disk tended to be dragged out by the solar wind plasma to become part of the nearby interplanetary magnetic field. The best correlation was obtained when the photospheric field was in the same direction throughout an area corresponding to at least two or three days rotation. These conclusions are consistent with the model suggested by Ahluwalia and Dessler[13] in which the sense of the interplanetary magnetic field filaments is related to the sense of photospheric magnetic field regions.

We thank Dr. Robert Howard for making the solar magnetograms available, and Dr. Howard and Dr. V. Bumba for several valuable discussions. One of us (J.M.W.) thanks the director of the Mt. Wilson Observatory for guest investigator privileges at the observatory.

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SUPPRESSION OF CONVECTIVE LOSSES FROM A STEADY-STATE PLASMA BY A POSITIVE-GRADIENT FIELD*

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The experiments of Baiborodov *et al.*[1] and of Perkins and Barr[2] have demonstrated stabilization of transient plasmas against convective "flute" instability losses by means of positive-gradient magnetic fields. In such fields the plasma is located in a region of nonzero field minimum, and is theoretically predicted to be hydromagnetically stable.[3,4] Both experiments cited above utilized a hexapole set of longitudinal line cusps superimposed on a simple magnetic mirror field to form the positive-gradient field. We here report the suppression of convective plasma losses by a dodecapole array of longitudinal line cusps in combination with a mirror field. In addition to the difference in the number of poles, our experiment contrasts with the previously reported experiments in the method of plasma formation (neutral-atom injection), in the higher plasma ion energy (20 keV), and in the fact that it is essentially a steady-state rather than a transient experiment. In a steady-state experiment the effect of convective losses is to limit the plasma density. We have found that with the convective losses suppressed, evidence appears for a lower order density limitation, by another instability mechanism. Preliminary observations show an apparent relation between this new limitation and the detection of coherent oscillations at the ion gyrofrequency.

The qualitative effect of adding the multipole line-cusp field to the mirror field is shown in Fig. 1. With a steady mirror field only and with a constant beam injection, the ion density is limited to a low value [first 0.2 sec of Fig. 1(b)] by the convective instability losses. Low-frequency oscillations characteristic of this instability[5] are evident on a capacitive probe [Fig. 1(a)]. At 0.2 sec the cusp field is turned on, the oscillation amplitude decreases (by a factor of at least 50, from other data), and the ion density increases markedly. At 0.49 sec the injected beam is switched off and the plasma decays with a char-

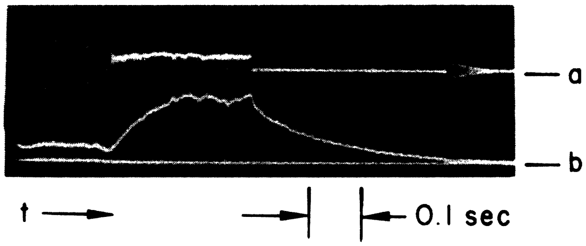

FIG. 1. Effect of adding 12-bar stabilizing field to the mirror field. Stabilizing field energized from 0.2 to 0.86 sec. (a) Envelope of low-frequency oscillation signal on a capacitive probe. The nonzero dc level before beam turnoff is a residual effect arising from beam-derived particle collection at the probe. (b) Relative trapped-ion density as monitored by the energetic-atom emission from the plasma.

acteristic time of 0.16 sec, in approximate agreement with the charge-exchange time based on independent estimates of the residual gas pressure [(1-2)$\times10^{-9}$ Torr]. At 0.86 sec, the line cusps are turned off (mirror field remains constant) and the low-frequency oscillations are seen to reappear, although the plasma is now at a density below the steady-state limited value.

The results can be made more quantitative by a comparison between experimentally measured densities and the density expected from plasma build-up calculations based on atomic processes only. Any ion losses in addition to charge exchange will be revealed by a steady-state density lower than the expected value.

The plasma is formed in the present experiment[6] by trapping a portion of a beam of 20-keV hydrogen atoms passed through the confining magnetic field. Gas-collisional ionization and Lorentz-force ionization of excited atoms are the trapping mechanisms. Aside from collective plasma losses, the dominant proton loss is charge exchange with the residual gas. Other atomic processes can be neglected. The equilibrium plasma ion density, spatially averaged, is expected to be approximately

$$n^+ = V^{-1}[Ln_0\sigma_{01} + f]I\tau, \qquad (1)$$

where I is the injected beam flux (atoms/sec), τ is the characteristic charge-exchange time, V is the plasma volume, L is the beam path length in the plasma, n_0 is the residual-gas density, σ_{01} is the ionization cross section, and f is the beam fraction ionized by the Lorentz force. In Fig. 2(a), n^+ is plotted as a function of $I\tau$, with solid line derived from Eq. (1). For all $I\tau < 0.1$ mA-sec, the curve is calculated for a neutral beam cur-

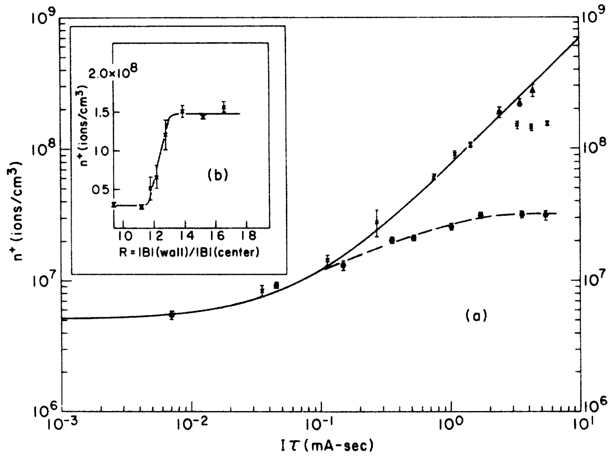

FIG. 2. (a) Ion density n^+ as a function of $I\tau$. Solid line: expected average density from Eq. (1); dash-solid circle-dash: average density, mirror field only; cross: average density, mirror field plus stabilizing field; triangle: central density, mirror field plus stabilizing field. Errors are standard deviation of the mean. (b) n^+ as a function of R for $I\tau \geq 4.0$ mA-sec.

rent of $I = 24$ mA (equivalent) to correspond to the value used in the experiment. A value of $f = 1.8 \times 10^{-5}$ is used, as estimated from earlier measurements of the excited-state population.[7] While Lorentz ionization by the dodecapole cusp field outside the confinement region lowers the Lorentz trapping rate by an estimated 30%, this is within the accuracy of our estimate of f and no specific correction is made for this effect.

The experimental ion density averaged over the plasma volume is determined from the integrated flux of energetic atoms detected at the plasma periphery after the injected beam is switched off. This method assumes that charge exchange is the only loss process during the plasma density decay, an assumption supported by the agreement between measured decay times and those calculated from gas-pressure measurements. In addition to the statistical errors shown in Fig. 2, systematic effects such as uncertainties in the plasma volume and in the calibration of the energetic-atom detector could cause errors of approximately 30% in the density measurements. The detector calibration was adjusted, within this range, to fit the data point at the lowest value of $I\tau$ to the solid curve.

When the plasma is formed in the simple mirror field, i.e., without energizing the line-cusp windings, a radical departure of average density from the expected value is evident in Fig. 2(a) for $I\tau > 0.3$. The density remains below 3.5×10^7 ions/cm^3 over the accessible range of $I\tau$, differing for large $I\tau$ by more than an order of magnitude from the expected value. Low-frequency electrostatic oscillations are also detected in these experiments and have been reported previously.[5] In reference 5 these oscillations were identified with the $m=1$ low-density convective instability mode calculated by Mikhailovski[8] and Post.[9] The oscillations are stable at very low density, reaching an unstable limit at a density which is predictable from the recent treatment of these effects by Kuo et al.[10] Indeed, Kuo and co-workers observe a similar density limitation in the Culham PHOENIX experiment. Our simple mirror-machine results will be described more completely elsewhere and we only make the point here that in the absence of the stabilizing field a flute instability due to the negative field gradient limits the density.

In order to reverse the field gradient, twelve longitudinal conductors are energized with current directions alternating in adjacent bars. The bars are located at a 14-cm radius and can be operated at a level up to 75 kA-turns/bar with a pulse length of 0.7 sec. Midplane magnetic mirror field is 5 to 8 kG; axial mirror ratio is 1.34. The strength of the stabilizing field relative to the mirror field can be gauged by the ratio $R = |B|(\text{wall})/|B|(\text{center})$, where $|B|(\text{wall})$ is the total field strength at the chamber wall ($r=12$ cm) and $|B|(\text{center})$ is the midplane field strength on the axis. As R is increased above 1.1, the measured density increases rapidly, saturating at $R=1.3$ [see Fig. 2(b)]. All density points plotted in Fig. 2(a), for stabilizing field on, are for values of R at or above the knee in the curve of n^+ vs R.

At values of $I\tau<2$, the measured average density equals the expected value assuming charge-exchange losses only, and the points in Fig. 2(a) lie on the calculated curve. Flute losses are apparently completely suppressed.

At values of $I\tau>2$ the average density falls below the theoretical curve, indicating either that the plasma is being spread axially beyond the region viewed by the detectors, or that ions are being lost to the walls by some instability mechanism, or both. In any event, the evidence strongly suggests the presence of cooperative effects. Time-correlated measurements give a clue to the nature of these cooperative effects. Repetitive bursts of 1-2 msec duration at the ion gyrofrequency are observed on a loop antenna, accompanied by the ejection of electrons through the mirror region. At the same time the positive plasma potential rises abruptly and the indicated ion density decreases by several percent, then slowly recovers. The spacing of the bursts is dependent on pressure, increasing to about 30 msec at the highest value of $I\tau$. The ion density variations are visible in Fig. 1(b). While further measurements are necessary to elucidate the mechanism, it appears that we may be encountering effects arising from electrostatic-resonance instabilities of the general type first proposed by Harris.[11] Such instabilities may occur in the presence of velocity-space anisotropy of the plasma or in plasmas with peaked ion-energy distributions. In the present experiment the injection mechanism initially creates a plasma which both is anisotropic and possesses a sharply peaked ion-energy distribution. At the low plasma densities thus far achieved, ion-ion scattering, which would ameliorate both these conditions, is unimportant and the theoretical conditions for electrostatic instabilities are strongly satisfied. However, we have not yet proved that electrostatic instabilities are in fact responsible for the new density limitation.

The ion density near the axis can be determined by measurement of the flux of slow ions through one mirror, these ions resulting from the charge-exchange process. Three central density points obtained in this way are shown in Fig. 2(a), with the assumption that the plasma is confined axially to the beam width. These points should lie on the calculated average-density curve because in this region, dominated by Lorentz trapping, the average and peak calculated densities are equal. The correspondence of these experimental points to the expected curve indicates that particles trapped at small radii are leaving solely via charge exchange, independent of the question of axial spreading. Apparently, convective phenomena are unimportant even in the central region, despite the fact that the field-gradient reversal occurs far from the axis (at $r\simeq 7.5$ cm). Thus, under the present plasma conditions, the dodecapole array, though having a substantially larger central region of negative-gradient field compared to the hexapole experiments,[1,2] appears to be effective in stabilizing the entire plasma against convective losses. The nature of the residual cooperative effects remains to be clarified.

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ULTRASONIC PROPAGATION NEAR THE CRITICAL POINT IN HELIUM

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A logarithmic singularity in the specific heat at constant volume C_v at the critical point has recently been reported to occur in argon[1] and oxygen.[2] Such a result is in direct contradiction to standard theory,[3,4] according to which C_v must remain finite at the critical point. Reconsideration of the theory in view of these data leads us to new inferences about the critical point. Among these is the prediction that the sound velocity exhibits anomalous behavior, which we have indeed observed.

The situation can be concisely discussed in terms of the stiffness matrix which, starting from the fundamental equation $U=U(S,V)$, can be written[4]

$$\begin{bmatrix} \left(\frac{\partial T}{\partial S}\right)_v & \left(\frac{\partial T}{\partial V}\right)_s \\ -\left(\frac{\partial P}{\partial S}\right)_v & -\left(\frac{\partial P}{\partial V}\right)_s \end{bmatrix} = \begin{bmatrix} \frac{T}{C_v} & \frac{T}{C_v}\left(\frac{\partial P}{\partial T}\right)_v \\ \frac{T}{C_v}\left(\frac{\partial P}{\partial T}\right)_v & \frac{1}{V\kappa_s} \end{bmatrix}. \quad (1)$$

Here κ_s is the adiabatic compressibility. This matrix can be diagonalized by a nonsingular linear transformation of the variables, yielding the diagonal elements

$$\lambda_1 = T/C_v, \quad \lambda_2 = 1/V\kappa_T. \quad (2)$$

According to standard theory, $\lambda_2 = 0$ and $\lambda_1 \neq 0$ at the critical point, ensuring that C_p, the coefficient of thermal expansion α, and the isothermal compressibility κ_T become infinite there.[4] The observation that $C_v \to \infty$ implies, on the other hand, that at T_c

$$\lambda_1 = \lambda_2 = 0. \quad (3)$$

Note that λ_1 and λ_2 are not eigenvalues, and inverting the order of the variables $U=U(V,S)$ we arrive at

$$\lambda_1' = 1/V\kappa_s, \quad \lambda_2' = T/C_p. \quad (4)$$

Now according to a well-known rule,[5] the number of positive, negative, and vanishing terms in the diagonalized form must be independent of the particular transformation used. Hence, Eq. (3) implies $\lambda_1' = \lambda_2' = 0$, so that κ_s must have a singularity at the critical point and the sound velocity $u = (\rho\kappa_s)^{-1/2}$ must vanish. Strictly speaking, the sound velocity should be regarded as complex in order to take dissipation into account. The quantity λ_1', however, is connected only with the real part, and as the critical point is approached the imaginary part will eventually become dominant. Hence the signal will be expected to fade out as the phenomenon of sound propagation becomes purely dissipative. High attenuation has in fact been previously observed in the critical region, e.g., in xenon.[6]

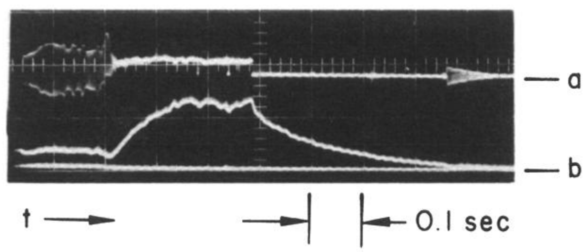

FIG. 1. Effect of adding 12-bar stabilizing field to the mirror field. Stabilizing field energized from 0.2 to 0.86 sec. (a) Envelope of low-frequency oscillation signal on a capacitive probe. The nonzero dc level before beam turnoff is a residual effect arising from beam-derived particle collection at the probe. (b) Relative trapped-ion density as monitored by the energetic-atom emission from the plasma.