REMARKABLE EFFECT OF MANY-PARTICLE COLLISIONS ON TRANSPORT PROPERTIES OF NONDEGENERATE PLASMAS

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As the preliminary study[1] has shown, the usual expression for the mean free path (mfp) of electrons in nondegenerate plasmas (and accordingly most other transport coefficients related to the dissipative nature of the system such as diffusion, viscosity, and thermal and electrical conductivity) is not appropriate for describing the actual state of affairs which is realized in nondegenerate plasmas. Almost all theories on transport properties so far developed have been based on the assumption of binary collisions, i.e., for sufficiently high-temperature plasmas the inelastic scattering between a (test) particle and the medium (electron gas) has been treated within the Born approximation [see Fig. 1(a)].

We first review the result of the usual theory following the language of the thermal Green's function.[2] Denoting the effective scattering cross section corresponding to the process in Fig. 1(a) by $\Gamma(p,q)dq$, in which one electron with momentum p is scattered to $p-q$ within a range dq, then[1]

$$\Gamma(p,q)=\frac{1}{\hbar}\mathrm{Im}\left[\beta^{-1}\sum_{\omega}G_0(p-q,\epsilon-\omega)\frac{v(q)}{\epsilon(q,\omega)}\right], \quad (1)$$

where β is the inverse temperature $1/\kappa T$; $G_0(p,\epsilon)=[\epsilon-\epsilon_0(p)]^{-1}$, the noninteracting Green's function; $v(q)=4\pi e^2\hbar^2/q^2$, the bare Coulomb interaction; and $\epsilon(q,\omega)$ is the wave number and frequency-dependent dielectric constant at finite temperatures;

$$\epsilon(q,\omega)=1+\hbar^2 v(q)\sum_{p}\frac{f(p+q)-f(p)}{\omega-\epsilon_0(p+q)+\epsilon_0(p)}.$$

Here $\epsilon_0(p)=p^2/2m$ and $f(p)$, the Fermi distribution function, reduces to the Maxwell distribution in the nondegenerate limit. In Eq. (1) $\omega=2\pi ir/\beta$ and the summation is taken over all integers r.

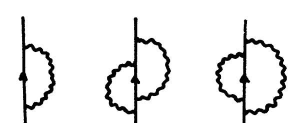

FIG. 1. Diagrams for the one-particle scattering process. Solid lines indicate propagation of electrons (or holes) and wavy lines correspond to the dynamically screened Coulomb interaction.

In order to obtain, for example, the mfp l_0, it is necessary to evaluate the average value of the energy transfer, $\epsilon_0(p)-\epsilon_0(p-q)$, over the scattering probability (1); thus

$$l_0^{-1}=\frac{m}{p}\sum_{q}\left\{\frac{\epsilon_0(p)-\epsilon_0(p-q)}{\epsilon_0(p)}\right\}\Gamma(p,q). \quad (2)$$

Consider first of all the scattering of one of the plasma electrons. Equation (2) gives

$$l_0^{-1}=\frac{4(2\pi)^{1/2}ne^4}{m^2 v_T v^3}\ln\frac{k_{\max}}{k_D}, \quad (3)$$

where n is the number density, $v=p/m$ the velocity of the particle, $v_T=(m\beta)^{-1/2}$, $k_D=(4\pi ne^2\beta)^{1/2}$, and $k_{\max}$ is to be taken as $mv_T/\hbar$ or $(\beta e^2)^{-1}$ depending on whether the Born approximation or the quasiclassical approximation is valid.[3] However, we may also use Eq. (2) to evaluate the mfp (l_0^B) of a beam of electrons injected into the plasma, provided that their density is much smaller than that of the plasma electrons. Thus

$$1/l_0^B=\frac{4\pi ne^4}{m^2V_0^4}\ln\frac{k_{\max}}{k_D}, \quad (4)$$

where V_0, the velocity of the beam, is much greater than the mean velocity of the plasma electrons (v_T). Equations (3) and (4) constitute essentially the usual formulas.[4]

Now the crucial point is that even if the Born approximation is valid, i.e., $e^2/\hbar v_T \ll 1$, the mfp given by Eq. (3) or (4) would not be adequate for existing plasmas just because there would appear an additional expansion parameter, namely, the number of particles in the force range of the screened Coulomb interaction, which is very large.

In fact, if we take the next-higher Born approximation [Figs. 1(b) and 1(c)], the cross section for a process in which an electron p is scattered to $p-q$ and then rescattered to $p-q-q'$ by the medium within ranges dq and dq' is given by

$\Gamma(p,q,q')dqdq'$, where

$$\Gamma(p,q,q') = \frac{1}{\hbar}\mathrm{Im}\Big[\beta^{-2}\sum_{\omega}\sum_{\omega'}G_0(p-q,\epsilon-\omega)G_0(p-q-q',\epsilon-\omega-\omega')\{G_0(p-q',\epsilon-\omega')+G_0(p-q,\epsilon-\omega)\} \times \frac{v(q)}{\epsilon(q,\omega)}\frac{v(q')}{\epsilon(q',\omega')}\Big]. \quad (5)$$

This consists of three types of scattering processes. One of these gives the vertex correction and one the self-energy correction to the Born (or binary) scattering process (1), both being of minor importance. The most important part is exclusively the three-body collision process which is illustrated in Fig. 2.

Corresponding to Eq. (2), taking the average of the energy transfer, $\epsilon_0(p)-\epsilon_0(p-q-q')$, over (5) gives the mfp l;

$$l^{-1} = \frac{m}{p}\sum_q\sum_{q'}\left\{\frac{\epsilon_0(p)-\epsilon_0(p-q-q')}{\epsilon_0(p)}\right\}\Gamma(p,q,q'). \quad (6)$$

Depending on whether the electron is a plasma electron or beam electron, Eq. (6) (its main term) yields[5]

$$l^{-1} = \frac{1}{9}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{e^2}{\hbar v_T}\right)^2\left(\frac{v_T}{v}\right)k_D \ln\frac{k_{\max}}{k_D} \quad (7)$$

or

$$1/l^B = \left(\frac{e^2}{\hbar v_T}\right)^2\left(\frac{v_T}{V_0}\right)^8 k_D \ln\frac{k_{\max}}{k_D}. \quad (8)$$

A remarkable result is obtained if one considers the ratio l_0/l. For an electron with the mean thermal velocity

$$l_0/l \approx \frac{n}{k_D^3}\left(\frac{e^2}{\hbar v_T}\right)^2 \sim \frac{1}{k_D r_B}, \quad r_B \equiv \frac{\hbar^2}{me^2}, \quad (9)$$

except for a numerical factor of order unity. The physical reason of this is clear: The factor $(e^2/\hbar v_T)^2$ expresses the higher order effect of the Coulomb interaction as a perturbation,[1] and the inclusion of three-body collisions gives the other factor, n/k_D^3, namely, the number of particles in the force range. For plasmas with $k_D \sim 10^4\ \mathrm{cm}^{-1}$ (and when $e^2/\hbar v_T \ll 1$ is satisfied so that perturbation theory may be applicable), our result (7) gives a very much shorter ($\sim 10^{-4}$ times) mfp than the usual result (3).

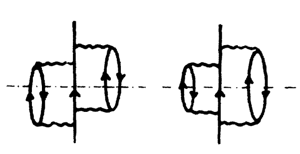

FIG. 2. Three-body collision processes resulting from Fig. 1(b) and (c). Dashed lines indicate intermediate states of real transitions in which energy is conserved.

For the beam electrons,

$$l_0^B/l^B = 1/k_D r_B (v_T/V_0)^4. \quad (10)$$

When applied to the ancient but notable Langmuir's experiment,[6] Eq. (10) explains the situation quite well. If we take $k_D \sim 10^2\ \mathrm{cm}^{-1}$ and $V_0 \sim 4v_T$, then $l_0^B/l^B \sim 10^4$,[7] which is just the required order to resolve the Langmuir paradox.[6,8]

Evaluation of the related transport coefficients is being planned along the present lines. A more serious problem left for the future is the examination of the convergence of the perturbation expansion with respect to n/k_D^3. If the actual plasmas are found to be the strong coupling limit of this parameter, perturbation approach can never be applied to them. A more detailed account of this work will be published elsewhere.

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[3]A. A. Vedenov and A. I. Larkin, Zh. Eksperim. i Teor. Fiz. 36, 1133 (1959) [translation: Soviet Phys. – JETP 9, 806 (1959)].

[4]See, for example, W. B. Thompson, An Introduction to Plasma Physics (Pergamon Press, New York, 1962), p. 150.

[5]The complete expression for l is given by

$$l^{-1} = (e^2/\hbar v_T)^2(v_T/v)^8 F(v/\sqrt{2}v_T)k_D \ln k_{\max}/k_D,$$

where

$$F(x) = \frac{2}{\sqrt{\pi}}\left(-x\,e^{-x^2} + \int_0^x dt\,e^{-t^2}\right) \times [-x^2 + x(1+2x^2)\,e^{-x^2}\int_0^x dt\,e^{t^2}].$$

The above expression for l^{-1} reduces to Eq. (7) in the limiting case $x \lesssim 1$ $[F(x) \sim (16/9\sqrt{\pi})x^7]$ and to Eq. (8) when $x \gg 1$ $[F(x) \to 1]$.

[6]I. Langmuir, Phys. Rev. 26, 585 (1925).

[7]A value for the mfp of the same order of magnitude as this has recently been derived from a quite different analysis based on the nonlinear treatment. See the review article by Y. Klimontvich and V. P. Silin, in Plasma Physics, edited by J. E. Drummond (McGraw-Hill Publishing Company, Inc., New York, 1961), Chap. 2.

[8]See the review article by J. E. Drummond in Plasma Physics, edited by J. E. Drummond (McGraw-Hill Publishing Company, Inc., New York, 1961), p. 2.

POLARIZED POSITRON ANNIHILATION IN FERROMAGNETS*

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It is well known that the measurement of the angular correlation of the two photons emitted during positron annihilation in solids can provide information concerning the momentum distribution of the electrons annihilating with the positron.[1] Positron annihilation with spin-aligned electrons in ferromagnets was first used by Hanna and Preston[2] for positron-spin analysis. They also indicated how such measurements can yield information regarding the momentum spectrum of the ferromagnetic electrons.[3] Their use of cylindrical geometry, particularly in its integral form, did not reveal some details of interest to the study of ferromagnetism; they also reported a null effect in nickel. Lovas[4] used parallel-slit geometry in a similar experiment, but with insufficient angular resolution and statistics. We have performed several measurements with improved statistics using a parallel-slit system.[5]

Recently Mijnarends and Hambro[6] also announced new, independent measurements in iron and interpreted their results as definite proof for the polarization of the conduction electrons. In this Letter we would like to present some of our new data in iron and nickel and to interpret the results in a rather different light. The experimental setup used was similar to one described previously,[7] with the addition of an electromagnet used to saturate the ferromagnetic samples. Positrons from an 8-mCi Na^{22} source were collimated by a circular aperture and allowed to strike the $\frac{3}{8}$-inch diameter samples. The magnetic field was set parallel (↑) or antiparallel (↓) to the incident positron momentum. The photons from positron-electron singlet annihilation were counted by the 7-inch long NaI crystals behind parallel lead-slit collimators. The slits subtended an angle of 0.75 mrad or 2 mrad in various runs. At each angle two counting rates, $N_\uparrow(\theta)$ and $N_\downarrow(\theta)$, were obtained with an automatic system by several cyclings through the predetermined angular range.

The model used in describing the observed

$$p(\theta)=\frac{N_\uparrow(\theta)-N_\downarrow(\theta)}{N_\uparrow(\theta)+N_\downarrow(\theta)}$$

assumes that the polarized positrons slow down to their ground state $\psi_+(\vec{r})$ in the crystal lattice but that their original spin direction is not changed appreciably.[8] Let the annihilation with the lth electron result in a momentum distribution

$$\rho_l(\vec{p})=\mathrm{const}\left|\int\psi_l(\vec{r})\psi_+(\vec{r})e^{-i\vec{p}\cdot\vec{r}}\right|^2,$$

where $\psi_l(\vec{r})$ is the lth-electron wave function; the positron-electron interaction is neglected. The parallel-slit setup integrates over dp_x and dp_y, and one obtains

$$n_l(p_z=mc\theta)=\iint\rho_l(\vec{p})dp_xdp_y,$$

m being the mass of the electron. Using the incident positron momentum direction as the axis of quantization, let P_p and P_l be the positron and the lth-electron polarizations ($P=0$, unpolarized particle, $P=\pm1$, completely polarized particle along and opposite the quantization axis). The probability for a spin-singlet overlap with the lth electron is then proportional to $\frac{1}{4}(1\mp P_lP_p)w_l$ and that for a spin-triplet overlap is proportional to $\frac{1}{4}(3\pm P_lP_p)w_l$ where $w_l=\int n_l(p_z)dp_z$; w_l is obviously proportional to $\int|\psi_+(\vec{r})\psi_l(\vec{r})|^2d\vec{r}$. Thus we find that with the superimposed magnetic field ↑ or ↓

$$N_{\uparrow,\downarrow}(\theta)=C_{\uparrow,\downarrow}\sum_l(1\mp P_lP_p)n_l(\theta). \quad (1)$$

It is to be noted that in ferromagnets the excess spin polarization is a negative number in the above notation. We shall return to the constants $C_\uparrow$ and $C_\downarrow$.