INFORMATION ABOUT THE TWO-BODY FORCE INSIDE NUCLEI FROM $(p, 2p)$ REACTIONS*

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A calculation of the $(p, 2p)$ reaction in which a $1p$ proton is knocked out of C^{12} has been performed. This is a direct-interaction calculation in the distorted-wave approximation in which no approximation is made (except the distorted-wave approximation itself) that cannot be directly related to a model for a simpler experiment. There are no free parameters.

The optical-model parameters are obtained by interpolation in various elastic scattering studies of protons on C^{12}, with the real parts of the potential compared by reducing them to $r_0 = 1.2$ fm according to the empirical rule $Vr_0^2 = \text{const}$. Spin-orbit coupling is not included in the optical model since polarizations have not been measured for $(p, 2p)$ experiments.

The bound-state wave function is obtained from the extreme single-particle model. Since the information about the shape of the two-body force comes from the shape of the angular correlation, the radial form of the bound-state wave function is most important. This is fixed in a calculation with a reasonable potential well by fixing the binding energy which is obtained from the $(p, 2p)$ experiment itself, and the rms radius of the charge distribution [in conjunction with s-state $(p, 2p)$ calculations] which is well known for electron scattering. In the calculation a square well was used.

$(p, 2p)$ calculations in which the above approximations have been made have previously been described by the present authors[1] in a paper referred to as I. The object of I was to survey the use of the distorted-wave Born approximation as a model for fitting data. The present object is to discuss a model in which there are no free parameters. The essential improvement in the present work is the inclusion of a model for the two-body t matrix in nuclear matter.

The model is a pseudopotential calculated to fit (p, p) scattering data at 90° at all energies up to 300 MeV, which are the energies most relevant to a $(p, 2p)$ calculation at 155-MeV incident energy. By using a pseudopotential, i.e., a potential for which the Born approximation fits the (p, p) data, we are adopting a momentum-space approach to the $(p, 2p)$ problem. We assume that a model that reproduces the correct distribution of momentum transfer components in the free-particle case will do the same for bound particles, both on and off the two-body energy shell.

The pseudopotential used is a linear combination of three Yukawa form factors,

$$t(r) = a_1 \frac{e^{-\mu_1 r}}{\mu_1 r} + a_2 \frac{e^{-\mu_2 r}}{\mu_2 r} + a_3 \frac{e^{-\mu_3 r}}{\mu_3 r}, \qquad (1)$$

where

$$a_1 = -83 \text{ MeV}, \; a_2 = +455 \text{ MeV}, \; a_3 = -1660 \text{ MeV},$$

$$\mu_1 = 0.73 \text{ fm}^{-1}, \; \mu_2 = 1.5 \text{ fm}^{-1}, \; \mu_3 = 3.0 \text{ fm}^{-1}.$$

The three terms correspond roughly to one-, two-, and four-pion components in a potential.

The $(p, 2p)$ matrix element is given by

$$T_L^M = \iint d^3r_1 d^3r_2 \chi^{(-)*}(\vec{k}_L, \vec{r}_1)\chi^{(-)*}(\vec{k}_R, \vec{r}_2) \times t(|\vec{r}_1 - \vec{r}_2|)\chi^{(+)}(\vec{k}_0, \vec{r}_1)\psi_L^M(\vec{r}_2), \qquad (2)$$

where $\vec{k}_0$, $\vec{k}_L$, and $\vec{k}_R$ are, respectively, the propagation vectors of the incident and the left and right outgoing particles. In the calculation $\vec{k}_L$ and $\vec{k}_R$ are symmetrical about the incident direction and coplanar with $\vec{k}_0$.

Because of the three-body final state we have used the expansion in $1/A$, where A is the mass of the residual nucleus, discussed in I. As in the C^{12} case in I, the angular dependence of the effective optical-model energies for the final-state wave functions has been taken into account by linear interpolation in three calculations in which the energies are correct for outgoing angles of 30°, 45°, and 60°. Relativistic effects and the Coulomb interaction between the final protons have been neglected. They were shown in I to involve shifts of the angular correlation curve of at most 2°.

In Fig. 1, the continuous line shows the result of the calculation for 155-MeV incident protons using the values $a = 3.5$ fm and $V_B = 38.4$ MeV for the radius and depth of the square-well potential used to parametrize the wave function. These values, together with those for the s state found in I, give agreement with the electron

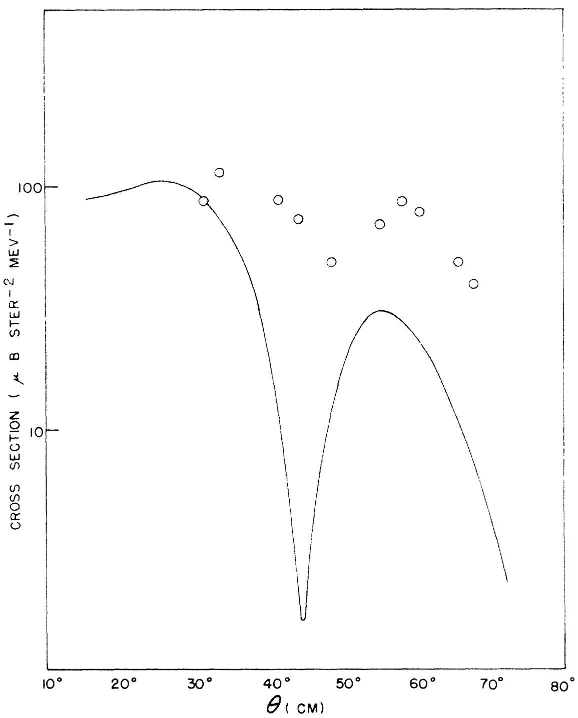

FIG. 1. Distorted-wave theory, with a pseudopotential that fits free (p,p) scattering, compared with experiment [J. P. Garron, J. C. Jacmart, M. Riou, C. Ruhla, J. Teillac, and K. Strauch, Nucl. Phys. 37, 126 (1962)] at 155 MeV.

scattering experiments for the rms radius of the charge distribution. The optical-model potentials for entrance and exit channels, respectively, were $V_0 = 17$ MeV, $W_0 = 17$ MeV, and $V_1 = 34$ MeV, $W_1 = 13$ MeV. The Eckart form factor in both cases was defined by the radius and surface thickness parameters $r_0 = 1.2$ fm, $b = 0.5$ fm.

It is immediately apparent from Fig. 1 that the calculation gives quite a poor fit to the experimental data. It is necessary to examine the approximations closely to see which have the biggest effect on the angular correlation shape and magnitude and to see if any information can be obtained from their breakdown.

The extreme single-particle approximation in which the cross section is assumed to be the sum of those for each of the four $1p$ protons separately is probably not very good for C^{12}, but it affects only the magnitude of the cross section. It is significant that this calculation reproduces the magnitude quite well, unlike previous calculations of reactions involving two-body collisions.

The striking discrepancy between theory and experiment is that the theory underestimates the large-angle contributions, which come (in the sense of the impulse approximation) from high-momentum two-body collisions.

Variation of the optical-model parameters was shown in I to have a small effect. In any case the parameters are quite realistic.

The coplanar symmetric case is specially suited for neglect of spin effects in the two-body force. In the impulse approximation (which is shown later to be quite good in this case as far as space effects are concerned) the two-body collisions are symmetric. Antisymmetry considerations forbid polarization. Experimentally the polarization is small for nearly symmetric collisions. The polarization caused by spin-orbit terms in the optical model is not large as has been shown for inelastic scattering by Haybron et al.[2] Spin and antisymmetry will be discussed further in a future publication.

The approximation of determining the radial form of the bound-state wave function by fitting the binding energy has been discussed recently by Austern[3] for stripping, which is a similar situation. He found it to be fairly good in cases where the parentage expansion is not too complicated. In the $(p, 2p)$ case the rms radius gives an additional constraint to the radial shape. To test the effect of introducing higher momentum components in the wave function without appreciably changing the rms radius, a 20% admixture of a $2p$ wave function in an infinite square well of the same radius was added to the $1p$ wave function. It changed the cross sections by about 1%.

We are left with the conclusion that the only approximation that could break down significantly (assuming the distorted-wave Born approximation is good) is the approximation that the effective two-body force in the presence of other nucleons is like that in free space. The long-range (one-pion-exchange) part is most likely to be affected. Figure 2 shows that reducing the one-pion-exchange term of the potential (1) by a factor of two relative to the core terms gives much better agreement with experiment. This force has roughly the same strength and average range as one found by Talman[4] to give a much better result than the one-pion Yukawa in a shell-model calculation of Ne^{20}.

The present calculation gives a check on the validity of the impulse approximation. This is interesting for two reasons. First, the impulse approximation has been used by many authors

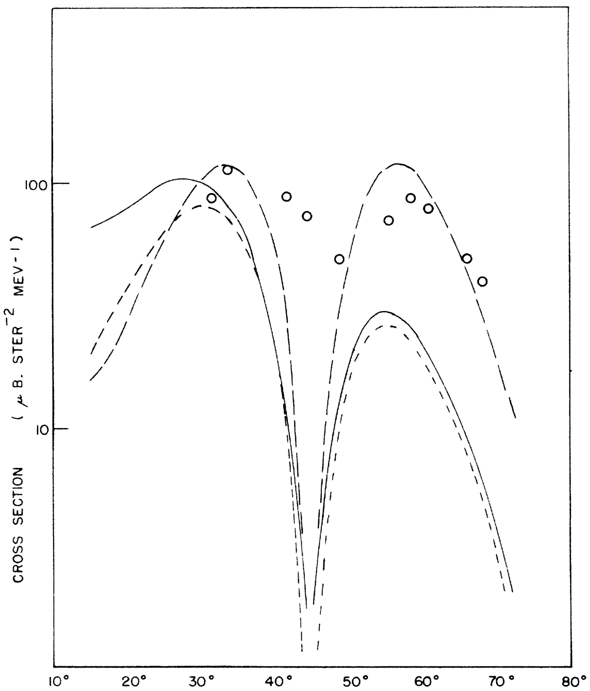

FIG. 2. Distorted-wave theory for 143 MeV in the infinite-mass approximation with k_L and k_R constant and correct at 45°. The solid curve is for the pseudopotential (1), the long-dashed curve is for the pseudopotential with one-pion component reduced by a factor 2 and normalized again to low-energy (p,p) scattering, the short-dashed curve is the impulse approximation for comparison with the solid curve. The experimental points for 155 MeV are illustrated.

to describe reactions in this energy range. Second, a reaction with a three-body final state can, in principle, give information about the two-body force off the energy shell. $(p,2p)$ is a specially simple case of a reaction with a three-body final state because one of the bodies is heavy. The perturbation expansion in $1/A$ of the wave function, which was discussed in I, has a first-order term which is separable into two-body wave functions.

The angular correlation was calculated assuming a core of infinite mass and with the approximation that the optical model energy of the final states is equal at all angles to that at 45°. (This approximation is not made in Fig. 1. It has a small effect and saves performing the computation once for each angle. We call this the "exact" result. The assumption of infinite mass means that the center-of-mass energy of 143 MeV is now equivalent to the laboratory energy.

The impulse approximation requires two steps. First, the matrix element (2) is replaced by

$$T_L^{M(i)} = \int \exp[i(\vec{k}_0-\vec{k}_L)\cdot r]t(r)d^3r\, T_L^{M(0)}, \quad (3)$$

where $T_L^{M(0)}$ is the distorted-wave matrix element in zero-range approximation. The first factor is still off the two-body energy shell because of the binding energy of the struck proton. It is replaced by the Fourier transform of $t(r)$ for a free two-body collision of a particle with momentum $\vec{k}_0$ with another of momentum $\vec{K}$, where $\vec{K}$ is the momentum transfer to the nucleus in the reaction. This assumes that the momentum transfer is due only to the original motion of the struck particle.

The dashed curve in Fig. 2 is the impulse-approximation result. It compares quite well with the "exact" result (full curve) except for small values of the momentum transfer $\vec{k}_0-\vec{k}_L$. This means that most of the off-shell effects are due to kinematics rather than distortion. The introduction of off-shell matrix elements by distortion at lower energies will be examined in a future publication.

Finally it should be noted that the $(p,2p)$ reaction is a much more sensitive probe for the space dependence of the two-body force in the presence of other nucleons than inelastic scattering. For inelastic scattering the effects of the two-body force and the bound-state wave functions are somewhat mixed up.[5] For $(p,2p)$ the peak height ratio is affected strongly by the two-body force and not by the wave-function properties while the over-all width of the angular correlation curve is determined much more sensitively by the wave function.

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[1]K. L. Lim and I. E. McCarthy, Phys. Rev. 133, B1006 (1964).

[2]R. M. Haybron, H. McManus, A. Werner, R. M. Drisko, and G. R. Satchler, Phys. Rev. Letters 12, 249 (1964).

[3]N. Austern, to be published.

[4]J. D. Talman, to be published.

[5]This is seen in unpublished calculations of inelastic scattering by K. A. Amos in which a finite range force with space exchange is used.