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CRITICAL-FIELD BEHAVIOR OF A MICROSCOPIC SUPERCONDUCTING BRIDGE*

R. D. Parks,† J. M. Mochel,‡ and L. V. Surgent, Jr.
Department of Physics and Astronomy, University of Rochester, Rochester, New York
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In a previous Letter[1] we reported anomalous structure observed in the resistivity vs magnetic-field curves of superconducting tin strips of microscopic widths, taken near the transition temperature. We interpreted these results in terms of free-energy effects associated with quantized vortices limited in size by the width of the strips. Anderson and Dayem[2] have since proposed an alternative explanation in which they assume that the structure we observed is due to voltages resulting from the motion of wave-function nodes (which they call "vortices") across the strip. On the basis of the data in reference 1, it is not possible to rule out this explanation, nor is it possible to derive from the data the quantitative thermodynamic properties of the sample. We attribute this now to sample inhomogeniety, indigenous to the sample configuration and the method of preparation, which resulted in broad resistive transitions and therefore a smearing of the observed structure. We have since developed new techniques for preparing different microgeometries in which the superconducting transitions are extremely sharp. In these samples well pronounced anomalies are observed in the critical-field behavior which can be interpreted only in terms of a reduction in the free energy of a microregion of the sample which occurs when quantized vortices are allowed to enter this region.

The samples are prepared in the following way. A thin fiber made from GE 7031 varnish is stretched over a glass microscope slide. The fiber is delicately severed with a microknife and metal (in this case tin) is then evaporated onto the resulting substrate. The assembly is then soaked in an ultrasonic bath of ethyl alcohol. This step removes the varnish fiber and the metal deposited on the fiber but not the metal deposited in the juncture where the cut was made. The resulting configuration is shown in Fig. 1. Superconducting bridges 1-10 μ wide and 1-50 μ long have been prepared in this manner.

A bridge sample prepared in the manner described above is immersed in liquid helium and the resistivity is measured using the conventional four-probe technique. Both dc and ac

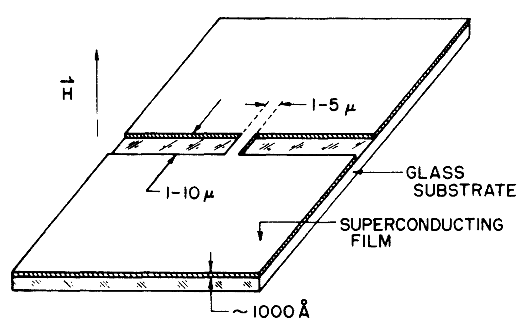

FIG. 1. Microbridge geometry used in experiment.

measurements are made. For the ac measurements lock-on techniques are employed which provide a sensitivity of 6×10^{-10} volts. In Fig. 2 the isothermal transition data is shown for a superconducting tin bridge 2.65 μ wide, 10 μ long, and 800 Å thick. Constant-temperature transition curves are taken at 5 millidegree intervals. These are obtained by plotting with an x-y recorder the voltage across the bridge in which a constant current is flowing, against the applied, perpendicular magnetic field. The data shown are from a 200-cps ac measurement. The results are identical for other frequencies (in the range available to us—10-10 000 cps) and for dc. The results are current independent for small currents but are affected as one might expect when the currents become large enough to depress or broaden the transition. We associate the dramatic drop in the resistivity on the 3.795°K isotherm at 3.65 gauss with the point at which quantized vortices may enter the microregion. In order to explain this we shall need to refer briefly to what Tinkham[3] has said about vortices in thin films and our extension[1] of his model to microgeometries.

According to Tinkham's model a thin film in a perpendicular magnetic field near T_c will support quantized current vortices in order to reduce the free energy due to the Meissner currents which must flow in the absence of the vortices. Near T_c where the condensation energy is small and the magnetic field is undistorted by the film, quantized vortices completely fill the space of the film. The overall radius of the vortices is determined by the relation

$$\pi R^2 H = n\varphi_0,$$

where $\varphi_0 = hc/2e$ is the flux quantum, H the applied magnetic field, and n an integer. In a qualitative application of the Ginzburg-Landau theory, Tinkham argues further that $n=1$ is the favored quantum state. Using this model we illustrate in Fig. 3(b) the vortex arrangement which should exist in the film near T_c at some field value smaller than $\varphi_0/\pi R_b^2$, where $2R_b$ is the width of the bridge. The vortices are too large to fit into the bridge. In Fig. 3(a) the vortex configuration corresponds to that in a field given by $H_b = \varphi_0/\pi R_b^2$, the field at which vortices of radius R_b can just fit into the bridge. At this field vortices readily enter the bridge in order to reduce the free energy. In Fig. 2 this occurs at the field value, 0.97 H_b. Numerous samples of the bridge geometry with widths in the range 1-5 microns have been studied. The fields at which vortices enter the bridges lie in the range $1.0\pm0.1\,H_b$. We estimate our overall accuracy to be $\pm0.1\,H_b$.

In order to determine that the results shown in

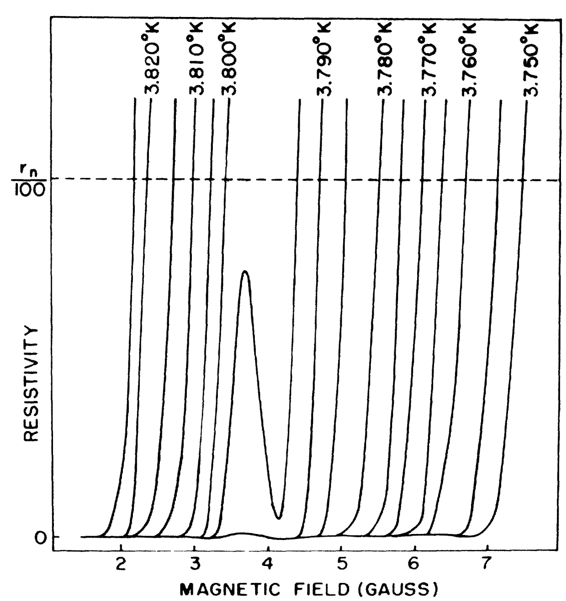

FIG. 2. x-y recorder tracings of the isothermal resistive transition curves for a tin bridge 2.65 μ wide.

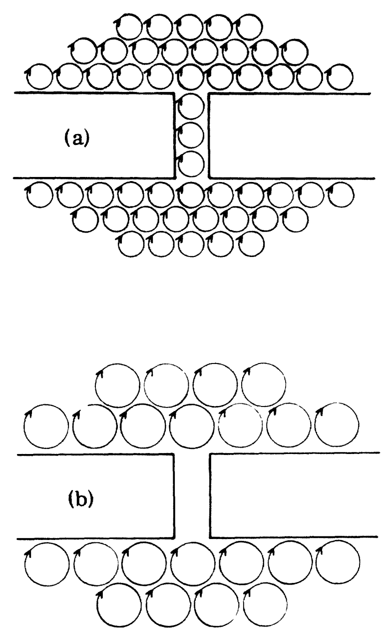

FIG. 3. (a) Vortex configuration in region of microbridge for a perpendicular magnetic field of magnitude $H_b = \varphi_0/\pi R_b^2$, where $2R_b$ is the width of the microbridge. (b) Vortex configuration for a field smaller than H_b.

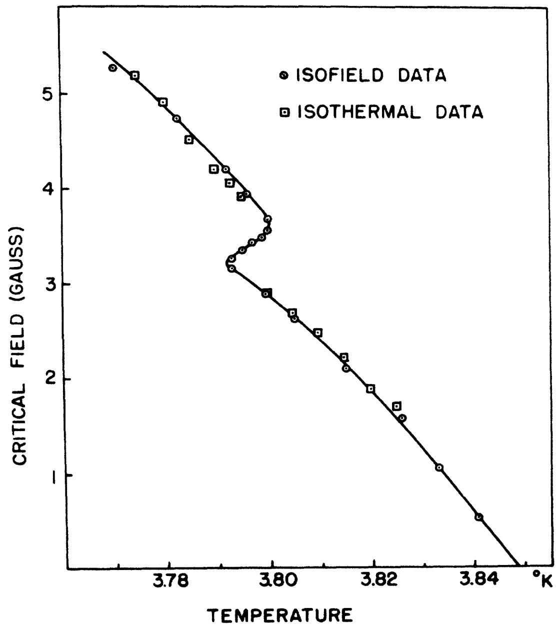

FIG. 4. Critical field vs temperature curve for a tin bridge 2.65 μ wide, obtained from both isothermal and isofield measurements.

Fig. 2 reflect only free-energy effects and not induction effects associated with the changing magnetic field, we made isofield transition measurements on the same sample. The resistivity of the sample was measured at different temperatures in constant magnetic fields. The critical-field data obtained from these measurements are shown in Fig. 4. The points were obtained by plotting the field and temperature corresponding to a resistivity ratio $r/r_N = 0.01$. The choice of r/r_N is arbitrary and identical results were obtained by using other ratios. From Fig. 2 critical-field data were obtained by plotting the intercepts of the $r/r_N = 0.01$ isoresistivity line with the transition curves. These results are plotted in Fig. 4, along with the isofield data. Within experimental error the critical-field curves obtained from the two types of measurements are identical. The bending over of the critical-field curve just before the anomaly is characteristic of the behavior of all of the samples that we have studied. This is due probably to the increase in free energy associated with the Meissner currents which must flow in the microgeometry before vortices are allowed to enter. From either the data in Fig. 2 or Fig. 4 it is possible to obtain directly the energetic advantage resulting from the presence of vortices in the 2.65μ microbridge. It corresponds to a change in the transition temperature of 0.015°K.

In samples of less favorable geometry we have observed structure at magnetic fields higher than H_b, as well as near H_b. The origin of this structure is not yet understood. It might result from $n = 2$ or higher order vortices, or, alternatively, it might be attributed to sample nonuniformity or sample configuration.

In summary we note that the results reported here strongly support Tinkham's qualitative vortex model. They cannot be understood in terms of the explanation offered by Anderson and Dayem.

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