of longer lifetime than they would be for uncorrelated pinners. The correlations between the pinners do not result from their mutual interaction, but from the entropy change due to their interaction with the dislocation lines. They extend over distances which are large compared with the range of interaction between pinning points (see Fig. 1).

The question of the entropy of dislocation lines has not received much attention in the past.[3] We think that it plays a significant role in the nucleation and the rate of clustering and precipitation on dislocation lines,[4,5] either of impurities[6] or of defects created by radiation damage.[7] It strongly increases the probability that a certain critical number of pinners come close to each other to form a nucleus and acts furthermore as a force on other pinners near or on the line to join existing clusters.

Internal friction experiments, especially the amplitude-dependent breakaway problem,[1] depend sensitively on the spatial distribution of the pinners on the line. Impurities already have the grouped or clustered distribution when the solid is cooled from higher temperature. Point defects on dislocation lines created by radiation damage at low temperatures will change from an uncorrelated distribution to the grouped distribution at that temperature for which pipe diffusion is possible.

Finally, it should also be mentioned that the formation entropy of vacancies near a free dislocation line is lowered by the amount of line entropy because of the pinning action of vacancies. It, thus, can even be negative. The formation entropy is not lowered near any already existing pinner. The vacancies, therefore, group mainly near impurities or jogs[8] or other nodal points.

To summarize, one can say the following: The dislocations establish correlations between mobile point defects in or close to the dislocation core. These correlations extend over distances which are large compared with the range of forces between the point defects. Dislocation modes differ from regular crystal modes by being sensitive to the position of point defects.

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NOTTINGHAM EFFECT IN FIELD AND *T-F* EMISSION: HEATING AND COOLING DOMAINS, AND INVERSION TEMPERATURE

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Electron emission is accompanied by energy exchanges between the conduction electrons and the cathode lattice, which become particularly important at the very high emission densities feasible with field and *T-F* emission cathodes. Their study is of basic interest as it provides a complementary check, through a direct measurement of the average energy of the emitted electrons, of the theory of field and *T-F* emission; it is also of practical importance because these energy exchanges control the cathode tip temperature and set an upper limit on the feasible emission density. This paper presents recent data confirming the theoretically predicted temperature dependence of the exchange and its reversal (from cathode heating to cooling) at high temperatures. A more comprehensive theoretical and experimental study is approaching completion.[1]

There are two main emission-induced energy-exchange phenomena. The familiar resistive or

Joule heating effect was first studied in the case of field emission by Dyke et al.[2] and Dolan, Dyke, and Trolan.[3] In the usual case where resistivity increases rapidly with temperature, resistive heating by itself leads to an inherently unstable situation at high emission densities.[1] Since stable high-density emission is observed,[4] there must exist another factor having a strong and stabilizing influence on cathode-tip temperature.

Such a stabilizing factor is provided by the energy exchange resulting from the difference between the average energy of the emitted electrons, $\bar{E}$, and that of the replacement electrons supplied by the circuit, $\bar{E}'$. In the case of thermionic emission this phenomenon, discussed by Richardson[5] and later by Nottingham,[6] is well known and produces cooling of the cathode by an average amount $\varphi + 2kT$ per emitted electron. The corresponding effect in field and T-F emission was first discussed by Fleming and Henderson,[7] who were unable to detect it experimentally, and has been a subject of controversy[6,7] with respect to the correct value of $\bar{E}'$ and hence the direction of the effect (cathode cooling occurs when $\bar{E} > \bar{E}'$, and heating when $\bar{E} < \bar{E}'$). Our data support the view of Nottingham, who took $\bar{E}'$ to be the Fermi energy E_{F} and, on that basis, predicted heating of the cathode in the case of field emission. Hereafter in this paper, the replacement electrons are assumed to have the Fermi energy, and the energy exchange just discussed is called the "Nottingham effect."

In pure field emission ($T = 0°\mathrm{K}$), energy levels above E_{F} are empty, all emitted electrons have less than Fermi energy, and the Nottingham effect necessarily produces heating of the cathode. However, if the cathode temperature T is increased (T-F emission), energy levels above E_{F} become populated and contribute preferentially to the emission, causing a decrease in the average heat transfer per emitted electron. If T exceeds an "inversion" temperature T_i [which depends on applied field and cathode work function —see Eq. (3) below], $\bar{E}$ becomes greater than E_{F}, reversing the direction of the effect from heating to cooling of the cathode.

Thus, the two main energy exchange phenomena are seen to have opposite behavior: Resistive heating, being proportional to ρI^2, increases rapidly with current and also with cathode temperature T in the usual case where resistivity increases with temperature; Nottingham heating, being proportional to $\bar{\epsilon} I$, increases less rapidly with I and decreases with T, becoming negative (i.e., cooling) at sufficiently high T. The relative magnitude of the two factors varies markedly with operating conditions. For an initially cold field emitter, Nottingham heating is the predominant effect at low emission densities. At high emission densities it is the triggering mechanism which raises the cathode tip temperature to where resistive heating becomes predominant. The Nottingham effect then changes to cooling and exerts a stabilizing, though not dominant, influence on tip temperature.

One of the present authors treated the combined effect of resistive and Nottingham phenomena in the special case of tungsten field emitters initially at room temperature[8]; this work was later extended to other work functions and confirmed experimentally.[9] Levine[10] gave a theoretical analysis of a similar problem. Drechsler[11,12] has recently reported significant departures from theoretical predictions for the temperature dependence of the Nottingham effect and for the value of inversion temperature for tungsten; however, the present work shows fairly good agreement with theory on these two points.

Calculation of the Nottingham effect rests on the determination of the total energy distribution of the emitted electrons for field and T-F emission. The situation is illustrated in Fig. 1. Young and Müller have derived the total energy distribution[13] and verified it experimentally at low temperatures.[14] In terms of $\epsilon = E - E_{\mathrm{F}}$, the total energy distribution is[15]

$$J(\epsilon)\delta\epsilon = J_{0F}\frac{e^{\epsilon/d}}{1+e^{\epsilon/kT}}\delta\left(\frac{\epsilon}{d}\right), \tag{1}$$

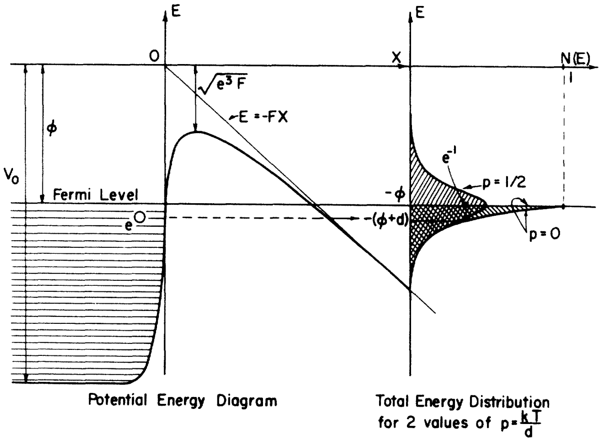

FIG. 1. Potential energy diagram and total energy distribution for field or T-F emitted electrons.

where J_{0F} is the field emitted current density at 0°K, and d (a parameter related to the rate of change of barrier tunneling probability with electron energy) is given by[13,15]

$$d = \tfrac{1}{2}\hbar e(2m)^{-1/2}F/\varphi^{1/2}t(y), \qquad (2)$$

where F is the applied field, φ is the work function, $y = (e^3F)^{1/2}/\varphi$, and $t(y)$ is a slowly varying tabulated function.[15] For pure field emission $J(\epsilon)$ is zero above E_F; below E_F, it decreases exponentially with E as illustrated in Fig. 1, and the cathode receives an average amount of energy d per emitted electron. For T-F emission ($T \neq 0$), the shape of the energy-distribution curve depends only on the parameter $p = kT/d$ and the average energy received per emitted electron is $-\bar{\epsilon} = df(p)$, where $f(p)$, can be obtained numerically by integration of Eq. (1). The total energy distribution becomes exactly symmetrical with respect to E_F when $p = \frac{1}{2}$; emission heating thus occurs for $p < \frac{1}{2}$, emission cooling for $p > \frac{1}{2}$, and the inversion temperature (corresponding to $p = \frac{1}{2}$) is given by

$$T_i = \frac{d}{2k} = \frac{\hbar e}{4k(2m)^{1/2}}\frac{F}{\varphi^{1/2}t(y)} \cong 5.32\times 10^{-5}\frac{F}{\varphi^{1/2}}, \qquad (3)$$

where T_i is in °K, F in V/cm, and φ in eV. This expression is more accurate than that of Levine[10] which does not include the image correction function $t(y)$.

The analytical treatment of T-F emission, leading to the above expressions, breaks down as $p = kT/d$ approaches unity. However, comparison with results of numerical integration shows this treatment to be quite accurate over the range $0 \leq p < \frac{2}{3}$ which includes the complete heating domain, part of the cooling domain, and the boundary between these domains which corresponds to $p = \frac{1}{2}$ and Eq. (3).

The main difficulty in measuring energy-exchange phenomena in field and T-F emission is the usually strong localization of these phenomena and of the associated temperature changes; this localization results from the cathode geometry (very sharp needle with a conical shank and a tip radius well below one micron) with which controlled field emission is most reliably obtained. A determination of both the magnitude and the location of the energy transfer requires measurement of the temperature at the emitting area itself, which is of the order of 10^{-9} cm^2. For this purpose,[9] temperature-sensitive coatings of materials which change cathode work function can be used to sense the local tip temperature. Measurements of this type, discussed in a forthcoming paper,[1] conclusively establish the existence of emission heating and cooling domains and, within the limited accuracy feasible, confirm the magnitude of the effect and localize the transfer of energy to within a few tip radii of the cathode tip. However, the complex experimental conditions (pulsed emission, large field, and temperature gradients near the tip, etc.) limit the accuracy of the results and, in the work presently reported, a more precise method has been used to measure the magnitude (but not location) of the energy exchange and the inversion temperature.

The method used here is a refinement of that of Drechsler,[11,12] who obtained field and T-F emission from random protrusions on the surface of very thin wires. The principle is to give the emitter-supporting filament sufficient thermal impedance that the small heat input resulting from dc emission at relatively low current level may be detected sensitively through the associated change in temperature and resistance of the filament. Reliance on emission from several random protrusions of unknown number, geometry, and location creates uncertainties in interpretation of the data which are avoided here by confining emission to a single field-emission needle (whose precise geometry has been determined in an electron microscope) mounted at the center of a smooth support wire 1 in. long and approximately 0.001 in. in diameter. The temperatures of the wire and cathode are controlled by adjusting a heating current passed through the wire, and are monitored by measuring the resistance R of a short central section of the wire. The thermal impedance of the wire is high enough that emission-induced power inputs can be measured with an estimated accuracy of 5 microwatts, permitting reliable measurements at low current levels where resistive heating is calculable and small and where, therefore, the Nottingham effect strongly predominates and is more effectively studied. The present embodiment also permits direct calibration of the wire resistance readings in terms of input power at the tip: For this purpose, the tip is bombarded by a very fine (0.020-in. diameter) electron beam generated by an auxiliary filament and focused at the cathode tip, and the change ΔR in support filament resistance is measured as a function of cathode temperature and bombarding beam power.

The method just described has been applied to

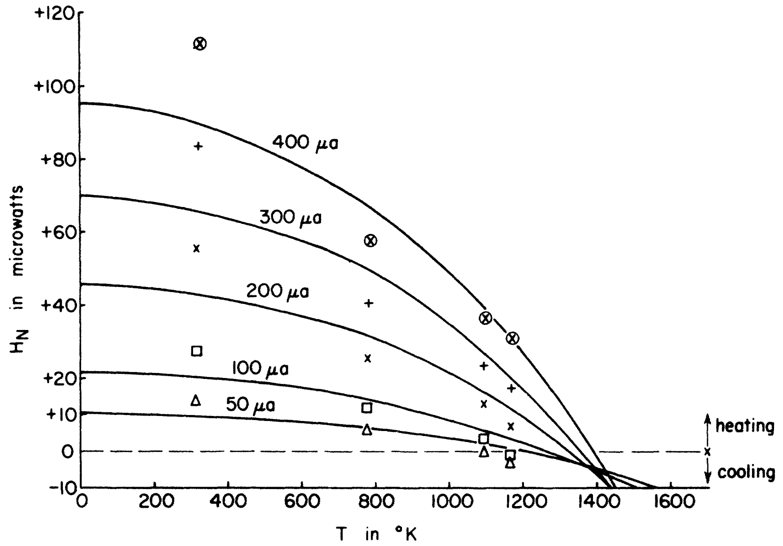

FIG. 2. Comparison of calculated (solid curves) and measured (data points) Nottingham power transfer H_N, for field and T-F emission from clean tungsten, at several emitted current levels.

clean tungsten emitters of effective work function $\varphi = 4.5$ eV. In order to extend the measurements to a variety of cathode work functions, the experimental tube is currently being modified to incorporate sources of electropositive and electronegative adsorbates.

The measurements to date have covered field and T-F emitted currents up to 500 μA and cathode temperatures up to 1170°K; the latter are just sufficient at low current levels to exceed the inversion temperature and produce emission cooling. Ultrahigh-vacuum techniques were used, the emission characteristics were stable and reproducible, and cumulative contamination over a complete experimental run always caused less than 1% change in tip work function; under these conditions, the rate of ion bombardment at the cathode was low enough that the corresponding power input could be neglected.

At each cathode temperature, the change ΔR in the resistance of the support filament was measured as a function of emitted current, then converted to emission-induced power input by the direct calibration technique discussed above. After small corrections for the emission-induced temperature drop from the tip to the base of the emitter and for the small resistive power input to the emitter, the data yield the Nottingham power input $H_N = Idf(p)$, since the known emitter geometry and work function permit calculation, for each set of I and T values, of the electric field F, therefore of d by Eq. (2) and of $f(p)$.

Figure 2 compares the calculated and measured values of $H_N(T)$ at several emitted currents. Fairly good agreement is found concerning the magnitude and the temperature dependence of the Nottingham power transfer, particularly since it appears that the systematically high val-

Table I. Inversion temperatures at various currents.

		Inversion temperature (°K)	
Emitted current (μA)	Cathode field (10^7 V/cm)	Calculated by Eq. (3) with $\varphi \cong 4.5$ eV	Derived from $H_N(T)$ data
50	4.79	1200	1092
100	5.02	1260	1160
200	5.30	1326	1250
300	5.42	1360	1360

ues measured at room temperature can be attributed to power-calibration difficulties which will be corrected in subsequent experiments.

The experimental values of inversion temperature which can be obtained by interpolation or extrapolation of the $H_N(T)$ data at various emitted currents show the expected trend with field at the cathode and agree within 10% with theoretical values derived from Eq. (3), as shown in Table I.

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ANOMALOUS DENSITIES OF STATES IN NORMAL TANTALUM AND NIOBIUM

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The conductance of tunnel junctions composed of normal metals has previously been thought to be independent of bias. However, we have studied junctions in which the tunneling was from normal Ta or Nb through thin insulating layers to normal Al and have found that the conductance exhibits a peak centered at zero bias. At helium temperatures this peak is characteristically a few millivolts wide and represents an increased conductance of the order of 10%. However, the effect is quite strongly temperature dependent.

The observed effect is tentatively ascribed to a logarithmic singularity in the density of electron states in Ta and Nb at their Fermi energies. Harrison[1] has considered tunneling between normal metals but his theory predicts no structure,[2] which probably indicates that the independent-particle model which was considered is not rigorously applicable to Ta and Nb in their normal states.

The tunnel junctions were made in a way similar to the Ta-I-Ag ("I" for "insulator") junctions in which the phonon effects in the superconducting state were measured.[3] The junctions had a resistance of ~1 Ω and an area of $\sim 7\times10^{-4}$ cm^2. The differential resistance (dV/dI) was measured with a signal of ≤ 35 μV, and it was plotted directly as a function of bias on an X-Y recorder. Measurements were made in a magnetic field to quench the superconductivity, with the tunnel current flowing perpendicular to the field.

Although the resistance was measured, the ensuing discussion will be in terms of conductance, $G(V)$, which is more closely related to the density of states. $G(V)$ showed a temperature-dependent peak superposed upon a broad symmetrical background $G_0(V)$ which was independent of temperature and was probably due to the profile of the tunnel barrier changing slightly with bias. This background is irrelevant to the discussion, so we consider $\Delta G(V) = G(V) - G_0(V)$, which is independent of it. In Fig. 1 are some typical results for a Ta-I-Al junction where $\Delta G(V)/G_0(0)$

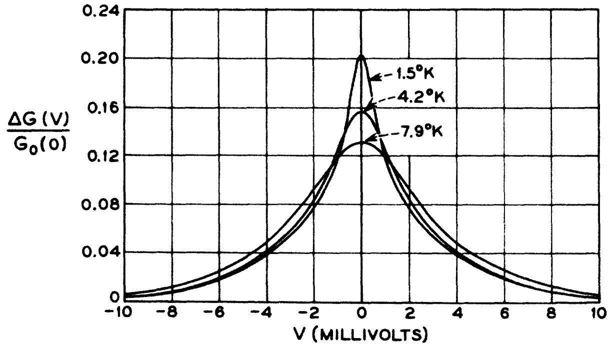

FIG. 1. $\Delta G(V)/G_0(0)$ as a function of bias for a Ta-I-Al junction at different temperatures.