ANALYSIS AND INTERPRETATION OF THE SLOW NEUTRON CROSS SECTIONS OF THE FISSIONABLE NUCLEI

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The interpretation of the resonances in the slow neutron cross sections of fissionable nuclei rests largely on the channel theory of fission.[1,2] At low energies there are a number of almost discrete fission thresholds for the decay of a compound nucleus in a state with angular momentum J and parity π. The ratio of the average fission width $\overline{\Gamma}_f^{(J\pi)}$ to level spacing $\overline{D}^{(J\pi)}$ of compound nucleus states at energy E is related to the fission channel thresholds $E_i^{(J\pi)}$ by $\overline{\Gamma}_f^{(J\pi)}/\overline{D}^{(J\pi)}=\sum_i P_i/2\pi$. The penetrability factor P_i for the fission channel i is[3]

$$P_i=\{1+\exp[2\pi(E_i^{(J\pi)}-E)/\hbar\omega]\}^{-1},$$

if the potential barrier at the saddle point is an inverted harmonic oscillator with circular frequency ω. The energy $\hbar\omega$ is expected to be of order 0.5 MeV; if the spacing of the threshold energies $E_i^{(J\pi)}$ is greater than this the energy dependence of $\overline{\Gamma}_f^{(J\pi)}/\overline{D}^{(J\pi)}$ will have, in Wheeler's phrase, a "carpeted staircase" appearance. The quantity $\sum_i P_i$ is usually denoted by N_{eff}, the effective number of channels of given spin and parity that are open at energy E.

The most studied fissionable nuclei are U^{233}, U^{235}, and Pu^{239}. The target U^{233} has spin and parity $\frac{5}{2}^+$. The compound states of U^{234} formed by s-wave neutron bombardment have spin and parity 2^+ and 3^+. Zero neutron energy corresponds to an excitation of 6.8 MeV, which is 1.5 MeV above the lowest fission threshold.[4] In this energy interval of 1.5 MeV it is possible, according to Wheeler,[2] that two channels open for the 2^+ levels (corresponding to a level of the lowest, $K=0$, rotational band and a gamma-vibrational level, $K=2$, with no rotation) and that one channel opens for the 3^+ levels (corresponding to a gamma-vibrational level with one unit of rotation). These considerations are confined to transition states at the saddle point with collective character. There may be, in addition, transition states with intrinsic nucleonic excitation. The value of N_{eff} averaged over the two spin states is thus expected to be at least 1.5.

U^{235} has spin and parity $\frac{7}{2}^-$; the U^{236} compound states have spin 3^- and 4^-. The neutron separation energy is 6.4 MeV, which is about 1 MeV above the lowest fission threshold.[4] At this energy there could be two open 3^- channels (a "sloshing" vibration, $K=0$, with rotation, and a bending vibration, $K=1$, with rotation) and one open 4^- channel (a bending vibration with greater rotation). The average value of N_{eff} should be 1.5.

Pu^{239} has spin and parity $\frac{1}{2}^+$ and the compound states of Pu^{240} have spin 0^+ and 1^+. The neutron separation energy is 6.38 MeV, 1.6 MeV above the lowest fission threshold.[4] The only expected open channel from among the collective family is the "ground," $K=0$, $J=0^+$ state. Thus, in the absence of channels with nucleonic excitation at the saddle point, N_{eff} should be unity for the 0^+ levels and zero for the 1^+ levels and its average value is 0.5.

The experimental data give results that are very different from these expectations. For U^{233} the average value[5] of N_{eff} is 0.65 ± 0.13, for U^{235} it is[6] 0.18 ± 0.03, and for Pu^{239} it is[7] 0.12 ± 0.03. Some data are available on the spins of the Pu^{239} resonances.[8] These suggest that $N_{\text{eff}}(J^\pi=1^+)=0.3\pm0.1$ and $N_{\text{eff}}(J^\pi=0^+)$ is very small.

It is our purpose to suggest that these discrepancies are due to the single-level nature of the analysis of the data. Many analyses are frankly single level in nature; they use the total width of a maximum in the cross section and the peak values of the partial cross sections to extract the neutron width Γ_n, fission width Γ_f, and radiative-capture width Γ_γ. Other analyses employ a many-level formalism, but they implicitly contain the assumption that a maximum in the cross section is associated with a single level in the reaction formalism. Only when the fitting gets into extreme difficulties is this assumption contravened and a "hidden level" invoked. The partial widths obtained differ little from those yielded by single-level fitting.

It is not generally realized how drastic level-level interference effects can be. An explicit two-level formula has been available for many years[9] but its properties do not seem to have been studied. Its major property is the following: If two levels (same spin and parity) are more

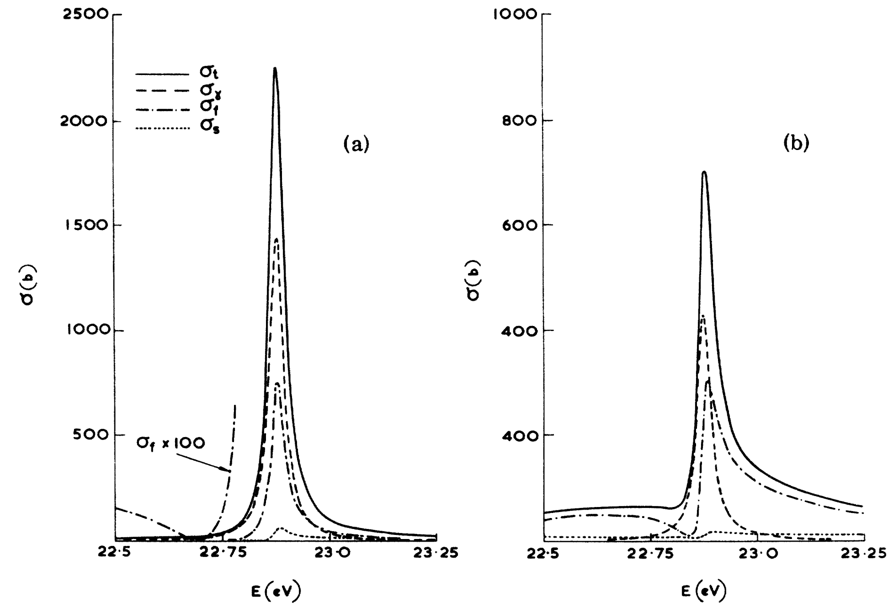

FIG. 1. Parameters for (a) are: $E_1=22.81$ eV, $(\Gamma_{1n}{}^0)^{1/2}=0.159$ $(\text{meV})^{1/2}$, $(\Gamma_{1f})^{1/2}=-0.728$ $(\text{eV})^{1/2}$, $\Gamma_{1\gamma}=0.025$ eV, $E_2=22.92(\Gamma_{2n}{}^0)^{1/2}=0.384$ $(\text{meV})^{1/2}$, $(\Gamma_{2f})^{1/2}=0.574$ $(\text{eV})^{1/2}$. In (b) there is a change of sign of one reduced width amplitude.

closely spaced than their widths they may give a single resonance with width that is less than the width of either level and with partial cross sections that can be very different from those that would be suggested by the partial widths of the levels. Examples of two-level interference are shown in Fig. 1. The spacing of the levels is 110 meV, their fission widths (in one channel) are 522 meV and 332 meV, radiation widths are 25 meV, and neutron widths are 0.12 meV and 0.71 MeV. A single-level analysis of the resonance in Fig. 1(a) gives $\Gamma_n=0.84$ meV. $\Gamma_\gamma=26$ meV, and $\Gamma_f=14$ meV. Figure 1(b) differs from the other in the sign of one partial-width amplitude. It is obvious that the single-level analysis would give an erroneous account of the parameters required for comparison with channel theory. Lest it be thought that the parameters of Fig. 1 are outrageous it should be pointed out that they occurred in the first run of a computer program which randomly selected reduced-width amplitudes and spacings for 20 levels from Porter-Thomas[10] and Wigner[11] statistical distributions. It was assumed that the mean fission width was $\frac{1}{2}\pi$ times the mean spacing, i.e., $N_{\text{eff}}=1$. When $N_{\text{eff}}=1$ the probability that the sum of the fission widths of two neighboring levels is greater than their spacing is 0.12.

The situation is even more complex if the cross section is formed from levels of two spins. The repulsion[11] between levels of like spin does not exist for unlike levels. In consequence, when N_{eff} is unity or greater it is very common for two levels to be closer than their widths and a single broader maximum to occur in the cross section. This and the interference effect give rise to a cross section in which the mean level spacing will be overestimated and the mean fission width underestimated. As an example we have calculated a cross section from Wigner-Eisenbud many-level formalism.[12,13] This is shown in Fig. 2. The resonance spacings were generated randomly from a Wigner distribution with mean value of 0.5 eV for each spin. The neutron reduced-width amplitudes were generated from a zero-mean Gaussian distribution[10] with a dispersion appropriate to a strength function of $\Gamma_n{}^0/\bar{D}=1\times10^{-4}$. The fission-width amplitudes for each channel were generated similarly. The mean fission widths were equivalent to $N_{\text{eff}}=3\times1$ for one spin and to $N_{\text{eff}}=1+2\times0.1$ for the other spin, giving an average of 2.1; such values could be expected from the channel theory. The radiation widths were constant at 25 meV.

A single-level analysis of the cross section in Fig. 2 gives "$\Gamma_f{}^{(J\pi)}$" = 137 meV, "$D^{(J\pi)}$" = 1.06 eV, and average "N_{eff}" = 0.8 ± 0.14, which is remarkably close to the U^{233} value. This is not the only similarity. The histogram of the

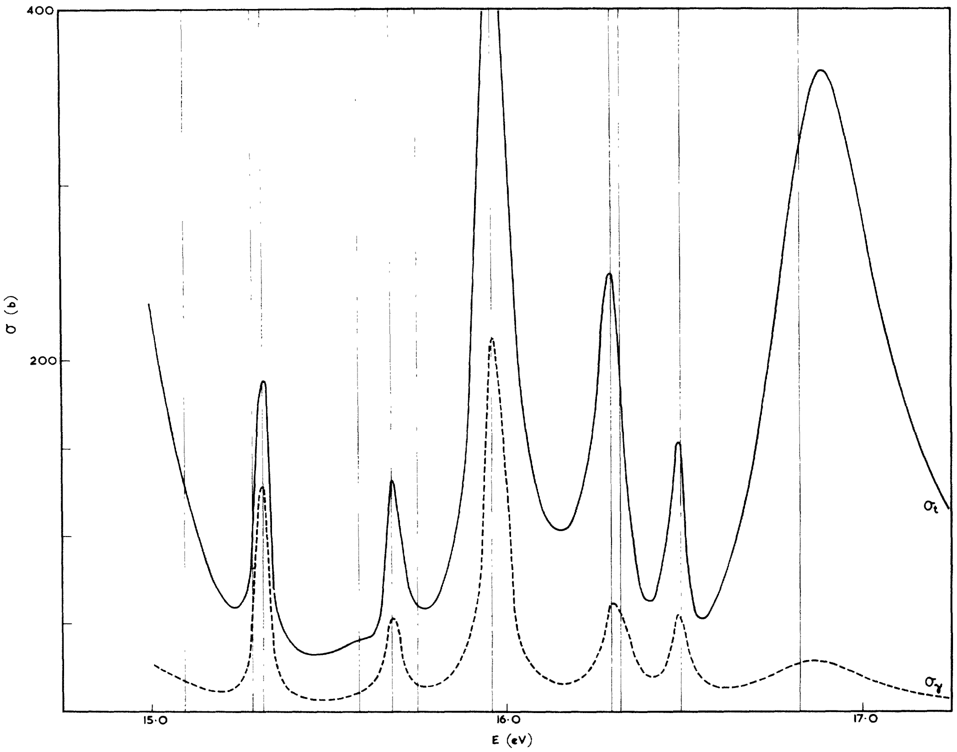

FIG. 2. Section of a computer-generated cross section; solid lines are σ_t, dashed lines are σ_γ. The vertical lines on the abscissa are the actual positions of levels.

"fission widths" of the simulated cross section can be fitted by a chi-squared distribution[10] with approximately four degrees of freedom. For U^{233} the distribution also corresponds to four degrees of freedom and for U^{235} to about two degrees of freedom. The "radiation widths" of the simulated case fluctuate from 17 to 88 meV. Corresponding fluctuations are found in published U^{233} and U^{235} data.[5,14] Many of the small-level spacings do not appear in the simulated cross section; in fact there are indications of only 22 resonances in the cross section whereas 34 were actually generated. Michaudon[15] has remarked that the level-spacing distribution obtained from the U^{235} cross section shows a dearth of small spacings; 20% more small spacings would be required to obtain agreement with a simple spacing superposition formula[16] for two spins. Finally, the simulated total cross section never falls to much less than 20 b in the minima; similar behavior is observed in the cross section of U^{233} and U^{235}. If $N_{\rm eff}$ were really less than unity the minima should fall almost to the potential scattering value of about 10 b. It thus seems extremely plausible that the U^{235} and U^{233} cross sections and possibly also the Pu^{239} cross section are fully consistent with the expectations of the channel theory of fission, and it is the assumptions made in analyzing the data that have been at fault. More quantitative statistical analyses are now being pursued in the hope that it may be possible to obtain better estimates of the number of open channels.

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MASS FORMULAS IN THE SU(6) SYMMETRY SCHEME*

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Recently it was proposed by Gürsey and Radicati[1] and Pais[2] that the SU(6) symmetry scheme incorporating spin and unitary spin may have important consequences in particle physics. They discussed, among other things, a possible mass formula and applied it to some low-dimensional representations. In this note[3] we propose that the SU(6) symmetry is broken analogously as in SU(3),[4,5] namely, the primary symmetry-breaking term in the Hamiltonian transforms like the $I=0, Y=0, J=0$ member of the $\underline{35}$ representation. The major result of this assumption is that in a given SU(6) representation, states with the same I, Y, and J belonging to different SU(3) multiplets are mixed in a definite way.

The 36 traceless operators $B_\mu{}^\nu$ of SU(6) are defined such that their representation in the six-dimensional vector space C_6 are given by

$$(B_\nu{}^\mu)_{ij} = \delta_{\mu j}\delta_{\nu i} - \tfrac{1}{6}\delta_{\mu\nu}\delta_{ij}, \quad (1)$$

$(\mu, \nu, i, j = 1, 2, \cdots, 6)$. These operators satisfy the commutation relations

$$[B_\nu{}^\mu, B_\beta{}^\alpha] = \delta_\beta{}^\mu B_\nu{}^\alpha - \delta_\nu{}^\alpha B_\beta{}^\mu. \quad (2)$$

The symmetry-breaking term is proposed to be $T_3{}^3 + T_6{}^6$, where

$$[B_\nu{}^\mu, T_\beta{}^\alpha] = \delta_\beta{}^\mu T_\nu{}^\alpha - \delta_\nu{}^\alpha T_\beta{}^\mu. \quad (3)$$

Note that the hypercharge operator Y is $-(B_3{}^3 + B_6{}^6)$. It can be shown that[6]

$$T_\nu{}^\mu = a_0\delta_\nu{}^\mu + a_1 B_\nu{}^\mu + a_2(B\cdot B)_\nu{}^\mu + a_3(B\cdot B\cdot B)_\nu{}^\mu + a_4(B\cdot B\cdot B\cdot B)_\nu{}^\mu + a_5(B\cdot B\cdot B\cdot B\cdot B)_\nu{}^\mu, \quad (4)$$

where the a_i's are constants depending only on the five Casimir operators of the group.

For the few low-dimension representations discussed below, only the first three terms in Eq. (4) are needed. Therefore, for those SU(6) supermultiplets we can write down the following mass formula:

$$M = M_0 + aY + b\{(B\cdot B)_{SU(4)} - 2Q(Q+1) - \tfrac{1}{2}Y^2\}. \quad (5)$$

For mesons mass squared is to be used in Eq. (5). The symbol $(B\cdot B)_{SU(4)}$ denotes the quadratic Casimir operator of the SU(4) subgroup which is considered by Gürsey, Pais, and Radicati.[7] $\vec{Q}$ is an angular momentum vector with components

$$Q_3 = \tfrac{1}{2}(B_3{}^3 - B_6{}^6),$$
$$Q_+ = B_3{}^6,$$
$$Q_- = B_6{}^3. \quad (6)$$

In the quark language,[8] $Q=\frac{1}{2}$ for the $S=\pm1$ quarks and $Q=0$ for the $S=0$ quarks. In Table I we shall give all the eigenvalues of $(B\cdot B)_{SU(4)}$ and Q of all the particles in the $\underline{20}$, $\underline{35}$, $\underline{56}$, and $\underline{70}$ representations.

Now let us discuss the $\underline{35}$ representation $[\underline{35} = (\underline{8}, \underline{1}) + (\underline{8}, \underline{3}) + (\underline{1}, \underline{3})]$ which has as members the pseudoscalar-meson octet and the vector-meson nonet. Since

$$\underline{35} \otimes \underline{35} = \underline{1} \oplus \underline{35} \oplus \underline{35} \oplus \underline{189} \oplus \underline{280} \oplus \underline{280}^* \oplus \underline{405}, \quad (7)$$

the matrix element

$$\langle \underline{35} | T_\nu{}^\mu | \underline{35} \rangle = a_0 + a_1 \langle \underline{35} | B_\nu{}^\mu | \underline{35} \rangle + a_2 \langle \underline{35} | (B\cdot B)_\nu{}^\mu | \underline{35} \rangle. \quad (8)$$