similar relation can also be obtained for $\lambda = \frac{5}{2}$, namely

$$m_{Y_0^*(1815)} = \tfrac{5}{2} m_{K^*(725)} = (25/4) m_{ABC}. \qquad (22)$$

Upon using $m_{ABC} = 290$ MeV,[15] the right-hand side becomes 1812.5 MeV, in very good agreement with the experimental value (≈1815 MeV).

A more detailed account of the present work will be given in a later paper.

I wish to thank Dr. G. C. Wick for informing me of his results prior to publication, and for suggesting their possible use in connection with the mass relations.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]T. F. Kycia and K. F. Riley, Phys. Rev. Letters 10, 266 (1963).

[2]R. M. Sternheimer, Phys. Rev. Letters 10, 309 (1963).

[3]R. M. Sternheimer, Phys. Rev. 131, 2698 (1963).

[4]R. E. Peierls and S. B. Treiman, Phys. Rev. Letters 8, 339 (1962).

[5]T. Takabayasi and Y. Ohnuki, Progr. Theoret. Phys. (Kyoto) 30, 272 (1963).

[6]We remark that the relation $m_f - m_\varphi = m_\varphi - m_\omega$ was noticed independently by the present author. Results similar to those of reference 5 were also obtained by R. Kumar, private communication.

[7]This equation for m_ρ was also proposed by Takabayasi [see reference 11, Eq. (28)].

[8]T. F. Kycia, private communication.

[9]M. Abolins, R. L. Lander, W. A. Mehlhop, N.-H. Xuong, and P. M. Yager, Phys. Rev. Letters 11, 381 (1963).

[10]G. C. Wick, private communication.

[11]T. Takabayasi, Nuovo Cimento 30, 1500 (1963).

[12]It may be noted that the recently reported $K\pi\pi$ resonance [T. P. Wangler, A. R. Erwin, and W. D. Walker, Phys. Letters 9, 71 (1964)] with mass 1175 MeV would correspond to $p = 0$, $q = 5$, i.e., $m = 5\kappa$. Moreover, the P_{11} pion-nucleon resonance [L. D. Roper, Phys. Rev. Letters 12, 340 (1964)] with mass 1485 MeV corresponds closely to $p = 4$, $q = 4$, for which Eq. (17) gives 1488 MeV. The recently discovered X meson, decaying into $\eta + 2\pi$, with mass $m = 960$ MeV, has the assignment $p = 7$, $q = 0$; we have $7m_\pi = 961.1$ MeV [G. R. Kalbfleisch et al., Phys. Rev. Letters 12, 527 (1964); M. Goldberg et al., Phys. Rev. Letters 12, 546 (1964)].

[13]Whenever they are available, we use the experimental mass values of A. H. Rosenfeld, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 783.

[14]This type of plot has been suggested by R. F. Peierls (private communication).

[15]N. E. Booth and A. Abashian, Phys. Rev. 132, 2314 (1963).

EMPIRICAL PION-NUCLEON SPECTROSCOPY

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The aim of this Letter is to show simple connections existing between the presently known pion-nucleon resonances (for simplicity, we call "resonances" all the maxima of the $\pi^\pm p$ cross sections). If these relationships have a wider validity, they enable us to predict where to expect other resonances and what are their angular momenta.

The different shapes of the maxima in the $T = \frac{1}{2}$ and $T = \frac{3}{2}$ cross sections [apart from the isobar $\Delta(\frac{3}{2}, \frac{3}{2})$] suggests that they could have a different nature. The simplest hypothesis is to consider the $T = \frac{1}{2}$ resonances as excited levels of the nucleon N and the $T = \frac{3}{2}$ resonances as excited levels of the isobar Δ. The general idea of this Letter is to make the isobar play a role similar to that of the nucleon.

In Table I, in addition to nucleon[1] N and isobar Δ, we have written the seven known resonances with their masses M_i ($i = 2T$), the corresponding momenta P_π of incident pions, and their angular momenta J_i if known. We have labelled the columns listing the resonances by $\lambda = 1, 2, 3, \cdots$, in order of increasing mass; when J is known we find that

$$\lambda = J - T + 1, \qquad (1)$$

the same relationship as stated by Kycia and

Table I. Masses,[a] corresponding pion momenta, and angular momenta[a] of known πp resonances. Values in parentheses are predicted by this paper.

	Nucleon	$T=1/2$ Resonances						
M_1 (GeV)	0.939	1.40	1.515	1.685	(1.87)[b]	2.19	(2.76)[b]	(4.23)[b]
P_π (GeV/c)	0	0.55	0.73	1.02	1.40	2.08	3.60	9.30
J_1	1/2	1/2	3/2	5/2	(7/2)[b]	(9/2)[b]	(11/2)[b]	(13/2)[b]
	Isobar	$T=3/2$ Resonances						
M_3 (GeV)	1.238	1.655	(1.75)[c]	1.92	(2.07)[c]	2.36	(2.87)[c]	(4.20)[c]
P_π (GeV/c)	0.30	0.98	1.18	1.48	1.80	2.50	3.90	9.0
J_3	3/2	(3/2)[c]	(5/2)[c]	7/2	(9/2)[c]	(11/2)[c]	(13/2)[c]	(15/2)[c]
$\lambda=J-T+1$		1	2	3	4	5	6	7

[a]Masses and angular momenta are given by M. Roos, Phys. Letters 8, 1 (1964); P. Bareyre et al., Phys. Letters 8, 137 (1964).
[b]Predicted by Eq. (3).
[c]Predicted by Eq. (2).

Riley.[2] For the $T=\frac{3}{2}$ row, we have only three resonances; with the rule (1), the 1.92-GeV resonance is in the $\lambda=3$ column. The two highest resonances (of which we do not know their angular momenta, so λ can be $4, 5, \cdots$) are tentatively written in the last column. If we put the "shoulder" of π^+p total cross section into the $\lambda=1$ column,[3] we find a quite simple relationship between the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ resonances in the same column,

$$M_1(\lambda)=1.111M_3(\lambda)-0.434 \text{ GeV} \qquad (2)$$

if M_i is expressed in GeV.[4] Equation (2) fits all the pairs of known resonances with the same λ and the pair (N, Δ) with an accuracy better than one percent (Table II).

This relationship predicts the existence of a $T=\frac{3}{2}$ resonance for $\lambda=2$ (or $J=\frac{5}{2}$) with a mass $M_3(2)=1.75$ GeV. The corresponding momentum of the incident π^+ is 1.18 GeV/c. The shape of $\sigma_{tot}(\pi^+p)$ in this momentum region does not exclude the possibility of a structure which, without appearing as a defined bump, could widen the adjacent maxima at 0.98 and 1.48 GeV/c. Besides, some preliminary results obtained at Saclay[3] indicate that the coefficients A_0 and A_2 of the angular distribution $(d\sigma/d\Omega^*)_{\pi^0}=\sum A_n \times\cos^n\Theta^*$ of π^0 in the reaction $\pi^+ + p \to \pi^+ + p + \pi^0$ show maxima at 0.98 and near 1.2 GeV/c; the analysis of this angular distribution admits an interpretation of these maxima as resonances $D_{3/2}$ and $P_{5/2}$ between π^0 and isobar Δ.

Table II. Comparison between M_3 calculated from Eq. (2) and M_3 experimentally observed.

M_1 (exp.)	M_3 (calc.)	M_3 (exp.)
0.939	1.236	1.238
1.40	1.651	1.655
1.515	1.754	...
1.685	1.907	1.92
2.19	2.362	2.36

Equation (2) permits one to find what $T=\frac{3}{2}$ resonance corresponds to a given $T=\frac{1}{2}$ resonance. A second observation might determine the sequence of $T=\frac{1}{2}$ resonances: If we plot the quantities $(1/M_1)^2$ versus λ we obtain a straight line for the first three resonances. The last one is also on this straight line if its angular momentum is $\frac{9}{2}$ (or $\lambda=5$). There is no experimental evidence as yet allowing us to attribute $J=\frac{9}{2}$ or $\frac{7}{2}$ for this resonance with certainty; however, if we believe that $J=\frac{9}{2}$ for this point and that it is permitted to extrapolate the straight line, we obtain (Fig. 1)

$$(1/M_1)^2 = [(0.588\pm0.006)-(0.076\pm0.003)\lambda] \text{ GeV}^{-2}. \qquad (3)$$

We note that this fit provides a limiting value 7 for λ; then

$$J_1 \leq 13/2 \text{ and } J_3 \leq 15/2.$$

If all the λ columns must be filled, we expect three new $T=\frac{1}{2}$ resonances and, with Eq. (2), three corresponding $T=\frac{3}{2}$ resonances. In Table I, we have added between parentheses the resonances predicted by Eqs. (2) and (3). Masses given in Table I for the expected new

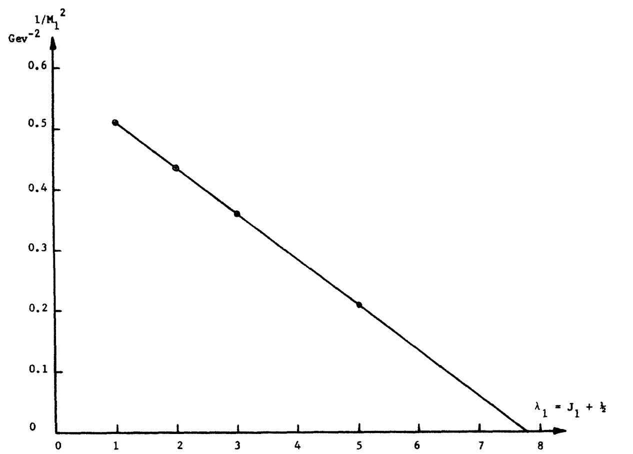

FIG. 1. Relation between masses and angular momenta for $T=\frac{1}{2}$ resonances.

resonances have an accuracy of the order of only 4 or 5%, because they depend on the precision of fit (3) and on determination of the last known resonance, 2.19 GeV.

A first indication of the possible validity of our scheme can be found in results about pion photoproduction at Cambridge[5]; there is some evidence for a $T=\frac{1}{2}$ resonance near 2.7 GeV which could fill the $(T=\frac{1}{2}, \lambda=6)$ box of our Table I.

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[1]We use for the masses of nucleon and pion the mean between the masses of their charged and neutral states $N=(m_p+m_n)/2=0.9388$ GeV, $m_\pi=(m_{\pi^\pm}+m_{\pi^0})/2=0.1373$ GeV.

[2]T. Kycia and K. Riley, Phys. Rev. Letters 10, 266 (1963).

[3]J. P. Merlo, private communication.

[4]We can also write Eq. (2) as

$$[M_1(\lambda)-N]=[(\Delta+m_\pi)/\Delta][M_3(\lambda)-\Delta].$$

R. M. Sternheimer, working on a similar mass formula, pointed out to us that this formula could be written with only his two constants $m_\pi=0.1373$ GeV and $\kappa=0.2347$ GeV $=N/4$:

$$M_1(\lambda)=(10/9)M_3(\lambda)-10m_\pi+4\kappa.$$

[5]R. Alvarez, Z. Bar-Yam, W. Kern, D. Luckey, L. S. Osborne, S. Tazzari, and R. Fessel, Phys. Rev. Letters 12, 710 (1964).

APPARENT REGULARITY IN THE MASSES OF THE MESONS*

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Table I shows a set of masses calculated from the formula

$$m^2=a^2[2n+\tfrac{1}{4}Y^2-I(I+1)], \qquad (1)$$

where $a=385$ MeV, n is an integer, and the hypercharge and isospin (Y, I) take on the values they assume in an SU(3) meson octet, namely $(0,0)$, $(\pm1,\frac{1}{2})$, and $(0,1)$. The most striking discrepancy in Table I, that the pion mass is given as zero, is related to the fact that $(472)^2$ is $\frac{3}{4}$ of $(544)^2$ or, to put it another way, that m_π^2 is small compared to the squares of the other particle masses and thus poorly determined. Aside from the pion, the worst agreement is for the K-meson mass (4.6%), all others being better than 3%. If the additive octet corrections δ indicated in Table I are applied (which may have the form $\pm5n$ MeV), the K-meson mass is still wrong by about 3.5%, but all other masses are within about 1%. For $I=Y=0$, Eq. (1) without correction yields the following masses for $n=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$: 385, [668], [861], 1019 MeV. The masses in brackets are unfamiliar to the author, but 385 MeV is the mass of the σ meson[1] and 1019 MeV is the mass of the φ meson. The assignment of the vector octets $(n=2,3)$ in Table I agree with that of Barger.[2] The only established meson known to the author and not yielded by Eq. (1) is the recently discovered "g" meson[3] at 960 MeV, which does not appear to be a vector particle (otherwise it would be given by $n=3$, $I=Y=0$).

Equation (1) incorporates a number of features noted, at least in part, by other authors. As previously remarked, it gives the vector octet