the σ resonance is invoked. The distribution in the decay normal is consistent with 0^-. It is also consistent with 1^+ and 2^-, but only under rather special circumstances. These findings agree with the work of the other groups[1,2] wherever the data can be compared. For $T=0$, and correcting for neutral Λ decays, the X^0 production cross section averaged over the 1.80- to 1.95-GeV/c momentum range is 54 ± 8 μb.

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[5]We wish to thank Dr. G. R. Kalbfleisch for making the data available to us.

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s-WAVE $\pi\pi$ INTERACTION AND $K_1^0K_2^0$ MASS DIFFERENCE*

Sharashchandra H. Patil†
University of California San Diego, La Jolla, California
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In a recent paper[1] the s-wave $\pi\pi$ interaction was analyzed with reference to various pieces of experimental data, in particular, the $K_1^0K_2^0$ mass difference which was assumed to be due to the 2π decay mode of K_1^0. Various models[1-3] have been discussed in connection with this mechanism of the mass difference. Recently, however, there has been an alternative proposition[4,5] that the mass difference comes from the pole due to π and η intermediate states in the self-energy graphs of K_2^0. In one version[4] of this scheme the K_2^0 is heavier than K_1^0, while in the other version[5] the K_1^0 is heavier than K_2^0 by almost an equal amount. In both the versions the magnitude of the mass difference is approximately proportional to $g_{K\pi}^2$ where $g_{K\pi}$ is the $K\pi$ weak coupling. If one calculates $g_{K\pi}^2$ from the $\mu^+\nu$ decays of K^+ and π^+ and assuming the $\Delta I=\frac{1}{2}$ rule, the $g_{K\pi}^2$ obtained is found to be too small[4] by a factor of about 30 to explain the $K_1^0K_2^0$ mass difference. On the other hand, arguments are presented in reference 4 to show that this coupling can be large.

In this paper we study the 3π decay mode of K_2^0 via a single-pion pole and hence try to get information about the $K\pi$ coupling directly without recourse to leptonic decay modes. We first assume that in the decay $K_2^0\to\pi^0\to\pi^++\pi^-+\pi^0$, the π^+ and π^- are in the $I=0$ s-wave state, and given the partial width for this decay we discuss the $K\pi$ coupling for various models of s-wave $\pi\pi$ interaction. In all the models we discuss, the $g_{K\pi}^2$ coupling comes out to be of the same order as obtained from $\mu^+\nu$ decay modes of K^+ and π^+, and hence in disagreement with the π,η pole mechanism for explaining the $K_1^0K_2^0$ mass difference. We also calculate the effect of p-wave $\pi\pi$ interaction in the form of the ρ meson and find its effect to be negligible.

Consider the decay width for the reaction

$$K_2^0\to\pi^++\pi^-+\pi^0 \tag{1}$$

via the π^0 pole diagram of Fig. 1. This is the most important diagram contributing to the above reaction from the dispersion-relation point of

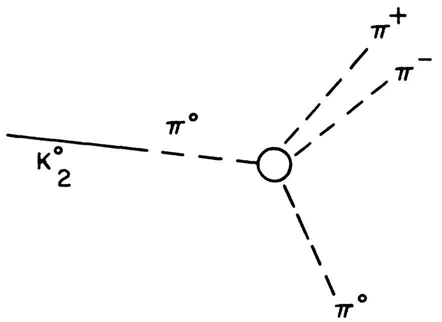

FIG. 1. Diagram for $K_2^0 \to \pi^+ + \pi^- + \pi^0$ via a single-pion intermediate state.

view. The η-pole diagram is neglected since η is coupled to the 3π state via the electromagnetic interaction. If the $\pi^+\pi^-$ are in an $I=0$ s-wave state, the decay width for the process (1) is

$$\Gamma = \alpha \int_4^{(m_K-1)^2} \left(\frac{s-4}{4s}\right)^{-1/2} \times \left[\left(\frac{s+m_K^2-1}{2m_K}\right)^2 - s\right]^{1/2} |T_0|^2 ds, \quad (2)$$

where $\alpha = 2g_{K\pi}^2 m_K^2/9\pi(m_K^2-1)^2$. The amplitude T_0 is $\exp(i\delta_0)\sin\delta_0$ and δ_0 is the $\pi\pi$ $I=0$ s-wave phase shift. All the quantities are in units of pion mass. If $g_{K\pi}$ is obtained from the $\mu^+\nu$ decays of K^+ and π^+, assuming $\Delta I = \frac{1}{2}$, we get $g_{K\pi}^2 = 1.37\times10^{-15}$. We now calculate $g_{K\pi}^2$, for various models of s-wave $\pi\pi$ interaction, so as to give the experimental partial width for Reaction (1). For the convenience of presentation of results we define R as the ratio of $g_{K\pi}^2$ calculated to that obtained from the $\mu^+\nu$ decays of K^+ and π^+.

If the s-wave $\pi\pi$ interaction is represented by the σ meson proposed by Brown and Singer,[6] the value of R obtained is of the order of unity which indicates that if the σ does exist, the $g_{K\pi}^2$ obtained from the $\pi^+\pi^-\pi^0$ decay of K_2^0 is an order of magnitude smaller than the number required to explain the $K_1^0K_2^0$ mass difference in terms of the π^0, η poles. Values of R for some values of the σ position and width are given in Table I.

Table I. The values of R for some values of the mass and width of the σ meson.

m_σ (MeV)	Γ_σ (MeV)	R
380	70	1.1
380	50	1.7
340	90	0.33

Now if the σ does not exist, one may attempt a scattering-length approximation following Chew and Mandelstam,[7] in which one takes the imaginary part of the left-hand cut to be zero. In that case we have

$$[\nu/(\nu+1)]^{1/2}\cot\delta_0 = (1/a) + h(\nu), \quad (4)$$

with

$$1/a = -1/5\lambda - h(\nu_0),$$

and

$$h(\nu) = \frac{2}{\pi}\left(\frac{\nu}{\nu+1}\right)^{1/2} \ln[\nu^{1/2} + (\nu+1)^{1/2}],$$

$$h(\nu_0) = (2\sqrt{2}/\pi)\tan^{-1}(2^{-1/2}),$$

where $\nu = s/4-1$ and, according to Chew and Mandelstam, λ is restricted by $-0.36<\lambda<0.3$. In order to obtain some information about λ we make use of the asymmetry in the $\pi^+\pi^-$ center-of-mass distribution in the ρ^0 production. The asymmetry, as defined by Bondâr et al.,[8] at the ρ-meson position is approximately proportional to $\sin^2\delta_0$, and so we put a limit of $|\delta_0| \geq 25°$ at the ρ-meson position, giving an asymmetry ≥ 0.08 compared to the experimental value of $\approx 0.35 \pm 0.1$. This restriction implies that $\lambda \leq -0.10$ or $\lambda \geq 0.07$. But for $\lambda \geq 0.07$ we get a negative phase shift, $\delta_0 \leq -25°$, between 400 and 700 MeV, and the resulting asymmetry is negative below 700 MeV which is inconsistent with the experimental data.[8] So from the asymmetry considerations we conclude that $\lambda \leq -0.10$. We have plotted the phase shift in Fig. 2 for some of the relevant values of λ, and the values of R for these cases are given in Table II. R is again of the order of unity and hence an order of magnitude too small to explain the $K_1^0K_2^0$ mass difference in terms of π^0, η poles. We also note that for $\lambda < -0.15$, there is a substantial enhancement of the s-wave amplitude which may correspond to the well-known ABC.

One may also attempt to obtain a unitary s-wave $\pi\pi$ scattering amplitude using dispersion relations. For this we set

$$\left(\frac{s-4}{4s}\right)^{-1/2}\exp(i\delta_0)\sin\delta_0 = ND^{-1}, \quad (5)$$

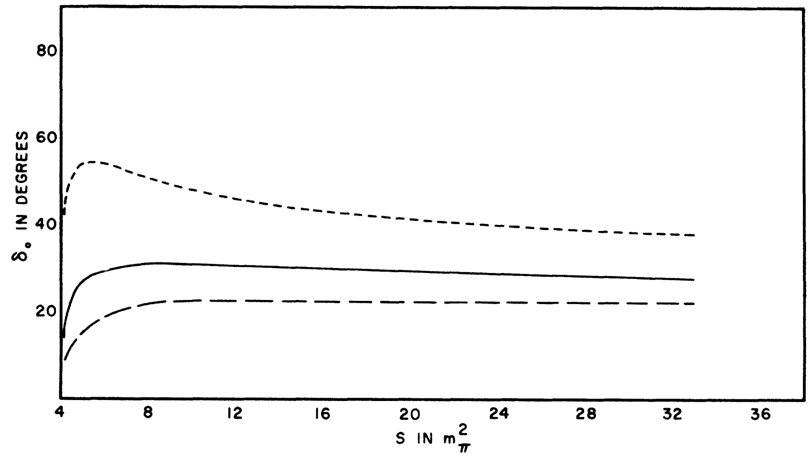

FIG. 2. The plot of phase shift δ_0 in the scattering-length model. Short-dashed line $\lambda=-0.27$, solid line $\lambda=-0.15$, and long-dashed line $\lambda=-0.10$.

in which case N has the left-hand singularities and D the right-hand singularities. They satisfy the following integral equations:

$$N(s)=B(s)+\frac{1}{\pi}\int_4^\infty \frac{s'B(s')-sB(s)}{s'(s'-s)}\rho(s')N(s')ds', \quad (6)$$

$$D(s)=1-\frac{s}{\pi}\int_4^\infty \frac{N(s')\rho(s')}{s'(s'-s)}ds', \quad (7)$$

where $\rho(s)=[(s-4)/4s]^{1/2}$ and $B(s)$ is the s-wave projection of the interaction,

$$B(s)=\tfrac{1}{2}\int_{-1}^{1}dzA(s,t), \quad (8)$$

where $t=-(s-4)(1-z)/2$ and A is regular for negative t but has singularities along the positive t axis. For the interaction we first include the ρ-meson exchange. In order to avoid the divergence of the integrals in (5) and (6) we use a Regge form of cutoff in which $A(s,t)$ is modified to

$$A(s,t)\to A(s,t)\exp[\alpha(t-m_\rho^2)\ln s]. \quad (9)$$

We determine α so that the s-wave phase shift at the ρ meson is $\geqslant 25°$. With this restriction we find $R<1$, which implies that we cannot explain the $K_1^0K_2^0$ mass difference in terms of the π, η poles. We have plotted the phase shift for $\alpha=0.01$ in Fig. 3; the corresponding value of R is 0.32. For the interaction, we add the f^0 exchange and find its effect to be negligible. We also included the exchange of an s-wave state in the form of σ and the effect of this turns out to be small.

For the sake of completeness we calculated the contribution to the width of decay (1) from the p-wave interaction in the form of the ρ meson for the pairs of final pions, and found the contribution to be about two orders of magnitude smaller than the contribution of the s-wave interaction of any of the above models.

In all the above models we thus find that R is of the order of unity, which implies that the coupling $g_{K\pi}$ calculated from the K_2^0 decay into the 3π is of the same order of magnitude as the one calculated from the $\mu^+\nu$ decays of K^+ and π^+. Within the framework of our models we then can-

Table II. The values of R for relevant values of λ.

λ	R
-0.27	0.5
-0.15	≈ 1.0
-0.10	2.0

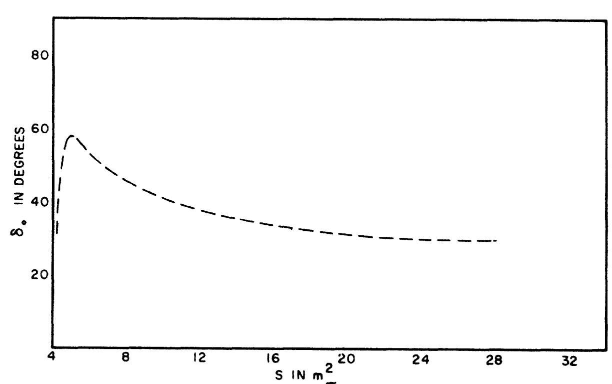

FIG. 3. The plot of phase shift δ_0 for the ρ-exchange model with $\alpha=0.01$.

not explain the $K_1^0K_2^0$ mass difference in terms of π, η poles as suggested in references 4 and 5.

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†Present address: Institute for Advanced Studies, Princeton, New Jersey.

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ERRATUM

NEW VALUE FOR THE FINE-STRUCTURE CONSTANT α FROM MUONIUM HYPERFINE STRUCTURE INTERVAL. W. E. Cleland, J. M. Bailey, M. Eckhause, V. W. Hughes, R. M. Mobley, R. Prepost, and J. E. Rothberg [Phys. Rev. Letters 13, 202 (1964)].

In the second line of the caption to Fig. 2, instead of "0°C," read "21.1°C." Nothing else in the Letter needs to be changed.