$3 \leq q^2 \leq 5$,

$$G_{Mn}/\mu_n \approx G_{Mp}/\mu_p \approx G_{Ep}, \quad (1)$$

$$G_{En} \approx 0. \quad (2)$$

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BROKEN SYMMETRIES AND WEAK INTERACTIONS*

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The observed dominance of $\Delta T = \frac{1}{2}$ processes in nonleptonic decays has been related[1] to the massive nature of the particles associated with the Z field, the charged vector field of weak interactions. The dynamical mechanism can be represented by the nonvanishing vacuum expectation value (vacuon) of the $T = \frac{1}{2}$, $Q = 0$, $CP = 1$ component of the phenomenological fields attached to 0^- particles (Φ), and to 0^+ particles (S). The pseudoscalar and scalar vacuons produce, respectively, parity-violating and parity-preserving $\Delta T = \frac{1}{2}$, $|\Delta Y| = 1$ mixing of baryon fields and of meson fields, which initiates the decays. Thus the pseudoscalar vacuon ($\langle\Phi_{23}\rangle = -\langle\Phi_{32}\rangle$) couples the vector field of K^* with the pseudovector field of π. This implies $K_1{}^0 \to \pi + \pi$ and also the s-wave part of hyperon pion decay. An analogous λ_5 mixing of baryon fields will not exist if the baryon coupling to 0^- mesons is pseudovector in form rather than pseudoscalar.

The p-wave part of hyperon pion decay and $K \to 3\pi$ should be explained analogously by scalar vacuon ($\langle S_{23}\rangle = \langle S_{32}\rangle$) mixing of K and π, K^* and ρ, and various baryons. For example, the decay $\Lambda \to N + \pi$ can occur either through the strong π coupling, $\Lambda \to \Sigma + \pi$, followed by the vacuon baryon mixing $\Sigma \to N$ and by the baryon mixing $\Lambda \to N$ followed by the strong interaction $N \to N + \pi$, or through the strong K coupling, $\Lambda \to N + \bar{K}$, followed by the vacuon meson mixing $\bar{K} \to \pi$. If, however, the breakdown of SU(3) symmetry is limited to the mass displacement described by the vacuon $\langle S_{33}\rangle$, no such decays occur, since the complete effect of $\langle S_{23}\rangle$ can be eliminated by a unitary transformation.[2] In the example of $\Lambda \to N + \pi$ the two contributions cancel in virtue of the coupling-constant equality, for π and K coupling with baryons, that expresses SU(3) symmetry. It is the failure of that equality, $f_K \neq f_\pi$, and of similar coupling-constant equalities that permits the scalar-vacuon mechanism to operate and to cause $\Delta T = \frac{1}{2}$ parity-conserving decays.

The idea of partially suppressed parity-conserving decays receives some support from the analysis of pionic hyperon decays. An essential aspect of the phenomena, which is omitted by the processes we have described, can be attributed to an effect that would be negligible were not the principal mechanism largely self-cancelling. This effect is the mixing of different SU(3) representations associated with the broken W_3 scheme. The baryon decays act to increase hypercharge. The parity-violating processes are described as $\Xi\to\Lambda+\bar{K}^*$, $\Sigma\to N+\bar{K}^*$, $\Lambda\to N+\bar{K}^*$, followed by $\bar{K}^*\to\pi$, which means $\bar{K}^{*0}\to\pi^0$ and $\bar{K}^{*-}\to\pi^-$. Positively charged pions cannot be produced and the s-wave decay $\Sigma^+\to N^0+\pi^+$ is forbidden. The analogous parity-preserving processes, $\Xi\to\Lambda+\bar{K}$, $\Sigma\to N+\bar{K}$, $\Lambda\to N+\bar{K}$, and $\bar{K}\to\pi$, also have this property. So does mixing of the baryon octuplet, which must produce the same processes as $\bar{K}$-π mixing since the two are capable of cancelling. Hence another p-wave mechanism is at work to generate $\Sigma^+\to N^0+\pi^+$ and extinguish the p-wave decay $\Sigma^-\to N^0+\pi^-$.

The ninth baryon Y^0, an SU(3) singlet, is mixed with N^0 by the weak scalar vacuon. There is also a strong coupling of Y^0 to the $T=0$ combination of Σ and π. The result is $\Sigma^+\to N^0+\pi^+$ and a contribution of equal magnitude to $\Sigma^-\to N^0+\pi^-$ which must cancel approximately the residual effect of the scalar vacuon within the baryon octuplet. This explanation of the structure of Σ decays connects the extent to which SU(3) coupling-constant relations are violated with the magnitude of the mass splitting that characterizes the breakdown of W_3 symmetry. There are, unfortunately, no reliable determinations of the K-baryon coupling constants. A large fractional deviation of the K and π coupling constants would not be expected if W_3 symmetry is more seriously broken than SU(3) symmetry. As a specific model we assume that the mass of the ninth baryon (in the algebraic sense that includes intrinsic parity) lies 2.34 BeV below the nucleon mass, so that Y^0 is the $\frac{1}{2}^-$ particle $Y_0^*(1405)$. In this model, singlet-octuplet mixing can account for the 8-MeV displacement of the Λ mass above the mass computed from $(\frac{1}{3})(2N+2\Xi-\Sigma)$.

Calculations.—The coupling of 1^- mesons to the baryon octuplet is represented by[4]

$$g_{U\Psi}^{(1)}\mathrm{Tr}\bar{\Psi}\gamma_\mu U^\mu\Psi+g_{U\Psi}^{(2)}\mathrm{Tr}\bar{\Psi}\gamma_\mu\Psi U^\mu,$$

and the analogous coupling to the 0^- meson octuplet is

$$g_{U\Phi}i\mathrm{Tr}(\Phi^\mu U_\mu\Phi-\Phi^\mu\Phi U_\mu).$$

The latter also produces the weak mixing of 1^- and 0^- mesons through the pseudoscalar vacuon mechanism,

$$g_{U\Phi}(-i\langle\Phi_{23}\rangle)([\Phi^\mu,U_\mu]_{23}-[\Phi^\mu,U_\mu]_{32}).$$

The implied weak coupling of π^- to baryon pairs illustrated by $f_{\Sigma^-}^{(s)}(1/m_\pi)\pi_\mu^-\bar{\Sigma}^+Y^\mu N^0$ is described by the coupling constants

$$f_{\Sigma^-}^{(s)}=\theta_-g_{U\Psi}^{(2)},$$

$$f_\Lambda^{(s)}=\theta_-6^{-1/2}(2g_{U\Psi}^{(1)}-g_{U\Psi}^{(2)}),$$

$$f_{\Xi^-}^{(s)}=\theta_-6^{-1/2}(2g_{U\Psi}^{(2)}-g_{U\Psi}^{(1)}),$$

where

$$\theta_-=g_{U\Phi}(m_\pi/m_{K^*}^2)(-i\langle\Phi_{23}\rangle).$$

These constants obey the sum rule[5]

$$2f_{\Xi^-}^{(s)}+f_\Lambda^{(s)}=(\tfrac{3}{2})^{1/2}f_{\Sigma^-}^{(s)}.$$

Numerical values obtained from the data[6] are $f_{\Sigma^-,\Lambda,\Xi^-}^{(s)}=(2.3,\ -2.7,\ 3.0)\times10^{-7}$. The sum rule is approximately obeyed. The comparison of $f_{\Sigma^-}^{(s)}$ and $f_\Lambda^{(s)}$ gives

$$-g_{U\Psi}^{(2)}/g_{U\Psi}^{(1)}\simeq1.1,$$

while $f_{\Sigma^-}^{(s)}$ and $f_{\Xi^-}^{(s)}$ require a ratio ~0.8. The analogous boson decay coupling can be reduced to

$$\theta_-g_{U\Phi}(1/m_\pi)(m_K^2-m_\pi^2)2^{1/2}K_1^0[\pi^-\pi^+-\tfrac{1}{2}(\pi^0)^2].$$

On comparing the empirical $K_1^0\pi^+\pi^-$ coupling constant with $f_{\Sigma^-}^{(s)}$, we obtain (in magnitude)

$$-g_{U\Phi}/g_{U\Psi}^{(2)}\simeq0.75.$$

Thus, the coupling constants $g_{U\Phi}$, $g_{U\Psi}^{(1)}$, and $-g_{U\Psi}^{(2)}$ are all equal, to within roughly 25%, and $\theta_-\sim\frac{1}{2}\times10^{-7}$.

The nine baryons represented by the fields Ψ, Y^0 are coupled to the spin-0 fields Φ and S through

$$(1/m_\pi)\Big[f_{\Phi\Psi}^{(1)}\mathrm{Tr}\bar{\Psi}\gamma_\mu i\gamma_5\Phi^\mu\Psi+f_{\Phi\Psi}^{(2)}\mathrm{Tr}\bar{\Psi}\gamma_\mu i\gamma_5\Psi\Phi^\mu+3^{-1/2}f_{\Phi Y}(\mathrm{Tr}\bar{\Psi}\Phi^\mu\gamma_\mu Y^0+\bar{Y}^0\gamma_\mu\mathrm{Tr}\Phi^\mu\Psi)\Big]$$

and

$$g_{S\Psi}^{(1)}\mathrm{Tr}\bar{\Psi}S\Psi+g_{S\Psi}^{(2)}\mathrm{Tr}\bar{\Psi}\Psi S+3^{-1/2}g_{SY}(\mathrm{Tr}\bar{\Psi}Si\gamma_5Y^0-\bar{Y}^0i\gamma_5\mathrm{Tr}S\Psi).$$

The scalar vacuon $\langle S_{33}\rangle$ is related to mass displacements within the baryon octuplet by

$$-g_{S\Psi}^{(1)}\langle S_{33}\rangle = \Xi - \Sigma = 125 \text{ MeV},$$

$$g_{S\Psi}^{(2)}\langle S_{33}\rangle = \Sigma - N = 250 \text{ MeV},$$

so that

$$-g_{S\Psi}^{(2)}/g_{S\Psi}^{(1)} = 2,$$

while singlet-octuplet mixing gives

$$\Lambda - \tfrac{1}{3}(2N + 2\Xi - \Sigma) = 2(\tfrac{1}{3}g_{SY}\langle S_{33}\rangle)^2/(\Lambda + Y^0)$$

or in magnitude,

$$g_{SY}\langle S_{33}\rangle = 300 \text{ MeV},$$

$$g_{SY}/g_{S\Psi}^{(2)} = 1.2.$$

The pion-nucleon coupling constant, $(f_{\pi NN})^2/4\pi = 0.08$, is obtained as $f_{\pi NN} = 2^{-1/2}f_{\Phi\Psi}^{(1)}$. The phenomenological analysis of nucleon-hyperon interactions suggests[7] that $f_{\pi\Sigma\Sigma}$ is about $\frac{1}{3}$ of $f_{\pi NN}$, or that

$$f_{\Phi\Psi}^{(2)}/f_{\Phi\Psi}^{(1)} \sim \tfrac{2}{3}.$$

The width of $Y_0^*(1405)$, $\Gamma = 50$ MeV, supplies the magnitude

$$f_{\Phi Y}/f_{\Phi\Psi}^{(1)} = 1.25.$$

The coupling constant that connects S with the 0^- meson octuplet, in $k_{S\Phi}\mathrm{Tr}\Phi S\Phi$, is determined by

$$-k_{S\Phi}\langle S_{33}\rangle = m_K^2 - m_\pi^2.$$

The weak scalar vacuon $\langle S_{33}\rangle$ produces mixings of the baryon fields and of the 0^- meson fields. The consequence is single-pion decay of hyperons, as illustrated by the coupling $f_{\Sigma^+}^{(p)}(1/m_\pi \times \pi_\mu{}^+\bar{\Sigma}^-\gamma^\mu i\gamma_5 N^0$. The coupling constants for the various charged-pion decays are given by

$$f_{\Sigma^+}^{(p)} = -\tfrac{1}{3}\theta_+ f_{\Phi Y} r,$$

$$f_{\Sigma^-}^{(p)} = \theta_+(\Delta f_{\Phi\Psi}^{(2)} - \tfrac{1}{3}f_{\Phi Y} r),$$

$$f_{\Lambda}^{(p)} = \theta_+ 6^{-1/2}(2\Delta f_{\Phi\Psi}^{(1)} - \Delta f_{\Phi\Psi}^{(2)}),$$

$$f_{\Xi^-}^{(p)} = \theta_+ 6^{-1/2}(2\Delta f_{\Phi\Psi}^{(2)} - \Delta f_{\Phi\Psi}^{(1)}),$$

where

$$\theta_+ = S_{23}/S_{33},$$

$$r = g_{SY}S_{33}/(Y^0 + N) = 0.13,$$

and

$$\Delta f_{\Phi\Psi} = f_{\pi\Psi} - f_{K\Psi}$$

expresses the violation of SU(3) coupling-constant relations. The p-wave (and s-wave) constants obey the sum rule

$$2f_{\Xi^-} + f_\Lambda = (\tfrac{3}{2})^{1/2}(f_{\Sigma^-} - f_{\Sigma^+}).$$

In virtue of the pure p-wave decay of Σ^+, the decay of Σ^- must be predominantly into the s-wave channel, whence

$$\Delta f_{\Phi\Psi}^{(2)}/f_{\Phi\Psi}^{(2)} \simeq \tfrac{1}{3}(f_{\Phi Y}/f_{\Phi\Psi}^{(2)})r = 0.08.$$

The experimental data are represented by $f_{\Sigma^+,\Lambda,\Xi^-}^{(p)} = (2.8, -1.5, -1.0)\times 10^{-7}$. The sum rule is satisfactorily obeyed and one obtains the well-defined ratio

$$\Delta f_{\Phi\Psi}^{(2)}/\Delta f_{\Phi\Psi}^{(1)} = 0.88.$$

It seems that SU(3) symmetry is somewhat better preserved for $f_{\Phi\Psi}^{(1)}$ than $f_{\Phi\Psi}^{(2)}$,

$$\Delta f_{\Phi\Psi}^{(1)}/f_{\Phi\Psi}^{(1)} \sim 0.06.$$

The relative magnitude of the weak and strong scalar vacuons is obtained as $\theta_+ = 3.7\times 10^{-6}$.

Two mechanisms are involved in the three-pion disintegration of the K particle. The dominant one combines a strong scalar $(\Phi^2)^2$ coupling with weak K-π mixing. The other is responsible for the energy dependence of the effective coupling constant. It uses 1^- mesons in the sequence $K \to \pi + K^*$, $K^* \to \rho$, $\rho \to \pi + \pi$, or the alternatives $K \to K + \rho$, $K \to \pi$, $\rho \to \pi + \pi$, and $K \to \pi$, $\pi \to \pi + \rho$, $\rho \to \pi + \pi$. The observed magnitude of the energy dependence in τ decay gives the fractional deviation between the coupling constants $g_{K^*\Phi}$ and $g_{\rho\Phi}$ as 2.7%. A similar order of magnitude is found for the fractional deviation between the $\bar{K}K\pi^2$ and $(\pi^2)^2$ coupling constants if one employs the value usually cited for the latter.

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EMPIRICAL SYSTEMATICS OF THE STRONGLY INTERACTING PARTICLES*

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In a recent Letter,[1] we have proposed a mass formula for the strongly interacting particles which has been found to be in good agreement with the mass values of all of the presently known particles and resonance states. This formula is given by

$$m = pm_\pi + q\kappa, \qquad (1)$$

where $m_\pi \equiv \frac{1}{2}(m_{\pi^+}+m_{\pi^0}) = 137.3$ MeV, and $\kappa \equiv \frac{1}{4}m_N = \frac{1}{8}(m_p+m_n) = 234.7$ MeV; p and q are integers which become negative in some cases. The constant κ has been previously introduced by Takabayasi and Ohnuki.[2] By means of Eq. (1), one can plot the various particles on a graph of q vs p. Such a plot is shown in Fig. 1 of reference 1.

As a special case of Eq. (1), there are eight observed particle states for which $m = pm_\pi$ ($p > 1$), i.e., $q=0$. These states are as follows: η [$p=4$], X(960) [7], A_1(1090) [8], $N_{3/2}^*$(1238) [9], $N_{1/2}^*$(1512) [11], $N_{1/2}^*$(1647) [12], $N_{3/2}^*$(1922) [14], and $N_{1/2}^*$(2197) [16], where the number in square brackets gives the value of p. We note that A_1 is the recently discovered $\pi\rho$ resonance,[3] and $N_{1/2}^*$(1647) is the possible resonance[4] in the reaction $\pi^- + p \to \Lambda + K^0$ at a pion energy $T_\pi = 839$ MeV. It is noteworthy that all eight states have strangeness $S=0$. Since the number of particles with $S=0$ represents about 60% of all particles, the probability that a random distribution of states would give the observed correlation with $S=0$ is $(0.6)^8 = 0.017$.

It was already noticed in our previous work[1] that it is essential to use the average pion mass for m_π in Eq. (1), i.e., $m_\pi \equiv \frac{1}{2}(m_{\pi^+}+m_{\pi^0})$. Thus if m_{π^+} or m_{π^0} were used instead of m_π, the simple relation $m = pm_\pi$ (p = integer) would not for those states for which m is known accurately, i.e., η, X(960), $N_{3/2}^*$(1238), $N_{1/2}^*$(1512), and $N_{3/2}^*$(1922).

It has been pointed out by Yang[5] that this result can be tested more accurately by taking the known experimental mass values $m_{\rm exp}$, and calculating the residue modulo M_π, i.e., $X \equiv (m_{\rm exp}/M_\pi) - N$, where N is the largest integer $\leq (m_{\rm exp}/M_\pi)$. We have used a rectangle of width $2\delta m_{\rm exp}/M_\pi$ for each particle, where $\delta m_{\rm exp}$ is the estimated uncertainty of $m_{\rm exp}$. The height of the rectangle is taken as $AM_\pi/\delta m_{\rm exp}$, where A is a constant, so as to give equal weight (equal area) to all particles. The sum of all rectangles gives the particle density ρ_p as a function of X. In the test, one uses for M_π the average pion mass $m_\pi = 137.3$ MeV, and a few values near m_π, e.g., m_{π^+} and m_{π^0}. One expects that for $M_\pi = m_\pi$, there will be a sharp and narrow maximum of ρ_p near $X=0$, whereas if M_π differs from m_π by even a small amount (~1-2 MeV), the maximum will rapidly disappear. The maximum is, of course, due to the eight particles for which $m = pm_\pi$.

The particles considered, and the values of $m_{\rm exp}$ and $\delta m_{\rm exp}$, are as follows: $ABC(290\pm10)$, $\sigma(379\pm4)$,[6] $\eta(549\pm2)$, $K^*(725\pm5)$, $\rho(750\pm5)$, $\omega(782\pm2)$, $K^*(888\pm3)$, $X(960\pm4)$, $\varphi(1019\pm2)$, $A_1(1090\pm15)$,[3] $K^*(1175\pm5)$,[7] $B(1220\pm10)$, $N_{3/2}^*(1238\pm2)$, $f(1255\pm5)$, $A_2(1310\pm15)$,[3] $\Xi(1319\pm2)$, $Y_1^*(1385\pm5)$, $Y_0^*(1405\pm5)$, $(\bar{K}K\pi)(1410\pm15)$,[8] $N_{1/2}^*(1485\pm5)$,[9] $N_{1/2}^*(1512\pm2)$, $Y_0^*(1520\pm5)$, $\Xi_{1/2}^*(1532\pm3)$, $N_{3/2}^*(1625\pm25)$,[10] $N_{1/2}^*$(1647