SPIN AND UNITARY SPIN INDEPENDENCE OF STRONG INTERACTIONS*

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In this note we pursue further the consequences of the assumption that strong interactions are spin and unitary spin (F-spin) independent.[1,2] In particular we discuss the meson-baryon vertex and some subgroups of SU(6).

Within a representation of SU(6), states of given four-momentum may be partially labeled by the eigenvalues of five commuting elements in the Lie algebra,[3] for which we may take (S_i = spin, $F_\lambda = F$ spin) S_3, F_3, F_8, S_3F_3, S_3F_8.

In this approach, a Fourier component $P_B{}^A(q)$ of the pseudoscalar octet ($A, B = 1, 2, 3$ are the SU(3)-tensor indices, q is the momentum) is united with a Fourier component $V_B{}^A(k,q)$ of the vector nonet ($\langle S_3\rangle \equiv k = -1, 0, 1$ is a polarization index) in the representation $\underline{35}$ of SU(6), described by a 6×6 matrix $M(q)$ given by

$$M(q) = \sigma_\mu M_\mu(q), \tag{1}$$

$$M_\mu = (iq_\mu/|q|)(F_A{}^B - \tfrac{1}{3}\delta_A{}^B F_C{}^C)P_B{}^A(q) + \sum_k n_\mu(k,q) F_A{}^B V_B{}^A(k,q), \tag{2}$$

with $\mu = 1,\cdots,4$. σ_μ are the Pauli spin matrices ($\mu = 1,2,3$) and the unit matrix ($\mu = 4$). $q_\mu/|q|$ and $n_\mu(k,q)$ form an orthogonal tetrad ($|q|^2 = q_0{}^2 - \vec{q}^2$). In particular, we have (see below)

$$V_3{}^3(k,q) = \varphi(k,q), \quad V_1{}^1(k,q) + V_2{}^2(k,q) = 2^{1/2}\omega(k,q) \tag{3}$$

where φ and ω stand for the corresponding vector mesons.

A matrix element of $M(q)$ is written as $M_\alpha{}^\beta(q)$, $\alpha, \beta = 1,\cdots,6$, and we have[4] $M_\beta{}^\beta(q) = 0$. With this notation, the $\underline{56}$ representation of SU(6) which unites[1,2] the baryon octet (b) and decuplet (d) is written as $B^{\alpha\beta\gamma}(q)$; the anti-particles are $\bar{B}_{\alpha\beta\gamma}(q)$. In both cases there is total symmetry in α, β, and γ.

We turn to the ($\bar{B}, B, M$) vertex in the pure SU(6) limit, where all B's have mass M_{00} and all M's have μ_{00}. This vertex will, for example, contain the minimal coupling of protons to ρ^0 which we normalize to $g\bar{p}\gamma_\mu p\rho_\mu{}^0$. The general ($\bar{b}, b, V$) vertex also contains $\sigma_{\mu\nu}q_\nu$ coupling, which we leave aside for the moment. At low energies this minimal part of the vertex will then contain only the s-wave ($\bar{b}bV$) and the p-wave ($\bar{b}bP$) coupling. The minimal vertex is unique[2] and has the form (for given Fourier components of the fields)

$$J_\gamma{}^\delta(-q)M_\delta{}^\gamma(q) = 6g\bar{B}_{\alpha\beta\gamma}(p)B^{\alpha\beta\delta}(p')M_\delta{}^\gamma(q),$$
$$q = p - p'. \tag{4}$$

$J_\gamma{}^\delta$ is the baryon part of the strong current. The full current, which also contains M terms, can be decomposed into an axial current octet $(\mathfrak{a}_\mu)_A{}^B$ and a vector current nonet $(\mathfrak{v}_\mu)_A{}^B$ by the method of Eq. (2). We have

$$\partial_\mu(\mathfrak{v}_\mu)_A{}^B = 0; \quad \partial_\mu(\mathfrak{a}_\nu)_A{}^B - \partial_\nu(\mathfrak{a}_\mu)_A{}^B = 0. \tag{5}$$

By simultaneous reduction in spin and F-spin we can decompose (4) into b, d and P, V. We state some results for this minimal vertex.

(1) ($\bar{b}bV$). Its strength is normalized as noted above. Hence, from the fact that ρ is coupled to the conserved isospin current, g is determined by the rate for $\rho \to 2\pi$. Thus[5]

$$\frac{g^2}{4\pi} \simeq \tfrac{1}{2}. \tag{6}$$

The coupling is pure F, as $\mathfrak{v}$ is conserved [see Eq. (5)].

(2) ($\bar{b}bP$). As noted before[1] this is a mixture of D and F. We call their ratio $(D/F)_A$ and find

$$(D/F)_A = \tfrac{3}{2}. \tag{7}$$

This ratio will reappear in the weak decays if we assume that the same axial vector current is involved in weak interactions.

In order to define the total strength of this coupling we again go to low energies and consider the p-wave term $g_A p^\dagger \vec{\sigma} p\cdot\nabla\pi^0/\mu_{00}$. We find

$$g_A = 5g/3. \tag{8}$$

As g is not renormalized, the same is true for g_A. This comes about because $\mathfrak{v}$ and $\mathfrak{a}$ currents can transform into each other in the SU(6) limit. In order to go from g_A to the pseudoscalar constant g_{PS}, we use the central mass values of SU(6). In this way we get

$$\frac{g_{PS}{}^2}{4\pi} = \frac{25}{9}\left(\frac{2M_{00}}{\mu_{00}}\right)^2 \frac{g^2}{4\pi}. \tag{9}$$

Using Eq. (6) and mean masses $M_{00} \simeq 1100$ MeV, $\mu_{00} \simeq 700$ MeV, we get $g_{PS}^2/4\pi \simeq 12.5$. We do not attach significance to the precise value, but believe that the estimate is fair and the result encouraging.

(3) $(\bar{d}bP)$. This d-decay vertex is also contained in Eq. (4). Its strength is related to the width Γ_{33} of $N_{33}{}^*$ by the following formula:

$$\Gamma_{33} = \frac{12}{25} \frac{g_{PS}{}^2}{4\pi} \frac{k^3}{m_{33}{}^2} \left[\frac{m_N m_{33}}{M_{00}{}^2}\right]. \tag{10}$$

The true masses m_N and m_{33} for nucleon and $N_{33}{}^*$ enter through the usual device of using the true phase space. g_{PS} is defined by Eq. (9). With the factor in square brackets $\simeq 1$, we get $\Gamma_{33} \sim 60$ MeV. This is of the right order, but these d widths cannot be too precise in the symmetry limit, as we know from[6] the properties of $Y_1{}^* \to \Sigma + \pi$.

(4) $(\bar{d}dP)$ and $(\bar{d}dV)$. The p-wave transition $N^{*++}(S_3 = \frac{3}{2}) \to N^{*++}(\frac{3}{2}) + \pi^0$ and the corresponding S-wave transition with a ρ^0 each have strength $3g$. Thus there is strong direct d-P and d-V interaction. It would be interesting to know whether this could explain to some extent the different value for $(D/F)_A$ found here as compared to other estimates.[7]

Remarks.—(i) The above considerations can be readily extended to include induced terms. Since in the low-energy limit the vertex is SU(6)-invariant for each partial wave, the static limit is obtained by taking the s-wave contribution of the minimal vector meson coupling together with the contribution of the induced pseudoscalar meson coupling. For the p wave the induced vector meson Pauli term completes the minimal pseudoscalar term.

(ii) For each partial wave the four-point function for B-M scattering contains only three independent amplitudes. Likewise, B-B scattering can be expressed in terms of four independent amplitudes. This implies a large number of selection rules.

We now turn to the discussion of an important subgroup of SU(6), which we denote by $W(Y) \otimes \mathrm{SU}(4)(T) \otimes \mathrm{SU}(2)(X)$. To define this subgroup we follow the usual procedure to study the algebra associated with the fundamental (6-dimensional) representation. Let

$$\lambda_\pm = \tfrac{1}{2}(1 \pm \xi), \quad \xi = (4/\sqrt{3})F_8 + \tfrac{1}{3}. \tag{11}$$

Thus $\xi^2 = 1$, $\lambda_\pm{}^2 = \lambda_\pm$, $\lambda_+\lambda_- = \lambda_-\lambda_+ = 0$. $W(Y)$ has the elements λ_+, λ_-. SU(4)(T) is generated by $\lambda_+ S_i$, F_i, and $\lambda_+ S_i F_k$ $(i, k = 1, 2, 3)$, and SU(2)(X) by $X_i = \lambda_- S_i$.

In Table I we list for some of the representations of SU(6) those representations of SU(4)(T) $\otimes$ SU(2)(X) which correspond to a definite eigenvalue of $\lambda_+ F_8$. We recall that ω, π, and ρ form the adjoint 15-dimensional representation of SU(4)(T) while φ is a scalar under SU(4)(T). Conversely, the requirement that the physical φ and ω belong to definite representations of SU(4)(T) defines the mixing of the "unphysical" SU(3) singlet $\omega^{(0)}$ and the octet member $\varphi^{(0)}$, for these physical mesons. Equation (3) is in accordance with this choice.

N appears in a 20 representation, together with N^*. This differs from Wigner's assignment[8] for the nucleon which was also provisionally used earlier.[1] It is most probable that Wigner's theory appears as a valid approximation to the SU(6) model in the nonrelativistic limit.

Table I. SU(4) multiplets in SU(6).

Representations[a] and dimensions of SU(6) $(\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5)$, D_6	Representations[b] and dimensions of SU(4)(T) $(PP'P'')$, D_4	Representation of SU(2)(X) X	G'-parity $G' = G\xi$	Particles
(10000), 6	$(\frac{1}{2}\frac{1}{2}\frac{1}{2})$, 4	0		
	(000), 1	$\frac{1}{2}$		
(10001), 35	(110), 15	0	−1	ω, π
			+1	ρ
	(000), 1	1	+1	φ
(30000), 56	$(\frac{3}{2}\frac{1}{2}\frac{1}{2})$, 20	0		N, N^*
	(000), 1	$\frac{3}{2}$		Ω^-

[a]Defined as in reference 1.
[b]Defined as in reference 11.

In the fourth column of Table I, we list for the mesons ω, π, ρ, and φ the eigenvalues of the operator $G' = G\xi$, where G is the usual G parity. Just as G is convenient for dealing with (π, ρ, ω), so G' will be convenient for dealing simultaneously with (π, ρ, ω) and φ. For these particles G' coincides with Bronzan and Low's number A.[9] The $(N, \bar{N})$ system is closed under G', and so are each of the systems $(K, \bar{K})$, $(K^*, \bar{K}^*)$, $(N^*, \bar{N}^*)$, and $(\Omega, \bar{\Omega})$. The other particles involved (for example the η) are all definite mixtures of states even and odd under G'. The behavior of all particles under G' is therefore fully specified and hence G' provides selection rules in the SU(4) limit. For example, the reactions $\pi + N \to N + n\pi + \varphi$, $N + N \to N + N + n\pi + \varphi$, and $N + \bar{N} \to n\pi + \varphi$ should be suppressed compared to the corresponding ω reactions. On the other hand, $\varphi \to K + \bar{K}$ and $K^- + p \to \Lambda + n\pi + \varphi$ are "allowed"; that is, not SU(4)-inhibited relative to ω. All these results appear to be in qualitative agreement with experiment. It may be stressed that these consequences of the theory carry no restrictions on particles present in intermediate states.

The reduction of the two-meson product (110) $\times$(110) in SU(4)(T) is worth noting. It yields (in terms of dimensions) $\underline{1} + \underline{15} + \underline{15} + \underline{20} + \underline{45} + \underline{45}^* + \underline{84}$. The $\underline{20}$, which is characterized by the representation $(2, 0, 0)$, has (T, S) content $(1, 1) + (1, 5) + (5, 1) + (3, 3)$ and thus contains an isoscalar of spin 2 which could be identified with the f^0(1250 MeV) with positive parity. If this is correct, then the $\underline{20}^+$ would also contain an isotriplet of axial vector mesons. However, as was the case for higher baryon resonances,[2] one must be prepared for a possible nonuniqueness. Thus in the present[10] case, the $\underline{84}$ also contains a $(1, 5)$.

Finally, we note that SU(6) invariance may also prove useful in the analysis of nuclear forces. In the static limit the mesons (ρ, ω, π) will still generate SU(4)(T)-invariant Majorana forces between nucleons. φ will not contribute in the limit of perfect symmetry, while a contribution to Wigner forces will arise from η exchange. While SU(4)(T) allows for an arbitrary mixture of Wigner versus Majorana forces, SU(6) invariance makes this mixture unique. It would be interesting to investigate the relationship with the Serber mixture[12] of nuclear forces.

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[1]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964).

[2]A. Pais, Phys. Rev. Letters 13, 175 (1964). In the fourth line from the end of this paper please read "$\underline{280} + \underline{280}^*$" for "$\underline{280} + \underline{280}$."

[3]The F_λ are normalized by the same convention as in M. Gell-Mann, Phys. Rev. 125, 1067 (1962), Eq. (4.16). For the S_i we have $[S_1, S_2] = iS_3$, cyclically.

[4]$M_\beta{}^\beta(q)$ is a separate representation of SU(6)(q) and describes a spinless unitary singlet.

[5]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

[6]V. Gupta and V. Singh, to be published.

[7]R. Cutkosky, Ann. Phys. (N.Y.) 23, 415 (1963); S. Glashow and L. Rosenfeld, Phys. Rev. Letters 10, 192 (1963); A. Martin and K. Wali, Nuovo Cimento 31, 1324 (1964).

[8]E. P. Wigner, Phys. Rev. 51, 106 (1937).

[9]J. E. Bronzan and F. E. Low, Phys. Rev. Letters 12, 522 (1964).

[10]The complete (T,S) contents are as follows: $\underline{45}$ (and $\underline{45}^*$) $= (1, 3) + (3, 1) + (3, 3) + (3, 5) + (5, 3)$; $\underline{84} = (1, 1) + (1, 5) + (5, 1) + 2\times(3, 3) + (3, 5) + (5, 3) + (5, 5)$. The $\underline{84}$ representation provides a first instance of multiple occurrence of the same (T,S) submultiplet.

[11]E. Feenberg and E. P. Wigner, Rept. Progr. Phys. 8, 274 (1941).

[12]J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley & Sons, Inc., New York, 1952), p. 170.

[13]After completion of this work, one of us (F.G.) was informed by Dr. B. Sakita that he, independently, had the idea of extending Wigner's supermultiplet theory to elementary particles and putting the mesons in the adjoint representation of SU(6).