(unpublished, but reported in reference 4) has shown for this experiment, performed at a relatively low photon energy of 1.23 BeV, that double-pion production from the electric dipole term and the Drell mechanism contribute about the same amount to the total cross section and together satisfactorily explain the data rates. No adjustment corresponding to this fact was made on the experimental cross sections in our analysis, since our conclusions do not depend very strongly on this particular datum.

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SCALING LAW FOR HIGH-ENERGY ELASTIC SCATTERING

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Foley et al.[1] have made extensive measurements of elastic p-p, $\bar{p}$-p, $\pi^{\pm}$-p, and $K^{\pm}$-p scattering in the momentum range 7 to 20 BeV/c. In order to compare the results for different incident particles, one may try a scaling law of the type suggested by an optical model[2],

$$d\sigma/dt = \pi R^4 f(R^2 t). \tag{1}$$

Here t is the square of the momentum transfer, in units of cm^{-2}, and to make the definition of the scaling length R explicit we take $f(0)=1$, so that

$$R^2 = (d\sigma/dt|_{t=0}/\pi)^{1/2}. \tag{2}$$

According to this scaling law, we are to compare values of $(d\sigma/dt)/(d\sigma/dt|_{t=0})$ as functions of the square of the momentum transfer measured in dimensionless units,

$$t' = (d\sigma/dt|_{t=0}/\pi)^{1/2} t. \tag{3}$$

If t is given in $(\mathrm{BeV}/c)^2$, rather than cm^{-2},

$$t' = 0.90416(d\sigma/dt|_{t=0})^{1/2} t, \tag{4}$$

with $(d\sigma/dt|_{t=0})$ in mb/$(\mathrm{BeV}/c)^2$.

Foley et al. have expressed the experimental results in the form

$$(d\sigma/dt)/(d\sigma/dt|_{t=0}) = \exp(-bt + ct^2), \tag{5}$$

with t in $(\mathrm{BeV}/c)^2$. In terms of the dimensionless variable, (5) takes the form

$$(d\sigma/dt)/(d\sigma/dt|_{t=0}) = \exp(-b't' + c't'^2), \tag{6}$$

with

$$b' = b/[0.90416(d\sigma/dt|_{t=0})^{1/2}],$$
$$c' = c/[0.81751(d\sigma'dt|_{t=0})]. \tag{7}$$

The upper limits of momentum transfers reached in the Brookhaven experiments were $t_{\max} \sim 0.8$-$1.1(\mathrm{BeV}/c)^2$, except for $\bar{p}$-p scattering where $t_{\max} \sim 0.5(\mathrm{BeV}/c)^2$. The corresponding values for t' range from $t_{\max}' = 4$ for K^+-p, to $t_{\max}' = 8$ for p-p scattering.

The values of the experimentally determined parameters are listed in Table I, and shown graphically in Figs. 1 and 2 as functions of p_0, the momentum of the bombarding particle.

Figure 1 shows that after the rescaling, the value of b', which describes the initial rate of decrease of the cross section, is virtually identical for bombarding protons or antiprotons, and also identical within the errors, but with a 50% larger value, for all the pseudoscalar mesons. The error on the K^--p scattering is so large that the result in this case is not compelling. The large errors for K^--p scattering are a reflection not merely of larger statistical errors of the measured cross sections; all the other cases are a combination of the results of two quite distinct experiments, a coincidence experiment to obtain the cross section for $t > 0.25(\mathrm{BeV}/c)^2$, and a magnetic analysis experiment for smaller t, while only the former was possible for K^--p scattering.

The behavior shown in Fig. 2 is similar in that the pseudoscalar mesons appear to have a common value of c', while the values for the proton are lower. However, the values of c' for $\bar{p}$-p and p-p do not agree. Foley et al. found that the $\bar{p}$-p data were adequately represented without a quadratic term ($c = 0$). In order to present the $\bar{p}$-p data in a form more comparable to the other cases, and in particular to obtain a probable error for the value of c, a least-squares fit with $c \neq 0$ was made to the experimental data for the cases $p_0 = 8.9$ and 12.0 BeV/c. The value of b' is not significantly changed from that given by the Brookhaven group for $c = 0$. For the case $p_0 = 7.2$ BeV/c, the accuracy of the low-momentum points (from the magnetic analysis experiment) is not sufficient to justify such a treatment, and the Brookhaven group's value has been used.

The similarity of the experimental curves for the pseudoscalar mesons can be demonstrated in another way that is not as incisive as our scaling law. Equation (5) can be rewritten as

$$(d\sigma/dt)/(d\sigma/dt|_{t=0}) = \exp[-bt + \alpha(bt)^2],$$

with $\alpha = c/b^2 = c'/b'^2$. The dimensionless parameter α describes the shape of the curve, b remaining as a scale factor. Some theoretical

TABLE 1. Values of the Parameters

	p_0 (Bev/c)	$d\sigma/dt\|_{t=0}$ mb/(Bev/c)^2	b (Bev/c)$^{-2}$	c (Bev/c)$^{-4}$	b'	c'
p-p	6.8	105.9 ± 2.4	9.78 ± 0.21	3.28 ± 0.31	1.050 ± 0.025	0.0378 ± 0.0037
	8.8	106.4 ± 2.7	9.62 ± 0.22	2.30 ± 0.31	1.032 ± 0.028	0.0265 ± 0.0037
	10.8	102.8 ± 3.1	9.79 ± 0.23	2.13 ± 0.34	1.069 ± 0.030	0.0254 ± 0.0041
	12.8	104.0 ± 4.3	10.03 ± 0.28	2.19 ± 0.38	1.091 ± 0.037	0.0259 ± 0.0046
	14.8	103.2 ± 4.8	10.37 ± 0.33	2.48 ± 0.46	1.128 ± 0.046	0.0293 ± 0.0056
	16.7	92.2 ± 5.5	9.79 ± 0.40	1.48 ± 0.59	1.130 ± 0.054	0.0197 ± 0.0079
	19.6	96.5 ± 7.0	10.48 ± 0.43	2.25 ± 0.56	1.180 ± 0.063	0.0285 ± 0.0073
$\bar{p}$-p	7.2	181 ± 16	13.15 ± 0.47	---	1.082 ± 0.062	---
	8.9	178 ± 7.5	12.98 ± 0.61	0.40 ± 1.49	1.074 ± 0.056	0.0027 ± 0.0102
	12.0	140 ± 9.8	11.67 ± 0.98	-2.77 ± 2.50	1.092 ± 0.091	-0.0242 ± 0.0215
π^+-p	6.8	43.7 ± 1.6	8.58 ± 0.23	2.24 ± 0.29	1.435 ± 0.048	0.0627 ± 0.0085
	8.8	41.0 ± 1.2	8.79 ± 0.23	2.38 ± 0.32	1.518 ± 0.046	0.0710 ± 0.0098
	10.8	38.1 ± 1.2	8.48 ± 0.25	1.79 ± 0.35	1.519 ± 0.050	0.0575 ± 0.0114
	12.8	37.8 ± 1.8	8.93 ± 0.27	2.36 ± 0.34	1.606 ± 0.058	0.0764 ± 0.0114
	14.8	37.5 ± 1.9	8.98 ± 0.31	2.27 ± 0.40	1.622 ± 0.069	0.0740 ± 0.0136
	16.7	32.6 ± 1.9	8.94 ± 0.35	2.82 ± 0.43	1.732 ± 0.085	0.1058 ± 0.0173
π^--p	7.0	42.6 ± 1.7	9.45 ± 0.25	2.75 ± 0.34	1.602 ± 0.053	0.0790 ± 0.0103
	8.9	41.4 ± 1.0	9.22 ± 0.21	2.54 ± 0.30	1.585 ± 0.039	0.0750 ± 0.0090
	10.8	41.8 ± 1.3	9.36 ± 0.23	2.45 ± 0.33	1.601 ± 0.046	0.0717 ± 0.0099
	13.0	42.4 ± 1.6	9.71 ± 0.26	3.02 ± 0.35	1.649 ± 0.055	0.0871 ± 0.0107
	15.0	42.4 ± 2.0	9.95 ± 0.29	3.31 ± 0.37	1.690 ± 0.065	0.0955 ± 0.0117
	17.0	35.2 ± 1.8	9.07 ± 0.37	2.12 ± 0.53	1.691 ± 0.081	0.0737 ± 0.0188
K^+-p	6.8	19.7 ± 4.3	6.13 ± 0.88	1.36 ± 0.77	1.527 ± 0.276	0.0844 ± 0.0513
	9.8	19.7 ± 1.5	6.23 ± 0.45	0.99 ± 0.55	1.552 ± 0.128	0.0615 ± 0.0345
	12.8	22.0 ± 1.4	6.93 ± 0.38	1.18 ± 0.45	1.634 ± 0.106	0.0656 ± 0.0254
	14.8	22.3 ± 1.6	7.06 ± 0.38	1.73 ± 0.42	1.653 ± 0.108	0.0949 ± 0.0240
K^--p	7.2	38.9 ± 11.0	10.2 ± 1.2	3.97 ± 0.92	1.862 ± 0.355	0.1248 ± 0.0473
	9.0	37.5 ± 10.8	10.5 ± 1.2	4.2 ± 1.0	1.896 ± 0.358	0.1370 ± 0.0525

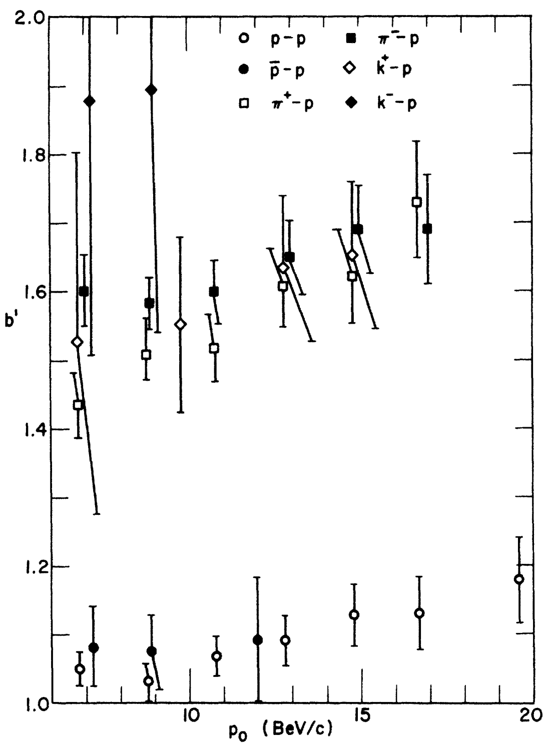

FIG. 1. The parameter b' as a function of the momentum of the bombarding particle.

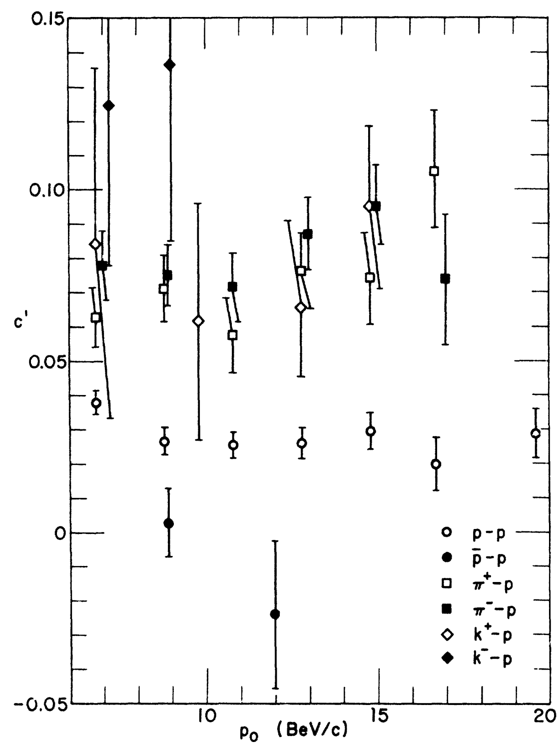

FIG. 2. The parameter c' as a function of the momentum of the bombarding particle.

discussion of α has been given by Van Hove.[3] The experimental values of α show no dependence on initial momentum p_0, except in the p-p case, where α appears to increase significantly at the lowest momentum, $p_0 = 6.8$ BeV/c. We have accordingly averaged the values of α over p_0, except for p-p, where the average is for $p_0 > 10$ BeV/c. The results are shown in Table II.

A puzzling question is why both b' and c' should agree for the mesons, while p and $\bar{p}$ give the same b' but quite different c'. An answer can be given in terms of the optical model used by the author to explain the p-p scattering for large momentum transfers,[2] beyond the t values for which expression (5) holds. The argument was made that p-p scattering represents a critical case:

Table II. Values of α.

	α
p-p	0.021 ± 0.002
π^+-p	0.030 ± 0.002
π^--p	0.030 ± 0.002
K^+-p	0.030 ± 0.006
K^--p	0.038 ± 0.009

the absorption being as large as possible without the appearance of a diffraction minimum. We have only to make the natural supposition that in the energy range being investigated the $\bar{p}$-p system has a somewhat larger absorption than the p-p system, and we are led to the expectation that the $\bar{p}$-p scattering will show a diffraction minimum, and to the conclusion that this is the reason for the very rapid drop-off of the observed $\bar{p}$-p cross section. Thus, for the two-nucleon system, with its large absorption, the coefficient c', or α, is unstable, a slight increase in absorption sufficing to reverse its sign. On the other hand, for the mesons the absorption is patently smaller, as indicated by the larger value of b', or the smaller value of σ_{el}/σ_{tot},[4] and a regularity in c' should not be so readily disturbed.

The most striking feature of our results is the appearance of a familial relation between π and K mesons. It is strange that it should appear in the form of a scaling law. Theoretical arguments have been made that at very high energies the real part of the elastic amplitude should vanish,[3] in which case our scaling factor becomes $t' = (\sigma_{tot}/4\pi)t$, and that the total cross sections should become equal for related groups of particles (Pomeranchuk theorems). In this limit the scaling law would be converted to a statement of the identity of the elastic scattering of related particles.

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TEST OF THE VALIDITY OF $\Delta S = \Delta Q$ RULE IN K^0 DECAY*

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The $\Delta S = \Delta Q$ rule is common to most theories of the strangeness-violating leptonic decays.[1] It predicts, for example, that the decay modes

$$K^0 \to \pi^- + \mu^+ + \nu,$$
$$K^0 \to \pi^- + e^+ + \nu,$$
$$\bar{K}^0 \to \pi^+ + \mu^- + \bar{\nu},$$

and

$$\bar{K}^0 \to \pi^+ + e^- + \bar{\nu}$$

are allowed while the reactions

$$K^0 \to \pi^+ + \mu^- + \bar{\nu},$$
$$K^0 \to \pi^+ + e^- + \nu,$$
$$\bar{K}^0 \to \pi^- + \mu^+ + \nu,$$

and

$$\bar{K}^0 \to \pi^- + e^+ + \nu$$

are forbidden. One of the consequences is that the time distribution of the leptonic decays of a beam which is an incoherent mixture of K^0 and $\bar{K}^0$ can be written

$$P(t) = \tfrac{1}{2}N_0\Gamma_{2L}[\alpha\exp(-t/\tau_1) + \exp(-t/\tau_2)], \quad (1)$$

where N_0 is the incident K^0 flux, τ_1 and τ_2 are the K_1^0 and K_2^0 lifetimes, respectively, and Γ_{2L} is the probability of leptonic decay for K_2^0, and α is unity. If the $\Delta S = -\Delta Q$ amplitudes are nonvanishing, Eq. (1) is still expected to hold, but $\alpha = |(1+x)/(1-x)|^2$, where x is the ratio of the $\Delta S = -\Delta Q$ and $\Delta S = +\Delta Q$ amplitudes, so that α is no longer equal to 1.

Recently the $\Delta S = \Delta Q$ rule has been put in question in two experiments which find $\alpha \neq 1$.[2] We report here briefly an experiment to check this question with greater sensitivity. Preliminary results of this experiment, showing no violation of the $\Delta S = \Delta Q$ expectations, were presented at the BNL[3] and Siena[4] conferences, and a more detailed account is being submitted for publication elsewhere. Results of Lagarrigue et al. in agreement with ours, but based on a substantially different technique, were also presented at Siena.[5]

The K^0's are produced in the annihilation of antiprotons, stopped in the Columbia-BNL 30-in. hydrogen bubble chamber. All V's within a 15-cm circle of stopped antiprotons, regardless of orientation, were searched for and those found were measured. Of 5058 measurements, 3592 passed the geometrical reconstruction program and were found to be within the fiducial region (15-cm-radius sphere, dip less than 60°) with respect to some antiproton annihilation. Of these, 3456 are classified as $K_1^0 \to 2\pi$ on the basis of kinematic fit without reference to the possible origin (1C), 19 are classified as π-p scatterings of such small momentum transfer (<50 MeV/c) that the recoil proton would be missed, and 51 fit Λ^0 or $K^\pm$ or $\pi^\pm$ or $\mu^\pm$ decay. The remaining 66 events are presumably three-