served values of the thermal conductivity of liquid helium below 0.6°K. In the present experiment the specular reflection from the walls serves only to contribute to the background signal.

A Model 50-A Min A tron impulse generator is used to produce two signals, one at 155 cps and the other at 310.4 cps. The lower frequency is filtered of all harmonics and sent into the carbon transmitter where it generates 310-cps local Joule heating of the liquid helium. The wavelength of the second sound arising from the Joule heat is 48 cm. Since this wavelength is much greater than the dimensions of the experimental chamber, it is clear that the observed enhancement of the received 310-cps signal could not be due to the specular reflection of thermal waves. After the received 310-cps signal is amplified, it is mixed with the higher 310.4-cps signal and a rectified signal of 0.4 cps is displayed on a strip chart recorder. The 0.33°K temperature was attained by means of a He^3 refrigerating system.[5]

The ability to focus and diffract phonons would make available a powerful new method for studying phonon spectra of solids.[6] The construction of a very high-frequency (10^{10}-10^{11} cps) phonon "monochromator," i.e., source of monoenergetic phonons analogous to optical monochromators, would become feasible. Phonon spectroscopy of solids would make it possible to study phonon-phonon, phonon-photon, phonon-electron, and phonon-imperfection interactions which play important roles in electrical and heat conduction, x-ray and neutron scattering, and other solid-state phenomena.

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*The study was supported by the U. S. Air Force Office of Scientific Research Contract No. AF49(638)-352, and by the National Aeronautics and Space Administration Grant No. NsG-130-61.

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THEORY OF COHERENCE OF LASER LIGHT

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In a recent Letter to this journal Jordan and Ghielmetti[1] have concluded from the analysis of interference and beat experiments[2,3] with two independent laser beams that the expectation value $\langle b^\dagger(t)\rangle$ of the field operator does not vanish for laser light whereas it does for light from thermal sources. In our present note we wish to show how this result can be derived from first principles. For this end we use a fully nonlinear theory of laser noise in contrast to the hitherto published theories which are basically linear.[4-11]

For our treatment we assume a homogeneously broadened Lorentzian line of width γ and resonance between atoms and the cavity mode under consideration. We describe the mode by a running wave in order to avoid an otherwise spatially inhomogeneous inversion of the atomic system. After splitting off the main time dependence $\exp(i\omega t)$, and after elimination of the atomic coordinates, the steady-state equation for the creation operator $b^\dagger$ of the cavity mode reads[12]

$$[\ddot{b}^\dagger + (\kappa+\gamma)\dot{b}^\dagger - Gb^\dagger + 2|g|^2 b^\dagger b b^\dagger] = i\sum\nolimits_{t_{\mu\nu}} g\delta(t-t_{\mu\nu})\delta\alpha_\mu^\dagger(t_{\mu\nu}), \qquad (1)$$

where $G=2|g|^2G_0-\kappa\gamma$. κ denotes the cavity linewidth. G_0 is the time average over the total number of the excited atoms and photons present, g is the coupling constant between the cavity mode and a single atom divided by $\hbar$. $\delta\alpha_\mu^\dagger(t_{\mu\nu})$ represents the jump of the electron excitation operator at atom μ due to the excitation by the pumping process at time $t_{\mu\nu}$. The essential feature of Eq. (1) can best be seen by interpreting[13] it as the equation of a classical particle moving in two dimensions under the action of the potential

$$V(|b|)=-\tfrac{1}{2}G|b|^2+\tfrac{1}{2}|g|^2|b|^4. \qquad (2)$$

For low inversion ($G<0$) the dashed line in Fig. 1 applies. After each excitation collision, represented by the right-hand side of Eq. (1), the light amplitude decreases to zero. This result is also obtained if one neglects $b^\dagger b$ in Eq. (1) completely or replaces it by its expectation value, as is done in linear theories. If at sufficiently high inversion G is positive, the solid line of Fig. 1 applies. In this case there exists a stable value r_0 for the absolute value of b. Since the potential (2) possesses rotation symmetry, the phase of $b^\dagger$ can still undergo an undamped diffusion process on account of the collision term on the right-hand side of Eq. (1). We have treated in detail both cases ($G \gtrless 0$).[12] Below threshold we obtain the same expressions as derived by Wagner and Birnbaum[5] with their model of fluctuating dipoles. Above threshold we put b^+ in the form $b = T\exp[i\varphi(t)][r_0+\rho(t)]$, where T denotes the time-ordered product and r_0 is a c number.[14] r_0^2 turns out to equal $G_0-\kappa\gamma/2|g|^2$ and coincides with the steady-state value of the photon number n determined in a previous paper.[15] The equation of motion can then be linearized and split into real and imaginary parts. The equation

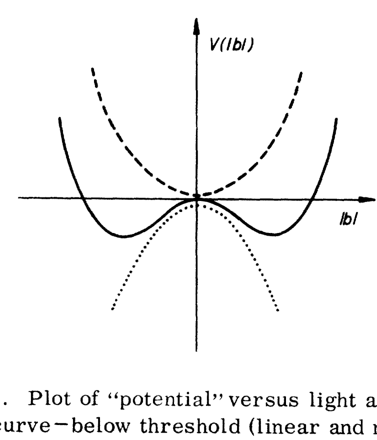

FIG. 1. Plot of "potential" versus light amplitude. Dashed curve—below threshold (linear and nonlinear theory); dotted curve—above threshold, linear theory leads to instability; solid line—above threshold, nonlinear theory.

for the phase reads

$$\ddot{\varphi}+(\kappa+\gamma)\dot{\varphi}=\frac{1}{r_0}\operatorname{Im}\{i\textstyle\sum_{t_{\mu\nu}}g\delta(t-t_{\mu\nu})\delta\alpha_\mu^\dagger(t_{\mu\nu})\}. \qquad (3)$$

We have absorbed the phase factor $\exp(-i\varphi)$ in the random phases of $\alpha_\mu^\dagger$ at each collision. The solution of Eq. (3) and of the corresponding one for ρ is straightforward and allows determination of $\langle b^\dagger(t)\rangle$ as well as $\langle b^\dagger(t+\tau)b(t)\rangle$. The angular brackets mean the quantum mechanical average as well as that over the Poisson distribution of excitation collisions and over all initial atomic phases after each excitation. We find the following results: $\langle\rho\rangle=0$ and $\langle\rho^2\rangle \lesssim N/(T_pP)$, where N is the total number of atoms, T_p the mean excitation collision time, and P the photon flux.[16] Somewhat above threshold $\langle\rho^2\rangle$ is thus many orders of magnitude smaller than r_0^2. Therefore the noise above threshold stems essentially from phase fluctuations for which we have

$$\langle b^\dagger(t+\tau)b(t)\rangle = r_0^2\langle\exp[-i\varphi(t+\tau)-i\varphi(t)]\rangle$$
$$= r_0^2\exp(-\Delta\nu\tau). \qquad (4)$$

The explicit determination of $\Delta\nu$ depends on the relative size of $1/T_p$, $\hbar r_0$, and γ. We quote here the results for large $\hbar r_0$ for the case that the lower optical level of an atom is emptied to the ground state before the next excitation collision of that atom takes place:

$$\Delta\nu=\kappa g^2N/[2PT_p(\gamma+\kappa)^2]. \qquad (5)$$

If the measurement of phase (e.g., by an interference experiment) starts at time $t=0$, $\langle b^\dagger(t)\rangle$ turns out to be

$$\langle b^\dagger(t)\rangle = r_0\exp(-\Delta\nu t)\cdot\exp i\varphi_0, \qquad (6)$$

where φ_0 is the (unpredictable) initial phase. Equation (6) represents the required result: If the observation time is small as compared to the reciprocal linewidth, there exists a nonvanishing expectation value of the light amplitude, its absolute value being equal to $\sqrt{n}$.

In conclusion, we would like to mention that our present analysis represents a deeper foundation of the nonlinear laser theory of Lamb[17] as well as Haken and Sauermann,[15] which neglects laser noise, but is capable of a detailed explanation of effects like hole burning[17,15,18] and coexistence of modes in a homogeneously broadened line.[19,15] An extension of our present analysis to multimode operation shows that one can recover these results including noise.

I am grateful to Professor H. Koppe,

Mr. H. Sauermann, and especially Dr. W. Weidlich, for valuable discussions.

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CRITICAL-FIELD BEHAVIOR OF A MICROSCOPIC SUPERCONDUCTING BRIDGE*

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In a previous Letter[1] we reported anomalous structure observed in the resistivity vs magnetic-field curves of superconducting tin strips of microscopic widths, taken near the transition temperature. We interpreted these results in terms of free-energy effects associated with quantized vortices limited in size by the width of the strips. Anderson and Dayem[2] have since proposed an alternative explanation in which they assume that the structure we observed is due to voltages resulting from the motion of wave-function nodes (which they call "vortices") across the strip. On the basis of the data in reference 1, it is not possible to rule out this explanation, nor is it possible to derive from the data the quantitative thermodynamic properties of the sample. We attribute this now to sample inhomogeniety, indigenous to the sample configuration and the method of preparation, which resulted in broad resistive transitions and therefore a smearing of the observed structure. We have since developed new techniques for preparing different microgeometries in which the superconducting transitions are extremely sharp. In these samples well pronounced anomalies are observed in the critical-field behavior which can be interpreted only in terms of a reduction in the free energy of a microregion of the sample which occurs when quantized vortices are allowed to enter this region.

The samples are prepared in the following way. A thin fiber made from GE 7031 varnish is stretched over a glass microscope slide. The fiber is delicately severed with a microknife and metal (in this case tin) is then evaporated onto the resulting substrate. The assembly is then soaked in an ultrasonic bath of ethyl alcohol. This step removes the varnish fiber and the metal deposited on the fiber but not the metal deposited in the juncture where the cut was made. The resulting configuration is shown in Fig. 1. Superconducting bridges 1-10 μ wide and 1-50 μ long have been prepared in this manner.

A bridge sample prepared in the manner described above is immersed in liquid helium and the resistivity is measured using the conventional four-probe technique. Both dc and ac

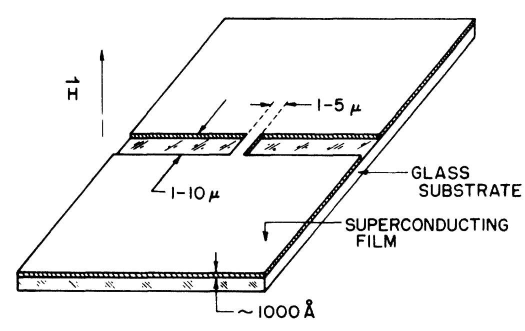

FIG. 1. Microbridge geometry used in experiment.