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LANDAU DAMPING AND FINITE-LENGTH EFFECTS IN UNIVERSAL PLASMA INSTABILITIES

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Excitation in controlled fashion of the universal instability in the plasma in a Q device[1] offers a means of investigating the effect of plasma length on this instability. This possibility derives from the fact that the oscillation amplitudes and frequencies, which can be measured with good accuracy, are affected by the length of the plasma. This Letter reports preliminary results of experiments in which we have investigated the proposed stabilization of the universal instability by ion Landau damping in a plasma whose dimensions satisfy[2] the length-to-radius criterion $L/R \leq 10$. We have also observed the effect of plasma length on the real part of the oscillation frequency, and phenomena that may be called "mode jumping" and "mode locking"; the last two effects are attributed to nonlinear mode interactions. All results reported here pertain to plasma densities $n \leq 7\times 10^{10}$ cm^{-3} and it is assumed that collisional effects can be neglected.

The experimental arrangement is similar to that described earlier[1] in which a thermal plasma column is produced in a magnetic field $[(1\text{-}4)\times 10^3$ G$]$ by contact ionization of atomic cesium on hot tungsten plates ($T\sim 2500\,°$K) which also provide the required electrons. In the present case, however, the plasma is terminated at one end by a cold plate (metal or ceramic) that moves in pistonlike fashion so that the plasma length can be varied from 12 to 90 cm. The presence of the cold plate causes the plasma to stream from the hot plate to the cold plate (where plasma is lost by surface recombination) and leads to the appearance of a thermoelectric potential.[3] Thermoelectric effects are avoided by using the ceramic plate or by allowing the metal plate to float electrically. The streaming process is essentially a free expansion and the streaming velocity is assumed to be of the order of the sound speed.[4] The sheath conditions are adjusted to keep the potential oscillations at "quasilinear" levels ($e\varphi/kT = 10^{-3}$-10^{-4}) so that stable narrow-line spectra can be observed with a spectrum analyzer connected to a floating probe.[5] In order to determine the qualitative nature of various phenomena rapidly, in certain cases the conventional spectrum-analyzer (SA) display has been replaced by a "raster" display, in which the SA signal intensity-modulates a slave oscilloscope whose horizontal sweep is synchronized with the SA sweep. A slow vertical sweep is applied simultaneously to the slave oscilloscope. If an experimental variable is now varied the resulting pattern is essentially a plot of this variable vs the oscillation frequency with time as a parameter as used in Fig. 3.

The stabilization of the universal instability by ion Landau damping when $L/R \leq 10$ can be understood from Fig. 1. In general ω/k_z, the phase velocity characteristic of the universal instability, satisfies $v_T^{(i)} \ll \omega/k_z \ll v_T^{(e)}$ and lies in the cross-hatched region on the right in Fig. 1. However, if k_z is increased or ω reduced so that $\omega/k_z \leq \alpha v_T^{(i)}$, where $\alpha\sim 3$, the phase velocity falls into the region where the negative slope of

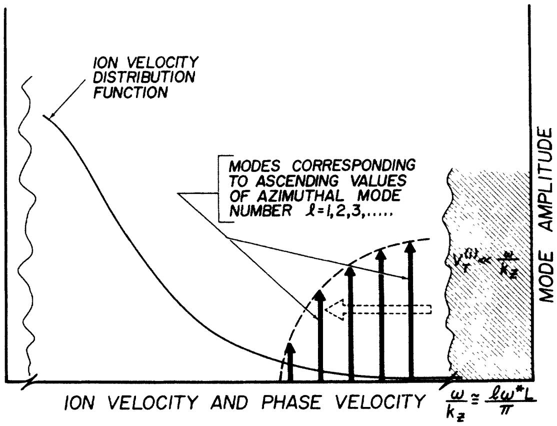

FIG. 1. Stabilization of the universal instability by ion Landau damping. The dotted arrow indicates the movement of the mode pattern when the phase velocity is reduced. The velocity distribution and amplitude locus (dashed line) remain fixed.

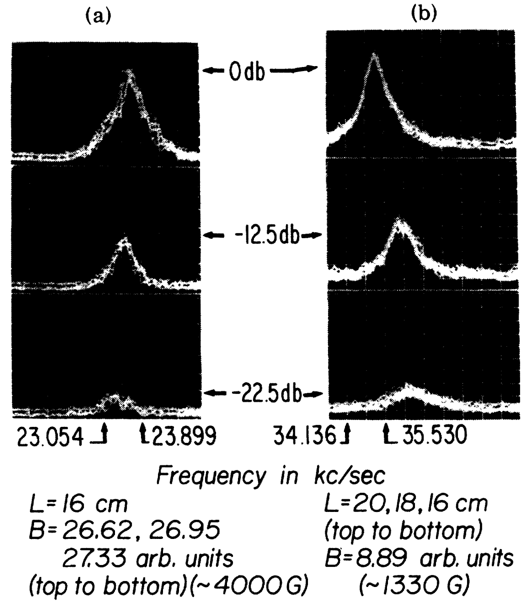

FIG. 2. Spectrum-analyzer presentation of the damping of a universal-instability mode by ion Landau damping. In Fig. 2(a) a reduction of frequency (phase velocity) of approximately 2 % (produced by increasing the magnetic field) reduces the mode energy by approximately 170 times (22.5 dB). In Fig. 2(b) a phase-velocity reduction of approximately 12 %, produced by shortening the column, reduces the mode energy by the same factor (0 dB = 200 μV).

the ion velocity distribution is no longer negligible and ion Landau damping can become important.[2] Making use of Eq. (3) below we can write the stabilization criterion

$$\frac{\omega}{k_z} \sim \frac{k_1}{k_z} v_T^{(i)} \left(\frac{\rho_L}{a}\right) \leq 3 v_T^{(i)}, \text{ i.e., } \frac{L}{R} \leq 10. \qquad (1)$$

In the present case $\omega/k_z = 2fL$ can be reduced by shortening the plasma column or by keeping the length fixed and increasing the magnetic field, thereby reducing the frequency as indicated by Eq. (1). The ion Landau damping then acts as a supplementary loss mechanism that serves to reduce the oscillation amplitude below the level set by the balance between various other (nonlinear) damping mechanisms and the inherent growth rate. For this reason the oscillation will be damped at values of α somewhat higher than that indicated above. Using the reasoning given by Landau and Lifshitz[6] and the fact that the Landau damping factor $\gamma_L \sim \exp\{-[v_\varphi/v_T^{(i)}]^2\}$, it can be shown that the mode energy varies as const $-\exp\{-[v_\varphi/v_T^{(i)}]^2\}$ in the critical region in Fig. 1 and that

$$v_T^{(i)} = v_\varphi \left(\frac{2dv_\varphi}{v_\varphi}\right)^{1/2} = 2\sqrt{2}Lf\left[\frac{df}{f} + \frac{dL}{L}\right]^{1/2}, \qquad (2)$$

where df/f and dL/L are respectively the fractional reductions in frequency and length required to reduce the mode energy to $(1-1/e)$ times its undamped value.

The results of experiments carried out to verify these considerations are shown in Figs. 2(a) and 2(b). In both cases the top picture shows the spectrum-analyzer presentation for a given mode when the experimental parameters are adjusted to make the phase velocity slightly greater than the value corresponding to the onset of Landau damping. In Fig. 2(a) the column length remains fixed but the phase velocity (frequency) is reduced by increasing the magnetic field as indicated in the figure caption. In Fig. 2(b) the magnetic field remains fixed but the phase velocity is reduced by decreasing the column length. (This results in a slight frequency increase but the phase velocity is still reduced because of the dominating effect of the increase in k_z; the origin of the frequency increase is discussed below.) It is evident from the figure that in both cases a small reduction in phase velocity results in a dramatic reduction in mode energy. On the other hand, increasing the length beyond the critical value has essentially no effect on mode amplitude or frequency. Using Eq. (2) and Fig. 2(a) we find $v_T^{(i)} \sim 7.5 \times 10^4$ cm/sec which, in this preliminary work, is regarded as satisfactory compared with the value 5.6×10^4 cm/sec $=(2kT/M)^{1/2}$ for $T=2500\,°\mathrm{K}$. We also note that $L/R\sim10$ in the damping region.

These results would appear to verify the finite-length stabilization criterion for a given mode of the universal instability (see below). The technique described can also be used to study the Landau damping mechanism itself. Investigations of Landau damping by other methods have been reported recently.[7,8]

The expected effect of length on the real part of the frequency can be obtained from an expression given by Mikhailovskii,[9]

$$\omega=\frac{\omega^*}{2}\left\{1+\left[1+\left(\frac{2k_z v_s}{\omega^*}\right)^2\right]^{1/2}\right\},\qquad(3)$$

where $\omega^*=k_\perp(ckT/Be)(n'/n)\cong k_\perp v_T^{(i)}(\rho_L/a)$; $k_\perp=l/r$ is the azimuthal wave number, $l=1,2,\cdots$; $k_z=\pi/L$ is the longitudinal wave number; v_s is the sound speed; c is the velocity of light; B is the magnetic field; n is the density; n' is the radial density gradient; a is the characteristic scale length; k is Boltzmann's constant; $v_T^{(i)}$ is the ion thermal velocity; ρ_L is the ion Larmor radius ($k_\perp\rho_L<1$). This expression, which predicts that the frequency should be a weakly diminishing function of plasma length under present conditions, gives good qualitative agreement (factor of two) with the frequency increments that are observed with changes in column length in Figs. 2(b) and 3 and in other experiments with longer columns ($L=90$ cm). However, it should be noted that there is a streaming velocity comparable with the thermal velocity in these experiments and it is possible that Eq. (3) should be modified appropriately. This subject is being investigated.

Although the length stabilization criterion has been verified for a single mode with L/R ratios found to be of the proper order of magnitude, examination of Fig. 3(a) shows that there is an additional effect that should be considered: When the plasma is made short enough to damp a given azimuthal mode by ion Landau damping, energy that had been feeding into this mode is fed to higher modes that are not yet damped. This "mode jumping" is evidently a nonlinear phenomenon that is not described by the linear theory and appears to be effective at the shortest plasma lengths available in the present experimental apparatus, with the $l=4$ mode being excited at a plasma length of 12 cm.

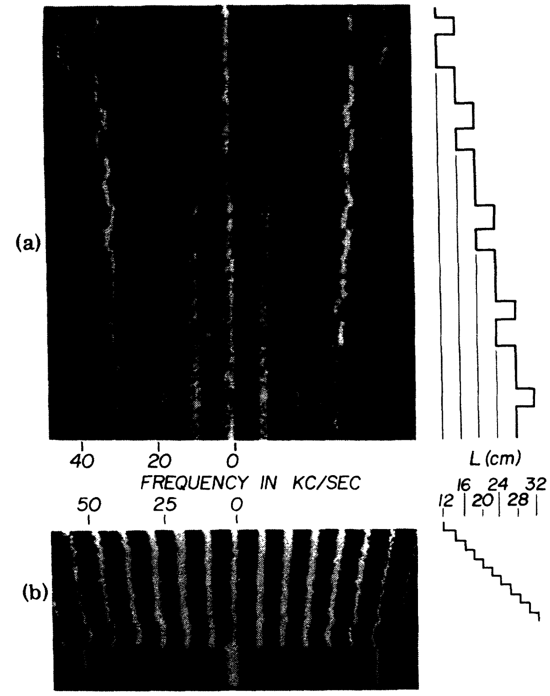

FIG. 3. "Raster" display of mode spectra as a function of plasma length. The variation of column length is shown by the sketch to the right of the pictures. Figure 3(a) shows the "mode jumping" caused by the nonlinear transfer of energy from damped modes to undamped modes. Figure 3(b) shows the "mode locking" attributed to nonlinear mode interactions. The length increments in Fig. 3(b) are 2 cm.

Another effect not described by the linear theory is shown in Fig. 3(b). This may be called "mode locking". In general, mode amplitudes are observed to be exponentially diminishing functions of azimuthal mode number.[1] It then appears that the establishment of the lowest azimuthal mode serves to set up boundary conditions such that the frequencies of the higher modes remain integral multiples of the lowest mode as k_z is varied, an effect not predicted by Eq. (3). This behavior can be described by a picture proposed by Kadomtsev in which all modes are at rest in the frame rotating with the electron drift velocity; modes with different l then tend to maintain their relative phases by strong mutual interactions.[10]

It should be noted that these nonlinear effects are observed at "quasilinear" amplitudes and may not obtain at higher nonlinear levels.

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OFF-DIAGONAL LONG-RANGE ORDER IN HEISENBERG FERROMAGNET

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Yang[1] has pointed out that the superfluid state of liquid helium is characterized by the existence of off-diagonal long-range order (ODLRO), i.e., the expectation values $\langle a_i^\dagger a_j\rangle$ do not vanish even when the distances between particles (i,j) become indefinitely large. Here the a_i, a_j are boson annihilation operators. Yang also proposes a characterization of superconducting states in terms of a somewhat different ODLRO for fermion operators. In both these cases, ODLRO has no classical analog. One may ask whether a similar characterization exists for the Heisenberg ferromagnet. There the boson or fermion operator is (for spin $\frac{1}{2}$) replaced by the Pauli spin reversal operator $b=\sigma_x+i\sigma_y$, and the expectation value of interest is $\langle b_i^\dagger b_j\rangle$. We shall say that the system has ODLRO if this expectation value is nonvanishing in the limit in which the distance between lattice sites i and j becomes infinite. If we consider the ferromagnet in the limit as the external field goes to zero, one finds from spin-wave theory that ODLRO exists well below the Curie temperature; and from rigorous high-temperature expansions one finds that, provided the range of direct interaction between spins is finite, there is no ODLRO well above the Curie point. From Yang's considerations one expects that the onset of ODLRO is associated with a phase transition. Indeed in the Green's function treatment of Tyablikov and Bogoliubov,[2] where $\langle b_i^\dagger b_j\rangle$ are evaluated at all temperatures by means of a random-phase approximation, the onset of ODLRO is found to occur at the Curie point. However, two features in the magnetic transition are essentially different from the superfluid or superconducting transitions:

(1) The quantities $\langle b_i^\dagger b_j\rangle$ do have a classical significance as correlation functions between transverse spin components. The real and imaginary parts are

$$\mathrm{Re}\langle b_i^\dagger b_j\rangle=\langle\sigma_x^i\sigma_x^j\rangle+\langle\sigma_y^i\sigma_j\rangle,$$

$$\mathrm{Im}\langle b_i^\dagger b_j\rangle=\langle\sigma_x^i\sigma_y^j\rangle-\langle\sigma_y^i\sigma_x^j\rangle.$$

(2) The ferromagnetic condensed phase is, of course, also characterized by a diagonal long-range order, the zero-field magnetization. In this respect the ferromagnet resembles the Ising model, for which the condensed phase is characterized by the diagonal long-range order.

In conclusion, we find that the ferromagnetic phase transition may be characterized by either type of long-range order.

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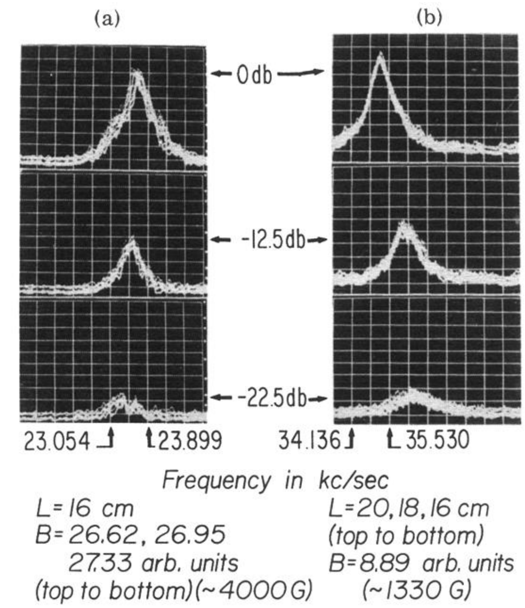

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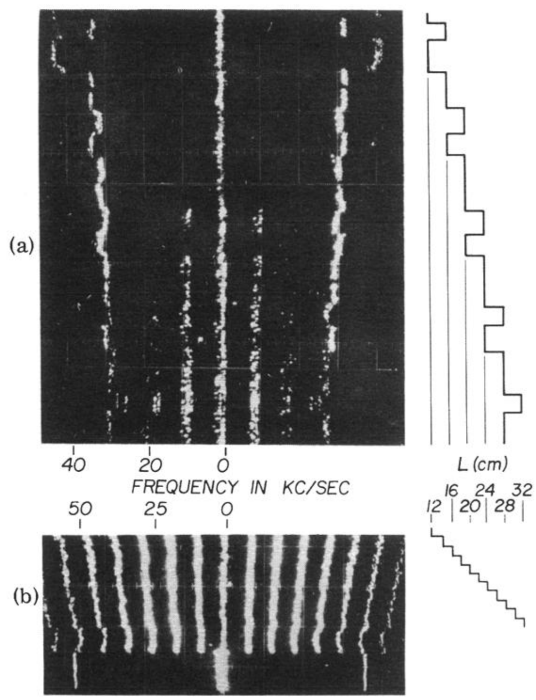

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