LONG-RANGE MAGNETIC COUPLING IN METALS

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Between two magnetic ions embedded in a normal metal an indirect exchange coupling exists. This well-known Ruderman-Kittel interaction has oscillating features. The theory of this interaction is founded on a second-order perturbation treatment of the free electron gas. Recent measurements[1-4] of coupling between ferromagnetic slabs through a thin film of nonmagnetic metal have exhibited nonoscillating interactions up to distances of several hundred angströms. In this paper we demonstrate how the existence of bound electronic states in the intermediate metal can lead to such nonoscillating couplings. This effect is due to matching of boundary conditions and thus cannot be obtained by usual perturbation techniques.

We give here an extremely simplified version of our model, which is sufficient, however, to show its chief features. We consider the contact between a ferromagnetic metal (F) and a normal metal (N) along the plane $z = 0$. (N) is characterized by a single parabolic band for both spin directions (+ and −).

The bands of (F) are split by an exchange interaction (with d electrons, for instance) (see Fig. 1).

For simplicity we take the same effective mass for all electrons, and the (−) band of (F) identical with the bands of (N).

The electron Hamiltonian in (F) is

$$H_0 = (\hbar^2/2m)k^2 + \eta\vec{M}\cdot\vec{\sigma} + \text{const.} \qquad (1)$$

$\vec{\sigma}$ = Pauli spin operator, and ηM gives the strength of the exchange interaction $[\eta M = (\hbar^2/2m)\alpha^2]$. The constant is introduced to adjust the bottom of the bands, as is indicated in Fig. 1, so that the (−) electrons are not affected by the contact. The wave functions for the (+) electrons are

(F): $\Psi_F(x,y,z)$

$$= [A\exp(ikz) + B\exp(-ikz)]\exp(-i\vec{k}_{\parallel}\cdot\vec{\rho}_{\parallel}); \qquad (2)$$

(N): $\Psi_N(x,y,z)$

$$= [C\exp(ik'z) + D\exp(-ik'z)]\exp(i\vec{k}_{\parallel}\cdot\vec{\rho}_{\parallel}). \qquad (3)$$

Taking into account the conservation of energy and of $k_{\parallel}$ we get the following relation between
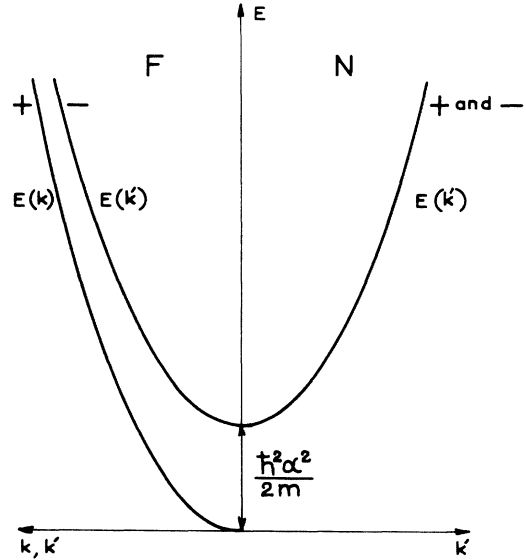

FIG. 1. $E(k) = (\hbar^2/2m)\vec{k}^2$, $\vec{k} = (k, \vec{k}_{\parallel})$;

$E(k') = (\hbar^2/2m)(\vec{k}'^2 + \alpha^2)$, $\vec{k}' = (k'; \vec{k}_{\parallel})$;

where k and k' are the projections of the vectors $\vec{k}$ and $\vec{k}'$ in the direction OZ; $\vec{k}_{\parallel}$ is the projection of $\vec{k}$ or $\vec{k}'$ on the plane $z = 0$; and $(\hbar^2\alpha^2/2m)$ is the exchange energy in (F).

k and k':

$$k' = (k^2 - \alpha^2)^{1/2}, \qquad (4)$$

where k' is real only when $k \geq \alpha$.

When $k < \alpha$ the electronic waves are evanescing: $\exp(-\kappa'z)$, where the attenuation factor $\kappa' = ik'$. The essential remark is the following: Among all the evanescing waves those for which $k \approx \alpha$ have a very small attenuation factor κ'. For a given value of the energy E, this will happen for all the waves such that

$$k_{\parallel}^2 = (2m/\hbar^2)E - k^2. \qquad (5)$$

When E is varied from 0 to E_F, one thus obtains a two-fold multiplicity of such waves whose composition gives an electronic density, slowly decreasing with the distance from the contact. More exactly, the density at a point z in (N) due to all evanescing waves is

$$\rho(z) = \tfrac{1}{2}\pi^2 \int_0^\alpha \exp(-2\kappa'z)(k^2/\alpha^2)(k_F^2 - k^2)dk. \qquad (6)$$

The asymptotic value of this integral for large z and for $\alpha z > 1$ is

$$\rho(z) = \frac{1}{8\pi^2} \frac{k_F^2 - \alpha^2}{\alpha} \frac{1}{z^2}. \tag{7}$$

The usual Ruderman-Kittel terms are included in the nonevanescing waves $(k > \alpha)$. The destructive interferences of the oscillating waves, due to the physical irregularities (roughness) of the contact, reduce rapidly to zero the contributions of the nonevanescing waves. On the contrary, the evanescing terms which have a constant sign give a slowly decreasing polarization of spin $(+)$, which appears as a tail in the polarization of (F).

Let us now introduce at z a ferromagnetic probe (S) of volume v, with the same Hamiltonian as (F). If v is small enough, so as not to create bound states in (N), the coupling energy W, with the electrons described in (7), may be obtained by first-order perturbation.

$$W = -\rho(z)(\hbar^2\alpha^2/2m)v \cos\theta. \tag{8}$$

θ = angle between the magnetization $\vec{M}v$ in (S) with that of (F), with the following numerical values: α = interatomic distance = 3 Å; $v = d^3$; $k_F d = 2$; $z = 30d$; and $E_F = 5$ eV. One obtains

$$W = 10^{-4}(\eta M/E_F)^{1/2} \text{ eV}. \tag{9}$$

The ratio between E_F and the exchange energy

ηM is not well-known; with a reasonable estimate $\eta M/E_F \approx 10^{-2}$, one gets $W = 10^{-5}$ eV, which corresponds to the coupling of one Bohr magneton with a field of 2000 Oe.

Of course, the model considered gives the maximum possible interaction. Usually the $(-)$ electrons will also have a tail in (N), whose effect is to oppose that of $(+)$ electrons.

Another way of estimating the strength of this coupling is to fill the space from z to infinity with probes like (S) (neglecting the fact, which will be considered elsewhere, that now bound states are associated with the probes). With the same numerical values one obtains $W \approx 0.6$ erg cm^{-2}, which is to be compared with the results quoted in references 1-4 which are in the range of 10^{-2} to 10^{-1} erg cm^{-2}.

A final remark: This coupling makes use of electrons of all energies between 0 and E_F, and thus should be independent of temperature and mean free path effects.

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[1]O. Massenet, thesis, Université de Grenoble, Grenoble, France, 1963 (unpublished).

[2]J. C. Bruyere, O. Massenet, R. Montmory, and L. Néel, Compt. Rend. 258, 841 (1964).

[3]J. C. Bruyere, O. Massenet, R. Montmory, and L. Néel, Compt. Rend. 258, 1423 (1964).

[4]O. Massenet and R. Montmory, Compt. Rend. 258, 1752 (1964).

OBSERVATIONS OF FINE STRUCTURE IN ISOBARIC ANALOG RESONANCES*

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Isobaric analog states have been previously identified as resonances of the compound system in proton reactions and scattering.[1] Current theoretical interest in the details of these resonances have led us to remeasure, in very fine detail, two resonances in Mo^{92} + p that are isobaric analogs of the first and second excited states of Mo^{93}.[2] The resonances occur at 5.3-MeV and 5.9-MeV proton energy and are s and d wave, respectively. Much of the theoretical interest is concerned with the question of whether or not the isobaric analog resonances characterized by isobaric spin quantum number T = $T_{target} + \frac{1}{2}$ are effectively spread due to a possible mixing with the more abundant states with

$T = T_{target} - \frac{1}{2}$. The proton resonances selected are particularly well suited for this study since they occur below the neutron threshold and the other possible reactions (e.g., p, α and p, γ) are comparatively unimportant. Our previous analysis of the 5.3-MeV resonance[3] was based on elastic-scattering data taken at only one angle with a relatively thick target. We have remeasured this resonance and the 5.9-MeV resonance at three angles using thin targets and very fine energy resolution. We find that there is considerable fine structure in the vicinity of the resonance, becoming much less prominent away from resonance. In addition, we are able to fit the observed resonance shape (after averaging the data