massive vector bosons. There are two $I=\frac{1}{2}$ vector doublets, degenerate in mass between $Y=\pm1$ but with an electromagnetic mass splitting between $I_3=\pm\frac{1}{2}$, and the $I_3=\pm1$ components of a $Y=0$, $I=1$ triplet whose mass is entirely electromagnetic. The two $Y=0$, $I=0$ gauge fields remain massless: This is associated with the residual unbroken symmetry under the Abelian group generated by Y and I_3. It may be expected that when a further mechanism (presumably related to the weak interactions) is introduced in order to break Y conservation, one of these gauge fields will acquire mass, leaving the photon as the only massless vector particle. A detailed discussion of these questions will be presented elsewhere.

It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar and vector bosons.[8] It is to be expected that this feature will appear also in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields.[9]

[1]P. W. Higgs, to be published.

[2]J. Goldstone, Nuovo Cimento 19, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).

[3]P. W. Anderson, Phys. Rev. 130, 439 (1963).

[4]In the present note the model is discussed mainly in classical terms; nothing is proved about the quantized theory. It should be understood, therefore, that the conclusions which are presented concerning the masses of particles are conjectures based on the quantization of linearized classical field equations. However, essentially the same conclusions have been reached independently by F. Englert and R. Brout, Phys. Rev. Letters 13, 321 (1964): These authors discuss the same model quantum mechanically in lowest order perturbation theory about the self-consistent vacuum.

[5]In the theory of superconductivity such a term arises from collective excitations of the Fermi gas.

[6]See, for example, S. L. Glashow and M. Gell-Mann, Ann. Phys. (N.Y.) 15, 437 (1961).

[7]These are just the parameters which, if the scalar octet interacts with baryons and mesons, lead to the Gell-Mann–Okubo and electromagnetic mass splittings: See S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964).

[8]Tentative proposals that incomplete SU(3) octets of scalar particles exist have been made by a number of people. Such a rôle, as an isolated $Y=\pm1$, $I=\frac{1}{2}$ state, was proposed for the κ meson (725 MeV) by Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 11, 42 (1963). More recently the possibility that the σ meson (385 MeV) may be the $Y=I=0$ member of an incomplete octet has been considered by L. M. Brown, Phys. Rev. Letters 13, 42 (1964).

[9]In the theory of superconductivity the scalar fields are associated with fermion pairs; the doubly charged excitation responsible for the quantization of magnetic flux is then the surviving member of a U(1) doublet.

SPLITTING OF THE 70-PLET OF SU(6)

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1. In a previous note,[1] hereafter called I, we proposed an expression for the mass operator responsible for lifting the degeneracies of spin-unitary spin supermultiplets [Eq. (31)-I]. The purpose of the present note is to apply this expression to the 70-dimensional representation of SU(6).

The importance of the 70-dimensional representation has already been underlined by Pais.[2] Since

$$\underline{35}\otimes\underline{56}=\underline{56}\oplus\underline{70}\oplus\underline{700}\oplus\underline{1134}, \tag{1}$$

it follows that $\underline{70}$ is the natural candidate for accommodating the higher meson-baryon resonances. Furthermore, since the SU(3)⊗SU(2) content is

$$\underline{70}=(\underline{1},\underline{2})+(\underline{8},\underline{2})+(\underline{10},\underline{2})+(\underline{8},\underline{4}), \tag{2}$$

we may assume that partial occupancy of the $\underline{70}$ representation has already been established through the so-called γ octet[2] $(\frac{3}{2})^-$. Recent experiments appear to indicate that some $(\frac{1}{2})^-$ states may also be at hand.[3] With six masses at one's disposal, our formulas can predict the masses of all the other occupants of $\underline{70}$ and also provide a consistency check on the input. Our discussion of the $\underline{70}$ representation thus appears to be of immediate physical interest.

The question of numerical predictions cannot be properly treated without a critical analysis of the available experimental input. Such an analysis, however, is outside the proper province of this note.[4]

2. The first problem at hand is the construction of basis tensors of the 70-dimensional representation and identification of the I, Y, J values associated with each component. The simplest procedure is to start with the reducible representation $\underline{15}\otimes\underline{6}=\underline{20}\oplus\underline{70}$ and remove the completely antisymmetric part. Alternatively one can start with $\underline{21}\otimes\underline{6}=\underline{56}\oplus\underline{70}$ and remove the completely symmetric part.

3. The $J=\frac{3}{2}$ tensors are pure under SU(3). For the $J=\frac{1}{2}$ tensors the SU(3) reduction is accomplished by separating out the part completely symmetric in SU(3) indices (decuplet), the part completely antisymmetric (singlet), and the part with "mixed symmetry" (octet). By taking appropriate linear combinations one can then write down all the eigenstates of the P chain. If SU(3) representations are labeled by the usual (p, q), then

$$C_2^{(3)}=\tfrac{2}{3}(p^2+q^2+pq+3p+3q). \tag{3}$$

4. Next one takes linear combinations of states in the P chain in order to obtain eigenstates of N and S. States which share the same values of Y and S can be combined into one or more sets, each set providing a basis for an irreducible representation of SU(4). These representations can be reduced with respect to $SU(2)_I\otimes SU(2)_N$; this reduction, in fact, provides a powerful check on the SU(4) assignments and the Wigner numbers[5] characterizing SU(4) representations. If the Wigner numbers are denoted by (Q, Q', Q''), then

$$C_2^{(4)}=Q^2+4Q+Q'^2+2Q'+Q''^2. \tag{4}$$

5. Our notation for particle states which are eigenstates of operators in the P chain is as follows[6]:

$$(\underline{1},\underline{2}):\ \Lambda_P', \tag{5}$$

$$(\underline{8},\underline{2}):\ \tilde{N}, \tilde{\Sigma}_P, \tilde{\Lambda}_P, \tilde{\Xi}_P; \tag{6}$$

$$(\underline{10},\underline{2}):\ \tilde{N}^*, \tilde{Y}_P^*, \tilde{\Xi}_P^*, \tilde{\Omega}; \tag{7}$$

$$(\underline{8},\underline{4}):\ N_\gamma, \Sigma_\gamma, \Lambda_\gamma, \Xi_\gamma. \tag{8}$$

For the eigenstates in the U chain, we have[6]

$$(1,0;\tfrac{3}{2},\tfrac{1}{2},\tfrac{1}{2}):\ N_\gamma, \tilde{N}, \tilde{N}^*; \tag{9}$$

$$(0,\tfrac{1}{2};1,1,1):\ \Sigma_\gamma \text{ or } \tilde{\Sigma}_U, \Lambda_U'; \tag{10}$$

$$(0,\tfrac{1}{2};1,0,0):\ \Lambda_\gamma \text{ or } \tilde{\Lambda}_U, \tilde{Y}_U^*; \tag{11}$$

$$(-1,0;\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}):\ \tilde{\Xi}_U; \tag{12}$$

$$(-1,1;\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}):\ \tilde{\Xi}_U^* \text{ or } \Xi_\gamma; \tag{13}$$

$$(-2,\tfrac{1}{2};0,0,0):\ \tilde{\Omega}. \tag{14}$$

Here the numbers in parentheses are $(Y, S; Q, Q', Q'')$ and "A or B" implies that A and B are states distinguished by J spin but totally identical with respect to $U(1)\otimes SU(2)_S\otimes SU(4)$.

Recoupling formulas, relating states in the two chains, are all of the form

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix}=\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}\begin{pmatrix} P_1 \\ P_2 \end{pmatrix}, \tag{15}$$

where $U_1=\Lambda_U'$, $\tilde{\Sigma}_U$, and $\tilde{\Xi}_U$ correspond, respectively, to $U_2=\tilde{\Lambda}_U$, $\tilde{Y}_U^*$, and $\tilde{\Xi}_U^*$. Similarly, we obtain the correspondences between P_1 and P_2 from the foregoing by changing the subscripts.

6. We have tabulated, in Table I, the quantum numbers associated with the states listed above. In order to use this table it is convenient, as in I, to start with the U chain as the basis and diagonalize the mass operator. One obtains in this way the masses of the real particles as well as the corresponding eigenvectors.

7. We obtain the following seven "sum" rules,[7] four additive and three multiplicative, connecting the 13 masses which occur in the 70-dimensional representation of SU(6) (notation: particle label ≡ particle mass):

$$3\Lambda_\gamma+\Sigma_\gamma=2(N_\gamma+\Xi_\gamma), \tag{16}$$

$$4(\tilde{Y}_R^*+\tilde{\Sigma}_R)-2(\tilde{N}^*+\tilde{N}+\tilde{\Xi}_R^*+\tilde{\Xi}_R)$$
$$=6(\tilde{N}^*-\tilde{N})-3(\tilde{Y}_R^*+\tilde{\Sigma}_R-\tilde{\Lambda}_R-\Lambda_R'), \tag{17}$$

$$2(\tilde{\Omega}-\tilde{N}^*)=3(\tilde{\Xi}_R^*+\tilde{\Xi}_R-\tilde{Y}_R^*-\tilde{\Sigma}_R), \tag{18}$$

$$2(\tilde{\Omega}-\tilde{N}^*)=3(\Sigma_\gamma+\Lambda_\gamma)-6N_\gamma, \tag{19}$$

$$\Lambda_R{}'\tilde{\Lambda}_R = (a+3b+\tfrac{3}{4}c-15e/2-\tfrac{3}{4}f)$$
$$\times(a+3b+\tfrac{3}{4}c-\tfrac{7}{2}e+5f/4)-9b^2, \qquad (20)$$

$$\tilde{Y}_R{}^*\tilde{\Sigma}_R = (a+9b+\tfrac{3}{4}c-15e/2+13f/4)$$
$$\times(a+9b+\tfrac{3}{4}c-\tfrac{7}{2}e+5f/4)-9b^2, \qquad (21)$$

$$\tilde{\Xi}_R{}^*\tilde{\Xi}_R = (a+9b+\tfrac{3}{4}c-d-\tfrac{7}{2}e+5f/4)$$
$$\times(a+9b+\tfrac{3}{4}c-d+\tfrac{1}{2}e-\tfrac{3}{4}f)-9b^2. \qquad (22)$$

Here the subscript R indicates real particles which are not eigenstates in either the P or the U chains. The parameters $a, b, \cdots, f$ are given by

$$a=\tfrac{1}{2}(N_\gamma+\Xi_\gamma)+(25/4)(\tilde{N}^*-\tilde{N})-(9/8)(\Sigma_\gamma-\Lambda_\gamma)-(5/4)$$
$$\times(N_\gamma-\tilde{N})-(29/8)(\tilde{Y}_R{}^*+\tilde{\Sigma}_R-\Lambda_R{}'-\tilde{\Lambda}_R), \qquad (23)$$

$$b=\tfrac{1}{4}(\tilde{Y}_R{}^*+\tilde{\Sigma}_R-\Lambda_R{}'-\tilde{\Lambda}_R)-\tfrac{1}{3}(\tilde{N}^*-\tilde{N}), \qquad (24)$$

$$c=\tfrac{1}{3}(N_\gamma-\tilde{N})-(\tilde{N}^*-\tilde{N})+\tfrac{1}{2}(\tilde{Y}_R{}^*+\tilde{\Sigma}_R-\Lambda_R{}'-\tilde{\Lambda}_R), \qquad (25)$$

$$d=N_\gamma-\tfrac{3}{2}\Sigma_\gamma+\tfrac{1}{2}\Lambda_\gamma, \qquad (26)$$

$$e=\tfrac{1}{2}(\tilde{N}^*-\tilde{N})-\tfrac{1}{4}(\Sigma_\gamma-\Lambda_\gamma)$$
$$-\tfrac{1}{4}(\tilde{Y}_R{}^*+\tilde{\Sigma}_R-\Lambda_R{}'-\tilde{\Lambda}_R), \qquad (27)$$

$$f=(\tilde{N}^*-\tilde{N})-\tfrac{1}{2}(\tilde{Y}_R{}^*+\tilde{\Sigma}_R-\Lambda_R{}'-\tilde{\Lambda}_R). \qquad (28)$$

8. Full details of the present note, as well as I, will be incorporated in a paper to be submitted elsewhere.

We wish to record our debt to Professor A. Pais for his continued interest and encouragement. We are grateful to Professor F. J. Dyson for some enlightening discussions. One of us (V.S.) wishes to thank Professor J. R. Oppenheimer for hospitality at the Institute for Advanced Study.

Table I. Quantum numbers of meson-baryon resonances in the 70-dimensional representation of SU(6).

Particle	Y	I	N	S	J	$C_2^{(3)}$	$C_2^{(4)}$
N_γ	1	1/2	3/2	0	3/2	6	39/4
Σ_γ	0	1	1	1/2	3/2	6	9
Λ_γ	0	0	1	1/2	3/2	6	5
Ξ_γ	−1	1/2	1/2	1	3/2	6	15/4
$\Lambda_{\mathrm{P}}{}'$	0	0	⋯	⋯	1/2	0	⋯
$\Lambda_{\mathrm{U}}{}'$	0	0	0	1/2	1/2	⋯	9
$\tilde{N}$	1	1/2	1/2	0	1/2	6	39/4
$\tilde{\Sigma}_{\mathrm{P}}$	0	1	⋯	⋯	1/2	6	⋯
$\tilde{\Sigma}_{\mathrm{U}}$	0	1	1	1/2	1/2	⋯	9
$\tilde{\Lambda}_{\mathrm{P}}$	0	0	⋯	⋯	1/2	6	⋯
$\tilde{\Lambda}_{\mathrm{U}}$	0	0	1	1/2	1/2	⋯	5
$\tilde{\Xi}_{\mathrm{P}}$	−1	1/2	⋯	⋯	1/2	6	⋯
$\tilde{\Xi}_{\mathrm{U}}$	−1	1/2	1/2	0	1/2	⋯	15/4
$\tilde{N}^*$	1	3/2	1/2	0	1/2	12	39/4
$\tilde{Y}_{\mathrm{P}}{}^*$	0	1	⋯	⋯	1/2	12	⋯
$\tilde{Y}_{\mathrm{U}}{}^*$	0	1	0	1/2	1/2	⋯	5
$\tilde{\Xi}_{\mathrm{P}}{}^*$	−1	1/2	⋯	⋯	1/2	12	⋯
$\tilde{\Xi}_{\mathrm{U}}{}^*$	−1	1/2	1/2	1	1/2	⋯	15/4
$\tilde{\Omega}$	−2	0	0	1/2	1/2	12	0

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[1]M. A. B. Bég and V. Singh, Phys. Rev. Letters 13, 418 (1964). The present note is meant to be read in conjunction with I.

[2]A. Pais, Phys. Rev. Letters, 13, 175 (1964).

[3]F. Bulos et al., Phys. Rev. Letters 13, 486 (1964) ($I=\tfrac{1}{2}$ state). D. Berley et al., Proceedings of the Dubna Conference, 1964 (to be published) ($I=0$ state). We thank S. F. Tuan for supplying us with these references. The old $Y_0{}^*$(1405 MeV) is another candidate with $I=0$.

[4]We hope to return to this point, in association with S. F. Tuan.

[5]E. Wigner, Phys. Rev. 51, 105 (1937). Our (Q, Q', Q'') are, respectively, identical to Wigner's (S, T, Y). We discard Wigner's nomenclature since it is likely to lead to disastrous confusion. The current notation of nuclear spectroscopists (P, P', P'') is also undesirable.

[6]We omit the subscripts P and U in those instances where the state is a simultaneous eigenstate of operators in the P and U chains, respectively.

[7]One of our sum rules, Eq. (16), has been obtained earlier by Kuo and Yao under more restrictive assumptions [T. K. Kuo and T. Yao, Phys. Rev. Letters 13, 415 (1964)].