our measurement with the value quoted above.

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[1]See, for example, R. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958); R. E. Marshak and E. Sudarshan, Proceedings of the Padua-Venice Conference on Mesons and Recently Discovered Particles, September 1957 (Società Italiana di Fisica, Padua-Venice, 1958); N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

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[6]The theoretical expectation for the internal bremsstrahlung is

$$\frac{\Gamma_1(\pi^+\pi^-\gamma)}{\Gamma_1(\pi^+\pi^-)}=\frac{\alpha}{\pi}\int_{k_{\min}}^{k_{\max}}\frac{dk}{k}\left(1-\frac{2k}{m_K}\right)\frac{\beta}{\beta_{\pi\pi}}\times\left[\frac{1+\beta^2}{\beta}\ln\frac{1+\beta}{1-\beta}-2\right],$$

where $\Gamma_1(\pi^+\pi^-\gamma)$ is the probability for radiative K_1^0 decay, $\Gamma_1(\pi^+\pi^-)$ is the probability for the decay $K_1^0\to\pi^+ +\pi^-$, k is the photon momentum, β is the π velocity in the $\pi\pi$ center of mass in the decay $K_1^0\to\pi^++\pi^-+\gamma$, and $\beta_{\pi\pi}$ is the velocity in the $\pi\pi$ center of mass in the decay $K_1^0\to\pi^++\pi^-$. We wish to thank Dr. Jonas Schultz, Dr. M. Bég, and Dr. R. Friedberg for making this result available to us.

[7]G. Zorn, T. Fujii, J. Jovanovic, and F. Turkot, Bull. Am. Phys. Soc. 9, 442 (1964), reported the value $\tau_{K_2^0}=(5.4\pm0.5)\times10^{-8}$ sec.

[8]D. Luers, B. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. 133, B1276 (1964). We use the value $\Gamma_{2(+-0)}/(\Gamma_{2L}+\Gamma_{2(+-0)})=0.171\pm0.023$.

[9]M. H. Anikina et al., Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 452.

[10]F. S. Shaklee, G. L. Jensen, B. P. Roe, and D. Sinclair, Bull. Am. Phys. Soc. 9, 34 (1964), report the branching ratios 4.6 ± 0.3 for K_{e3} and 3.0 ± 1 for $K_{\mu3}$.

EMPIRICAL SYSTEMATICS OF THE STRONGLY INTERACTING PARTICLES*

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The purpose of this note is to point out a set of mass relations involving the mesonic resonances and the baryon isobars. This work can be regarded as an extension of previous results on empirical mass relations involving the baryon isobars.[1-3]

We note that to a very good accuracy, the mass of the well-known $N_{3/2}^*$ isobar with $I=J=\frac{3}{2}$ can be written as

$$m(N_{3/2}^*)=9m_\pi=1236\text{ MeV},\qquad(1)$$

as compared to the experimental value, 1238 ± 2 MeV. Here and throughout this paper, we denote by m_π the average $\frac{1}{2}(m_{\pi^\pm}+m_{\pi^0})$ of the masses of $\pi^\pm$ and π^0; thus $m_\pi=137.3$ MeV. With this definition of m_π, we have[4] $m_\eta=4m_\pi$ for the η meson. Upon making use of the results of Kycia and Riley,[1] one obtains by means of Eq. (1) $m(N_{1/2}^*, 1512\pm2)=11m_\pi$; $m(N_{3/2}^*, 1920\pm15)=14m_\pi$; $m(N_{1/2}^*, 2190\pm20)=16m_\pi$ for the higher nucleon isobars belonging to the $N_{3/2}^*(1238)$ isobar system.[1,3]

We now consider the results of Takabayasi and Ohnuki[5] who have shown that the masses of the isotopic spin $I=0$ mesons with strangeness $S=0$ are equally spaced,[6] at intervals of κ, where $\kappa\cong235$ MeV. With $m_\eta=4m_\pi$, one obtains

$$m_n(I=0)=4m_\pi+n\kappa,\qquad(2)$$

where $n=0$ for the η meson, $n=1$ for ω, $n=2$ for φ, and $n=3$ for f.

In order to obtain an equation for the mass m_ρ of the ρ meson, we make use of the relation previously obtained by Sternheimer [reference 3, Eq. (11)]:

$$m_\rho+m_\omega=m_f+2m_\pi.\qquad(3)$$

Upon using Eq. (2) for m_ω and m_f, one obtains[7]

$$m_\rho = 2m_\pi + 2\kappa. \tag{4}$$

It has been pointed out by Kycia[8] that $m_B - m_\rho \approx 2\kappa$, where m_B is the mass of the B particle[9] ($m_B \cong 1220$ MeV). Hence for the $I=1$ mesons, we may have the equation

$$m_n(I=1) = 2m_\pi + 2n\kappa, \tag{5}$$

where $n=1$ for ρ, $n=2$ for B.

We note that to a very good approximation, the mass of the $K^*(888)$ meson is given by

$$m_{K^*(888)} = m_\rho + m_\pi. \tag{6}$$

This relation was derived by using a mass transformation recently obtained by Wick.[10] From Eqs. (4) and (6), we find

$$m_{K^*(888)} \cong 3m_\pi + 2\kappa. \tag{7}$$

It has been noted by Takabayasi[11] that the value of κ is essentially $\frac{1}{4}m_N$, where $m_N \equiv \frac{1}{2}(m_p + m_n)$. Thus $\kappa \equiv 234.7$ MeV. If we regard the $Y_0^*(1815)$ state as the combination[2] of a nucleon and a $\bar{K}^*(888)$ particle, we can write for its mass

$$m_{Y_0^*(1815)} \cong m_N + m_{K^*(888)} \cong 3m_\pi + 6\kappa. \tag{8}$$

Takabayasi[11] has also pointed out that the difference $m_{Y_0^*(1815)} - m_\Lambda$ is closely given by 3κ, and that $m_{Y_0^*(1405)} \cong 6\kappa$. Upon using these results, and the mass differences shown in Fig. 1 of reference 3, one obtains the following expressions for the various Y_0^* and Y_1^* states, and for the Λ and the ABC particle:

$$m_\Lambda = 3m_\pi + 3\kappa, \tag{9}$$

$$m_{Y_1^*(1385)} = 5m_\pi + 3\kappa, \tag{10}$$

$$m_{Y_0^*(1520)} = 6m_\pi + 3\kappa, \tag{11}$$

$$m_{Y_1^*(1660)} = 7m_\pi + 3\kappa, \tag{12}$$

$$m_{\mathrm{ABC}} = m_{Y_0^*(1815)} - m_{Y_0^*(1520)} = 3\kappa - 3m_\pi. \tag{13}$$

Upon using Eq. (4) for m_ρ, one can obtain expressions for the nucleon isobars $N_{1/2}^*(1688 \pm 3)$, $N_{3/2}^*(1650 \pm 25)$, and $N_{3/2}^*(2360 \pm 25)$, by means of the Kycia-Riley scheme.[1] We have also found the following empirical relations:

$$m_K + m_{K^*(725)} = m_B, \tag{14}$$

$$m_\Sigma + m_\eta + m_\pi = m_\Sigma + 5m_\pi = 2m_N, \tag{15}$$

$$m_\Xi + m_\varphi + m_\pi = 2m_{N_{3/2}^*(1238)}. \tag{16}$$

These relations enable us to obtain expressions for the mass of Σ, Ξ, K, and $K^*(725)$. We note that Eq. (15) was obtained by using the transformation of Wick[10] mentioned above.

All of the expressions for m obtained here are of the following general form:

$$m = pm_\pi + q\kappa, \tag{17}$$

where p and q are integers. Thus p and q are in the nature of quantum numbers pertaining to the mass formula. It should be pointed out that the masses of all of the well-known particles and resonance states can be expressed in the form of Eq. (17). The details of the derivation of the p and q values, and the relation of p and q to the quantum numbers I, J, B, and S of the particles will be discussed in a forthcoming paper.[12]

For comparison with experiment, Table I gives the experimental mass values and the values calculated from Eq. (17) for some of the particles. The estimated uncertainties of the experimental values[13] have also been indicated. It is seen that the differences between calculated and experimental values are generally less than the experi-

Table I. Mass values and (p,q) assignments for some of the mesonic resonances and baryon states. (All mass values are in MeV.)

Particle	(p,q)	m_{exp}	m_{calc}
η	(4, 0)	550 ± 2	549.2
f	(4, 3)	1255 ± 5	1253.3
ABC	(-3, 3)	~290	292.2
ρ	(2, 2)	750 ± 5	744.0
B	(2, 4)	1220 ± 10	1213.4
K	(7, -2)	495.9 ± 0.6	491.7
$K^*(888)$	(3, 2)	888 ± 3	881.3
$N_{3/2}^*(1238)$	(9, 0)	1238 ± 2	1235.7
$N_{1/2}^*(1512)$	(11, 0)	1512 ± 2	1510.3
$N_{1/2}^*(1688)$	(2, 6)	1688 ± 3	1682.8
$N_{3/2}^*(1920)$	(14, 0)	1920 ± 15	1922.2
Λ	(3, 3)	1115.4 ± 0.15	1116.0
Σ	(7, 1)	1193.4 ± 0.3	1195.8
Ξ	(13, -2)	1319 ± 2	1315.5
$Y_1^*(1385)$	(5, 3)	1385 ± 5	1390.6
$Y_0^*(1815)$	(3, 6)	1815 ± 20	1820.1

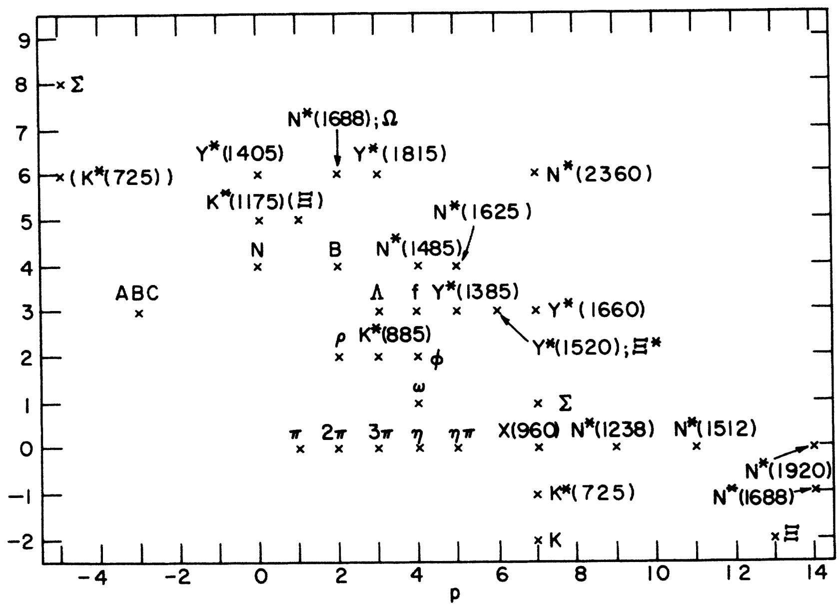

FIG. 1. Plot of the strongly interacting particles in terms of the values of the coefficients p and q of Eq. (17).

mental errors, and in no case does the difference exceed 7 MeV.

Figure 1 shows a plot of the strongly interacting particles in terms of the coefficients p and q of Eq. (17). This figure[14] also includes the "quasiparticles" 2π, 3π, and $\eta\pi$, which have been used by Kycia, Riley, and Sternheimer[1-3] in a classification of the baryon isobars. Figure 1 shows the existence of 12 sequences of particles, where a sequence is defined as a group of three or more particles with the same mass spacing Δm. The values of Δm are nm_π, $n\kappa$ $(n=1,2,3)$, and $\Delta m = \kappa - m_\pi$. These sequences interrelate the masses of the mesons and baryons. For a few cases, two alternative (p,q) assignments are shown.

A statistical study has been made to determine the probability P that a random distribution of masses would give the same agreement with Eq. (17) as the actual experimental values. As would be expected, this probability is negligibly small $(P=6\times10^{-4})$.

The model from which the preceding equations might be derived is somewhat different from the compound model of references 1-3. In these papers, it was assumed that the isobars are essentially compound particles (in the manner of a quasinucleus) with very small binding energy. As a result, the isotopic spin, baryon number, and strangeness could be deduced from the properties of the constituent particles. In the present work, one no longer has the direct concept of a compound particle, while at the same time retaining the property that the mass can be expressed as the sum of an integral multiple of smaller mass units (e.g., m_π and κ).

We have also obtained other mass relations of the type discussed above. As examples, we mention the following:

$$2m_K + m_{K^*(888)} = 2m_N, \tag{18}$$

$$3m_K = m_N + m_\eta = 4\kappa + 4m_\pi, \tag{19}$$

both of which are very accurately satisfied (within $\lesssim 2$ MeV).

Somewhat independently of the preceding results, it has been found that for several pairs of strongly interacting particles (a,b), we have the relation

$$m_b = \lambda m_a, \tag{20}$$

where λ is a simple fraction (rational number). The accuracy with which Eq. (20) is satisfied is very good in most of the cases; e.g., $m_{\Xi_{1/2}^*(1530)} = \frac{3}{2}m_\varphi$; $m_{K^*(725)} = \frac{5}{2}m_{\mathrm{ABC}}$; $m_{N_{3/2}^*(1238)} = \frac{5}{2}m_K$; $m_{N_{3/2}^*(1920)} = \frac{7}{2}m_\eta$. In connection with the Λ mass, one can apply the relation (20) twice in succession (with $\lambda = \frac{3}{2}$), so as to obtain

$$m_\Lambda = \tfrac{3}{2}m_\rho = (9/4)m_K. \tag{21}$$

The relation $m_\Lambda/m_K = 9/4$ holds to excellent accuracy ($m_K = 496$ MeV, $m_\Lambda = 1115.4$ MeV). A similar relation can also be obtained for $\lambda = \frac{5}{2}$, namely

$$m_{Y_0^*(1815)} = \tfrac{5}{2} m_{K^*(725)} = (25/4) m_{\mathrm{ABC}}. \qquad (22)$$

Upon using $m_{\mathrm{ABC}} = 290$ MeV,[15] the right-hand side becomes 1812.5 MeV, in very good agreement with the experimental value (≈1815 MeV).

A more detailed account of the present work will be given in a later paper.

I wish to thank Dr. G. C. Wick for informing me of his results prior to publication, and for suggesting their possible use in connection with the mass relations.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

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[7]This equation for m_ρ was also proposed by Takabayasi [see reference 11, Eq. (28)].

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EMPIRICAL PION-NUCLEON SPECTROSCOPY

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The aim of this Letter is to show simple connections existing between the presently known pion-nucleon resonances (for simplicity, we call "resonances" all the maxima of the $\pi^\pm p$ cross sections). If these relationships have a wider validity, they enable us to predict where to expect other resonances and what are their angular momenta.

The different shapes of the maxima in the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ cross sections [apart from the isobar $\Delta(\frac{3}{2},\frac{3}{2})$] suggests that they could have a different nature. The simplest hypothesis is to consider the $T=\frac{1}{2}$ resonances as excited levels of the nucleon N and the $T=\frac{3}{2}$ resonances as excited levels of the isobar Δ. The general idea of this Letter is to make the isobar play a role similar to that of the nucleon.

In Table I, in addition to nucleon[1] N and isobar Δ, we have written the seven known resonances with their masses M_i ($i=2T$), the corresponding momenta P_π of incident pions, and their angular momenta J_i if known. We have labelled the columns listing the resonances by $\lambda = 1, 2, 3, \cdots$, in order of increasing mass; when J is known we find that

$$\lambda = J - T + 1, \qquad (1)$$

the same relationship as stated by Kycia and