ues measured at room temperature can be attributed to power-calibration difficulties which will be corrected in subsequent experiments.

The experimental values of inversion temperature which can be obtained by interpolation or extrapolation of the $H_N(T)$ data at various emitted currents show the expected trend with field at the cathode and agree within 10% with theoretical values derived from Eq. (3), as shown in Table I.

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ANOMALOUS DENSITIES OF STATES IN NORMAL TANTALUM AND NIOBIUM

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The conductance of tunnel junctions composed of normal metals has previously been thought to be independent of bias. However, we have studied junctions in which the tunneling was from normal Ta or Nb through thin insulating layers to normal Al and have found that the conductance exhibits a peak centered at zero bias. At helium temperatures this peak is characteristically a few millivolts wide and represents an increased conductance of the order of 10%. However, the effect is quite strongly temperature dependent.

The observed effect is tentatively ascribed to a logarithmic singularity in the density of electron states in Ta and Nb at their Fermi energies. Harrison[1] has considered tunneling between normal metals but his theory predicts no structure,[2] which probably indicates that the independent-particle model which was considered is not rigorously applicable to Ta and Nb in their normal states.

The tunnel junctions were made in a way similar to the Ta-I-Ag ("I" for "insulator") junctions in which the phonon effects in the superconducting state were measured.[3] The junctions had a resistance of ~1 Ω and an area of ~7×10^{-4} cm^2. The differential resistance (dV/dI) was measured with a signal of ≤35 μV, and it was plotted directly as a function of bias on an X-Y recorder. Measurements were made in a magnetic field to quench the superconductivity, with the tunnel current flowing perpendicular to the field.

Although the resistance was measured, the ensuing discussion will be in terms of conductance, $G(V)$, which is more closely related to the density of states. $G(V)$ showed a temperature-dependent peak superposed upon a broad symmetrical background $G_0(V)$ which was independent of temperature and was probably due to the profile of the tunnel barrier changing slightly with bias. This background is irrelevant to the discussion, so we consider $\Delta G(V) = G(V) - G_0(V)$, which is independent of it. In Fig. 1 are some typical results for a Ta-I-Al junction where $\Delta G(V)/G_0(0)$

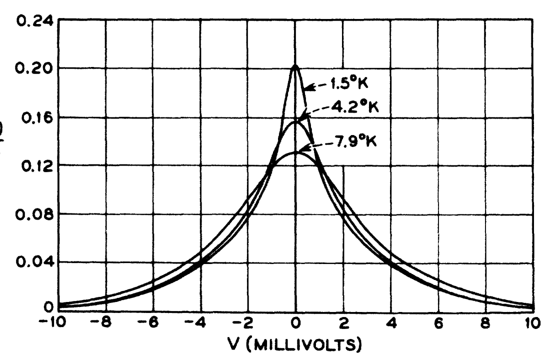

FIG. 1. $\Delta G(V)/G_0(0)$ as a function of bias for a Ta-I-Al junction at different temperatures.

is plotted as a function of bias (V) at three different temperatures. Two of the curves were measured at temperatures below the superconducting transition temperature (T_C) and in a magnetic field of 9 kG, while the other was measured in zero field at $T > T_C$. The effect of the magnetic field was measured at 4.2 and 1.5°K. At 4.2°K there was no observable change for fields between 2 and 20 kG. However, at 1.5°K there was a slight change; as the field increased from 4 to 20 kG the minimum broadened slightly with the half-width changing by ~10%. Similar results were obtained with Nb-I-Al junctions.

We note several striking features about these curves in Fig. 1. They are symmetrical within the experimental error about zero bias; as the temperature is lowered the conductance difference $\Delta G(0)$ increases while the half-width decreases. This behavior is quite continuous through the transition temperature. In Fig. 2, $\Delta G(0)/G_0(0)$ is shown as a function of $\log T$, and we see that the experimental points lie quite reasonably on a straight line.

To determine whether the effect was associated with the Ta or the Al, an Al-I-Al junction was measured and $G(V)$ showed only the broad temperature-independent variation. We estimate that any peak in $\Delta G/G_0(0)$ for Al is <0.1%. We can therefore assume that Al behaves as a simple metal with a density of states which is essentially constant near its Fermi energy. It has not been possible to conceive of a mechanism associated with the insulating barrier which could give rise to the effect, so we are really forced to assume that it is due to the density of states in the Ta and Nb. This assumption, as we shall see below, leads to an internally consistent description of the observed behavior. Ta-I-Ag and Nb-I-Ag junctions were found to behave similarly, but their resistances were a factor of 10^2 higher than the ones with Al, and although the half-widths of the peaks were the same at a given temperature, the values of $\Delta G(0)/G_0(0)$ were 5-10 times smaller.

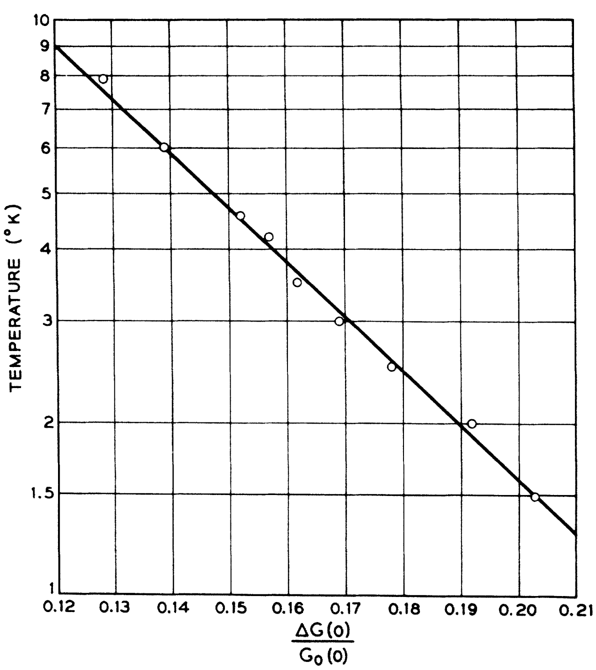

FIG. 2. $\Delta G(0)/G_0(0)$ as a function of temperature for a Ta-I-Al junction.

To analyze the results we use the assumption that has worked so successfully for superconductors[4]: that the probability of tunneling is simply proportional to the density of states (ρ). The current is then given by

$$I(V) = C\int_{-\infty}^{\infty} \rho_{\text{Al}}(E+eV)\rho_{\text{Ta}}(E) \times\left[f\left(\frac{E+eV}{kT}\right)-f\left(\frac{E}{kT}\right)\right]dE, \quad (1)$$

where C is a constant and f is the Fermi function $f(x) = (\exp x + 1)^{-1}$ with energies measured from the Fermi energy.

We assume that the density of states in the Al is constant, but that in the Ta it varies with energy, asymptotically approaching logarithmic behavior at low energies and tending to a constant, ρ_0, at high energies. Physically it would seem reasonable that the logarithmic behavior be truncated at a very low energy of the order of kT, but this would not qualitatively change the results, so for simplicity we take the density of states to be given approximately by

$$\begin{aligned} \rho(E)/\rho_0 &= [1-a\ln(|E|/E_0)], \quad && |E|<E_0, \\ &= 1, && |E|>E_0, \end{aligned} \quad (2)$$

where a and E_0 are constants.

Substituting this density of states into Eq. (1) and differentiating with respect to V, we find[5] for $eV < E_0$,

$$\Delta G(V)/G_0(0) = ag(eV/kT) - a\ln(kT/E_0), \quad (3)$$

where

$$g\left(\frac{eV}{kT}\right) = \int_{-\infty}^{\infty} \ln\left(\frac{|E|}{kT}\right) f'\left(\frac{E+eV}{kT}\right) d\left(\frac{E}{kT}\right).$$

At $V=0$, $g = 0.125$, so the temperature dependence

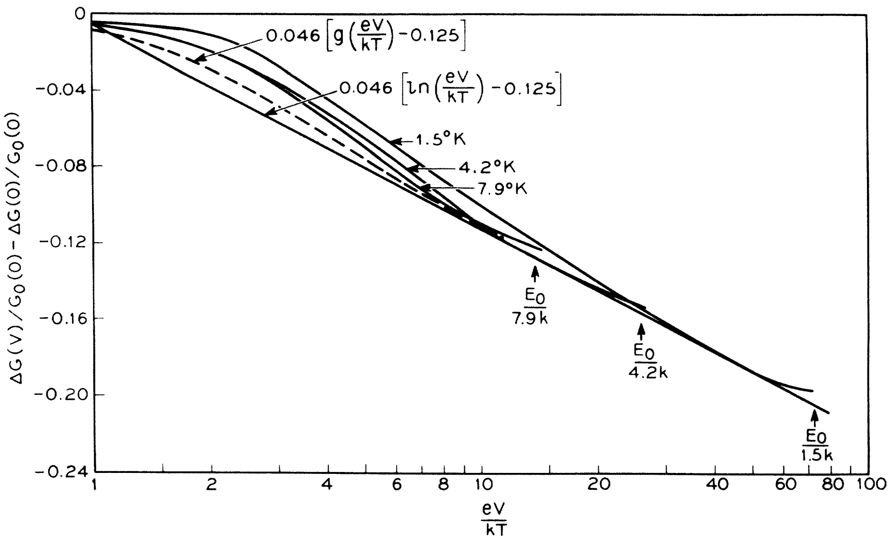

FIG. 3. $[\Delta G(V)-\Delta G(0)]/G_0(0)$ as a function of eV/kT for a Ta-I-Al junction at different temperatures, and the curve calculated from Eq. (5).

of the conductance at zero bias is given by

$$\Delta G(0)/G_0(0)=a[(0.125-\ln(kT/E_0)]. \tag{4}$$

This is the temperature dependence we found in Fig. 2. The corresponding values of the constants are $a=0.046$ and $E_0=9.4$ meV.

From Eqs. (3) and (4) we see that

$$\frac{\Delta G(V)-\Delta G(0)}{G_0(0)}=-a\left[0.125-g\left(\frac{eV}{kT}\right)\right]. \tag{5}$$

In Fig. 3 we show $[\Delta G(V)-\Delta G(0)]/G_0(0)$ and also $-0.046[0.125-g(eV/kT)]$ against eV/kT. For $eV/kT\gg 1$, $g(eV/kT)\sim -\ln(eV/kT)$, and this asymptotic limit is shown by the straight line in Fig. 3. The differences between the three experimental curves are within the experimental error, so we see that $[\Delta G(V)-\Delta G(0)]/G_0(0)$ is a universal function of eV/kT, and that it has a very similar form to the curve described by Eq. (5) with the same value of a as found by fitting Eq. (4) to the data in Fig. 2. The experimental results, however, do lie consistently above the calculated line, but it is not clear whether this is a significant deviation or a consequence of subtracting a slightly incorrect background. Equation (5) should hold only for $eV<E_0$, and using the value of E_0 found from Eq. (4) we get reasonable cutoff values; these are indicated by arrows in Fig. 3 for the three temperatures.

Harrison[1] has considered tunneling between normal metals on an independent-particle model. With this premise, variations in the densities of states do not affect the tunnel current except at energies near band edges. The Fermi levels of Ta and Nb are almost certainly not at band edges, and so we conclude that the theory breaks down because the independent-particle model is not applicable to Ta or Nb. Somewhat similar behavior occurs in Esaki tunnel diodes, where both peaks and dips in the conductance have been noted.[6] Studies[7] of this structure indicate that the behavior of the peaks is similar to those described above for Ta and Nb junctions.

To conclude, we have found an anomalous behavior of the conductance of Ta and Nb tunnel junctions. The temperature and voltage dependence was shown to be consistent with a logarithmic singularity in the density of states at the Fermi energy, but as yet there is no explanation for such an anomalous density of states.

I would like to thank P. W. Anderson, S. Englesberg, and W. L. McMillan for their valuable suggestions, and R. A. Logan and J. M. Rowell for their helpful discussion and for conveying their results to me prior to publication.

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values of $[\Delta G(V)-\Delta G(0)]/G_0(0)$ as it would give the smooth departure from $-\ln(eV/kT)$ behavior at $eV \sim E_0$ in Fig. 3.

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CONDUCTANCE ANOMALIES IN SEMICONDUCTOR TUNNEL DIODES

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In narrow p-n junctions, where current flow is by tunneling across the forbidden gap, the conductance at zero bias exhibits structure which corresponds to a conductance dip, peak, or combination of these shapes.[1-3] The detailed structure depends on the choice of semiconductor, the specific dopants and doping level, and the temperature of measurement, and varies with magnetic field strength. The conductance peaks which may be formed in both Si and Ge diodes are strikingly similar to those described in the preceding Letter by Wyatt[4] for tunneling through an insulating film between the transition metals Nb or Ta and a metal.

At present the most extensive data have been obtained with Si diodes. These diodes were made by alloying Al or Cu containing 1% of boron into n-type wafers of Si where the p-type regrowth has an acceptor concentration of 2.3×10^{19} cm^{-3}. The doping level in the n side was varied from $\sim 1 \times 10^{19}$ cm^{-3} to 1.5×10^{20} cm^{-3} by choice of starting material, the diodes changing from backwards to Esaki diodes over this range. Reciprocal conductance vs voltage plots were obtained by applying a constant ac current to the diode and measuring the voltage developed across the diode as a function of dc bias. Typically, the ac voltage generated was less than 70 μV. The diodes were immersed in liquid helium which could be pumped from 4.2 to 0.8°K.

Conductance peaks were observed in all junctions made by alloying Al-B to Sb-doped crystals. Typical results for a diode with $n = 8 \times 10^{19}$ cm^{-3} are shown in Fig. 1 where the conductance, $G(V)$, normalized to the background conductance at zero bias, $G_0(0)$, is plotted against bias at 4.2 and 0.84°K. The value of $G_0(0)$ changes by only ~4% in this temperature range.

Wyatt[4] has shown that the similar peak obtained with Ta structures may be explained by assuming a logarithmic singularity at the Fermi level in the density of states of Ta. This gives rise to a conductance change $\Delta G(V) = G(V) - G_0(V)$ whose temperature dependence at $V = 0$ is given by

$$\Delta G(0)/G_0(0) = a[0.125 - \ln(kT/E_0)], \qquad (1)$$

where a is a constant and E_0 is the cut-off energy for the logarithmic singularity. Using the data of Fig. 1 at 4.2 and 0.84°K, and similar data at intermediate temperatures, a plot is shown in Fig. 2(a) of $\Delta G(0)/G_0(0)$ versus $\log T$, and the reasonable fit to a straight line gives $a = 0.024$ and $E_0 = 2.3$ MeV. The peaks observed here are considerably narrower than in the Ta and Nb structures where $E_0 = 9.4$ MeV. As discussed by Wyatt,[4] the data should lie on a universal curve given by

$$[\Delta G(V) - \Delta G(0)]/G_0(0) = -a[0.125 - g(eV/kT)], \qquad (2)$$

where $g(eV/kT)$ is a known function whose value approaches the limit $-\ln(eV/kT)$ for $eV/kT \gg 1$. Figure 2(b) shows a plot of $[\Delta G(V) - \Delta G(0)]/G_0(0)$ against $\log(eV/kT)$ using data obtained at five

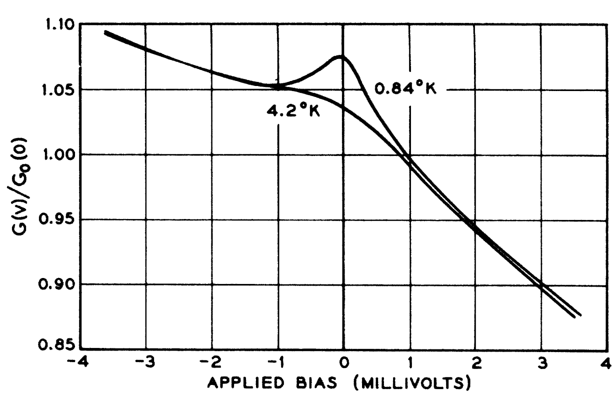

FIG. 1. The conductance $G(V)$ normalized to the zero-bias background conductance $G_0(0)$ plotted against bias at 4.2 and 0.84°K.