existence of stable $_\Lambda He^6$ and $_\Lambda Li^6$ are uncertain.[13] The mass of $_\Lambda Li^{7*}$, a member of an isobaric triplet, is to be just near the sum of the masses of Li^6 and Λ.

(h) Since for isobaric multiplets placed in the same row of column 3 in Table I there holds a relation $I + \frac{1}{2}Y = \text{const}$, their masses are equally spaced, according to the Gell-Mann–Okubo mass formula.[1,3] Therefore, all the masses of isobaric multiplets in the representations $D(0,6)$, $D(0,9)$, $D(1,10)$, $D(0,12)$, $D(1,13)$, $D(0,15)$, $D(0,18)$, and $D(0,21)$ can be predicted. [For $D(1,10)$ and $D(1,13)$, the masses of three isobaric multiplets are known. This is sufficient to determine the masses of all the others.]

(i) Hypernuclei with $s = |S| > 2$ may be produced by reactions such as

$$(\pi \text{ or } Z^A) + Z_1^{A_1} \to {}_S Z_2^{A_2} + sK + \cdots,$$

$$(\bar{K} \text{ or } \Sigma) + Z_1^{A_1} \to {}_S Z_2^{A_2} + (s-1)K + \cdots, \text{ etc.},$$

where $_S Z^A$ stands for a hypernucleus with the designated quantum numbers. Most hypernuclei belonging to the unitary multiplets which were explicitly mentioned in (h) are stable. Some preliminary reports[14] have been made on the existence of hypernuclei with $S = -2$.

It would be interesting to investigate heavier hypernuclei with large nucleon number and hypernuclei which correspond to the excited states of nuclei. They not only afford an enormous number of experimental tests of the unitary symmetry model, but also constitute a chart of hypernuclear spectroscopy.[15]

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[2] Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[3] S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962). Our notation is related to that of this reference by $\lambda_1 = f_1 - f_2$, $\lambda_2 = f_2 - f_3$, $\lambda_1' = f_1' - f_2$, and $\lambda_2' = f_2' - f_3$. Incidentally, Eq. (13) in the reference should read $Y = (f_1' + f_2') - \frac{2}{3}(f_1 + f_2 + f_3)$.

[4] H. Weyl, The Classical Groups (Princeton University Press, Princeton, New Jersey, 1939).

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[6] F. Ajzenberg-Selove, Nuclear Spectroscopy (Academic Press, New York, 1960).

[7] E. H. S. Burhop, D. H. Davis, and J. Zakrzewski, Progr. Nucl. Phys. 9, 157 (1964). Some other references can be found in this review article.

[8] M. M. Block, C. Meltzer, S. Ratti, L. Grimellini, T. Kikuchi, L. Lendinara, and L. Monari, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 458.

[9] D. H. Davis, R. Levi Setti, and M. Raymund, Nucl. Phys. 41, 73 (1963).

[10] R. H. Dalitz, Nucl. Phys. 41, 78 (1963).

[11] If $_{\Lambda}Li^8$ and $_{\Lambda}Be^8$ are associated with the unitary multiplet of Be^{8*}(17.63 MeV; 1^+, $T=1$), they have spin unity. [In this case, the representation is $D(2,11)$ with dimension 270.] However, the prediction of parity is in disaccord with that of reference 10.

[12] R. J. Oakes, Phys. Rev. 131, 2239 (1963); I. S. Gerstein, Nuovo Cimento 32, 1706 (1964). The former discusses the unitary multiplet of the ground state $^3S_1$ of the deuteron, i.e., $D(0,3)$ of Table I in our text, and the latter that of the virtual state $^1S_0$, which representation is $D(2,2)$ with dimension 27. The author is indebted to Professor B. W. Lee for kindly bringing the author's attention to these articles.

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[15] R. J. Oakes has discussed hypernuclei with $A \leq 5$ in the unitary symmetry model [Proceedings of the Athens Topical Conference on Recently Discovered Resonant Particles (Ohio University, Athens, Ohio, 1963), p. 226.] For a review of the experimental situation on hyperfragments, see Proceedings of the International Conference on Hyperfragments, St. Cergue, 1963 (CERN Report No. 64-1, 1964), unpublished.

SU(6) AND ELECTROMAGNETIC INTERACTIONS

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1. The purpose of this note is to discuss some properties of the electromagnetic vertex of baryons under the assumption that the effective electromagnetic current associated with the strongly interacting particles transforms according to the adjoint representation of the group[1-3] SU(6). In particular we show that, in the limit where SU(6) is broken by electromagnetism only, all of the following quantities can be expressed uniquely in terms of the proton magnetic moment $\mu(p)$: (a) the magnetic moments of all baryon octet members, (b) those of the spin-$\frac{3}{2}$ decuplet, (c) all allowed transition moments between octet and decuplet. We recall[1,2] that the octet and the decuplet are united in the 56-dimensional representation of SU(6) and that $\underline{56}^* \otimes \underline{56}$ contains $\underline{35}$ only once. All our results about baryons stem from this single occurrence of $\underline{35}$.