[10]T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960).
[11]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963); see also M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1958).
[12]H. H. Bingham, talk presented to Royal Society Discussion on Leptonic Interactions, Feb. 27, 1964 (unpublished); and H. Ticho, Proceedings of the Sienna International Conference on Elementary Particles (Società Italiana di Fisica, Bologna, Italy, 1963).
[13]J. J. Sakurai, Phys. Rev. Letters 12, 79 (1964).
[14]C. Bhalla, private communication, reinterpretation of neutron and O^{14} lifetime experiments.
[15]Average of data from R. W. Birge, D. H. Perkins, J. E. Peterson, D. H. Stork, and M. H. Whitehead, Nuovo Cimento 4, 834 (1956); G. Alexander, R. H. W. Johnston, and C. O'Ceallaigh, Nuovo Cimento 6, 478 (1957); and B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters 7, 346 (1961).
[16]M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report No. CTSL-20, 1961 (unpublished).
[17]G. Feinberg and L. Lederman, Ann. Rev. Nucl. Sci. 13, 431 (1963).
[18]W. Barkas and A. Rosenfeld, University of California Radiation Laboratory Report No. UCRL-8030 (Rev.) (unpublished).
[19]Y. Yamaguchi, CERN Technical Report No. 59-35, 1959 (unpublished), p. 342.
[20]C. Ryan, private communication.
[21]The appropriate SU(3) coefficients are written out explicitly in notes on a lecture by N. Cabibbo at the summer school in Erice, Italy, May 1963 (unpublished).
[22]G. Zweig, to be published.

NATURALLY OCCURRING ZERO-MASS PARTICLES AND BROKEN SYMMETRIES*

G. S. Guralnik†
Imperial College of Science, London, England
(Received 1 June 1964)

In a recent Letter[1] it was pointed out that the Goldstone theorem[2,3] is not valid in superconductivity in the presence of long-range interactions. It was then speculated that the proofs of this theorem in relativistic theories are also misleading and that the consistency of theories with broken symmetry may follow through the existence of spurious states $|0'\rangle$ which are not the limiting states of any branch of the excitation spectrum, and hence cannot be considered as particle states. While we have no quarrel with this possibility, we would like to point out that there is a large class of theories which will support a broken symmetry through relations of the sort

$$\langle 0,\eta|\varphi(x)|\eta,0\rangle = C(\eta) \tag{1}$$

explicitly because of the natural presence of zero-mass particles in the theory, and where the objections raised in this Letter[1] are not valid. We have introduced a numerical parameter η into the above equation to distinguish explicitly the broken-symmetry vacuum states of this equation from the usual vacuum states $|0\rangle$ for which the expectation value of $\varphi(x)$ vanishes. $C(\eta)$ is a nonvanishing numerical function of η whose exact form depends on the method of construction of $|\eta,0\rangle$ from $|0\rangle$. Our approach will involve a change in viewpoint from that normally associated with broken symmetries. Usually, one regards the broken symmetry requirements as doing violence to the basic structure of the theory in such a way as to pick an alternative solution to the field equations and in so doing inducing new zero-energy states in order to guarantee the consistency of requirement (1) with the operator symmetries of the problem. It is an explicit mark of usual theories that they will not support condition (1) when the interaction is turned off and that the perturbation solutions exploit the nonlinearity of the interaction. This is not the case in some examples we present.

We shall show that there is a class of theories in which it is possible to construct exactly the state $|\eta,0\rangle$ satisfying Eq. (1) from the state $|0\rangle$ for which

$$\langle 0|\varphi|0\rangle = 0.$$

This is done without modifying the energy spectra or the multiplicity of states and hence without introducing any spurious states of the sort $|0'\rangle$. A remarkable result that we obtain is that ordinary electrodynamics with no bare photon mass belongs to this class. We then conclude as an exact consequence of the field equations that there is a zero-mass particle present in interacting electrodynamics which is identified as the photon.

The mechanism to be examined is quite simple. Suppose that in addition to the normally conserved quantities such as energy-momentum and angular momentum there exists a Hermitian operator

$$L_\eta(x^0) = \int d^3x\, l_\eta(x) = \int d^3x\, \eta(x) l(x) \tag{2}$$

which, in the limit that the function $\eta(x)$ is constant, is independent of time and hence commutes with the energy operator p^0. We shall denote the operator in this limit as L_η. The limit $\eta(x)$ = constant is taken by assuming that it has nearly the constant value η over a spatial volume, with a sharp dropoff to zero at the edges of the volume. We then let the volume become infinite. This device insures that the operator defined by (2) converges for $\eta(x)$ not constant. We shall not bother with details of this procedure, but assume that it is possible to proceed formally. It is emphasized, however, that care must be exercised when derivatives of η are taken. Assume for some field operator $\varphi(x)$ that

$$i[L_\eta, \varphi(x)] = C(\eta). \tag{3}$$

Here C is a numerical function. For convenience we have suppressed all indices referring to degrees of freedom. Usually the existence of L_η is connected with the invariance of the Lagrangian under the transformation $\varphi \to \varphi + \eta$. l_η is then the time component of a conserved current which is essentially the momentum canonically conjugate to φ.

It is now possible to conclude that there are zero-mass particles present in the theory. We use the usual set of states $\{|a\rangle\}$ containing the vacuum for which the physical symmetries have the property $S|0\rangle = 0$. Note that $L_\eta|0\rangle \neq 0$. It is emphasized that these states occur in a perfectly normal type of theory and hence presumably do not contain spurious states of the type $|0'\rangle$. Therefore, we may proceed without fear in the fashion ordinarily used to prove the Goldstone theorem. From (3) it follows that

$$i\int d^3y \langle 0|[l_\eta(y), \varphi(x)]|0\rangle = C(\eta).$$

If we write the Lehmann representation for

$$\langle 0|[l_\eta(y), \varphi(x)]|0\rangle,$$

it is seen after performing the spatial integration to produce $C(\eta)$ that the requirement that this integrated quantity is independent of $y^0 - x^0$ guarantees that a zero-mass particle is present. This is the essential observation[2,3] made in proofs of the Goldstone theorem. Henceforth, we shall denote the application of such a procedure as operation G.

This use of (3) to prove the presence of zero-mass particles is very direct, but does not establish the connection with ordinary broken symmetry theories. To do this note that from (3) it follows (η = constant) that

$$\langle 0|\exp(iL_\eta)\varphi(x)\exp(-iL_\eta)|0\rangle = C(\eta). \tag{4}$$

If a new set of states is defined by

$$|\eta, a\rangle = \exp(-iL_\eta)|a\rangle,$$

Eq. (4) may be rewritten as

$$\langle 0, \eta|\varphi(x)|\eta, 0\rangle = C(\eta).$$

Note that since η is constant the states $|\eta, 0\rangle$ have the same energy as the states $|a\rangle$. If the additional assumption is made that there is a continuous symmetry S which essentially has the behavior

$$i^{-1}[S, \varphi] = \varphi,$$

it follows that

$$\begin{aligned}\langle 0, \eta|i^{-1}[S, \varphi]|\eta, 0\rangle &= i^{-1}\textstyle\sum_a[\langle 0, \eta|S|\eta, a\rangle\langle a, \eta|\varphi|\eta, 0\rangle \\ &\quad - \langle 0, \eta|\varphi|\eta, a\rangle\langle a, \eta|S|\eta, 0\rangle] = C(\eta).\end{aligned} \tag{5}$$

Since the states $|\eta, a\rangle$ are characterized by the same spectrum as the states $|a\rangle$, and this spectrum does not have any spurious states $|0'\rangle$ of the sort discussed by Klein and Lee,[1] operation G may be applied without fear to Eq. (5). Thus, we have established in the usual manner that a zero-mass particle is present which carries the same quantum numbers as $\varphi(x)$. It is now possible to understand the physics involved in forming the state $|\eta, a\rangle$. The operator $l_\eta(x)$ of Eq. (2) creates particles of zero mass with the same quantum numbers as $\varphi(x)$. The integration over space in (2) insures that l_η is an operator which creates particles of zero energy. Thus the operator $\exp(il_\eta)$ when applied to $|a\rangle$ modifies this state by the superposition of an infinite number of zero-energy particles.

This transformation does not change the physical information contained in the states, but merely relaxes the usual requirement that $S|0\rangle = 0$. A trivial but extremely enlightening example of a "broken symmetry" occurring through the possibility of finding a conserved operator is the free spin-zero charged field described by the Hermitian operator

$$\varphi = \begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix}$$

through the Lagrange density

$$\mathcal{L} = \varphi^{\mu}\partial_{\mu}\varphi + \tfrac{1}{2}\varphi^{\mu}\varphi_{\mu} - \tfrac{1}{2}\mu^{2}\varphi\varphi$$

with the usual canonical commutation relations

$$i^{-1}[\varphi(y), \varphi^{0}(x)] = \delta^{3}(x-y), \text{ etc., } x^{0} = y^{0}.$$

From the field equations it is seen that the requirement $\langle 0|\varphi|0\rangle \neq 0$ necessitates that $\mu^2 = 0$. With $\mu^2 = 0$, $\mathcal{L}$ is invariant under the transformation $\varphi \to \varphi + \eta$. We may transform this information to the states $|a\rangle$ by going through the procedure outlined in the preceding paragraphs. With $\mu^2 = 0$ the field equations require that $\partial_{\mu}\varphi^{\mu} = 0$. Consequently $\int d^3x\, \varphi^0(x)$ is a constant of the motion. Making the identification

$$L_{\eta} = \int d^3x\, \varphi^0(x),$$

it follows from the canonical commutation relations that

$$i[L_{\eta}, \varphi(x)] = \eta.$$

Hence a zero-mass particle is present. As we already have seen, we define the states

$$|\eta, a\rangle = \exp(-i\int d^3x\, \eta\varphi^0)\,|a\rangle.$$

Note that the previous interpretation of an infinite superposition of soft quanta is valid here. It only remains to observe that the conserved charge

$$Q = i\int d^3x\, \varphi q \varphi^0,$$

where $q = \sigma_2$, has the property that

$$[Q, \varphi] = -q\varphi.$$

The next example is the free electromagnetic field. From the requirements of classical gauge invariance, it would seem legitimate to demand that

$$\langle 0|A^{\mu}(x)|0\rangle = C^{\mu},$$

where C^{μ} is a number. As a matter of convention, the states and operators of quantum electrodynamics are usually chosen to exclude this possibility. The possibility may be reintroduced by noting that since

$$\partial_{\mu}F^{k\mu} = 0,$$

the quantity

$$l_{\eta} = \int d^3x\, \eta_k F^{k0} \tag{6}$$

is a constant of the motion in the limit that η_k is constant. The form that $C^{\lambda}(\eta)$ takes, where

$$i[L_{\eta}, A^{\lambda}(x)] = C^{\lambda}(x),$$

is dependent on the particular gauge of A. In general, $C^{\lambda}(\eta)$ is a nonvanishing linear function of η and we conclude a zero-mass particle is present. In the Lorentz gauge it is easily established that $C^{\lambda}(\eta) = \eta^{\lambda}$ if we define $\eta^0 = 0$. In radiation gauge one obtains similar results. This emphasizes the fact that this effect is the result of the physical transverse particle. More explicit analysis of the structures involved makes this point transparent. Proceeding as before, the new states $|\eta, a\rangle$ are defined by

$$|\eta, a\rangle = \exp(-i\int d^3x\, \eta_k F^{k0})\,|a\rangle.$$

The symmetry S that is broken in this case is Lorentz invariance since

$$i^{-1}[J^{\mu\nu}, A^{\lambda}(x)] = g^{\mu\lambda}A^{\nu}(x) - g^{\nu\lambda}A^{\mu}(x) + \partial^{\lambda}\Lambda^{\mu\nu} + (x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu})A^{\lambda}.$$

In this expression the explicit form of $\Lambda^{\mu\nu}$ depends on the gauge chosen for A^{λ}. Taking the vacuum expectation and passing to the limit η = constant we find

$$\begin{aligned}\langle 0\eta|i^{-1}[J^{\mu\nu}, A^{\lambda}(x)]|\eta 0\rangle &= g^{\mu\lambda}C^{\nu} - g^{\nu\lambda}C^{\mu} \\ &\quad + \partial^{\lambda}\langle 0, \eta|\Lambda^{\mu\nu}|\eta, 0\rangle.\end{aligned} \tag{7}$$

In all conventional gauges

$$\partial^{\lambda}\langle 0, \eta|\Lambda^{\mu\nu}|\eta, 0\rangle = 0.$$

In a more explicit analysis of the left-hand side of (7) it is necessary to exercise some care in passing to the limit η = constant when using conventional forms of $J^{\mu\nu}$. We may apply operation G to Eq. (7) and again conclude there are zero-mass photons present.

A far less trivial example is the case of interacting electrodynamics. There $\partial_{\mu}F^{k\mu} = j^k$ and hence the quantity

$$\exp(L_{\eta}) = \int d^3x\, \eta_k [F^{k0} - x^k j^0] \tag{8}$$

is independent of time in the limit η_k = constant. This relation is a simple generalization of (6) to the case of interacting electrodynamics.

It is particularly convenient since in most gauges

$$[j^0(x), A^\lambda(y)]_{x^0=y^0}=0.$$

We may therefore carry over the preceding analysis because

$$[\exp(L_\eta), A^\lambda(y)]=[L_\eta, A^\lambda(y)].$$

The form of Eq. (7) is unchanged in the presence of interactions and hence we may again apply operation G and conclude that there is a zero-mass particle present in ordinary electrodynamics with the same quantum numbers as $A^\lambda(x)$. We emphasize that this particle is not always directly associated with the "free-field photon." In the case that the coupling is very large,[4] it might be possible to produce zero-mass "positronium" which will dominate contributions to the consistency of Eq. (7). If zero-mass Fermi particles are present, we may have essentially the same situation. In the Schwinger model the photon has no dynamical degrees of freedom and a radiation gauge formulation fails for a conserved current. However, a zero-mass particle is present because of arguments related to the ones given above.[5] It would be particularly interesting if these arguments could be extended to the Yang-Mills field. To the present, such attempts have been unsuccessful. It should be pointed out that any interference that transformation (8) makes with Lorentz invariance when used to form $|\eta, a\rangle$ is trivial and may be handled with ease in more explicit calculations. A detailed paper discussing gauge problems, the consistency of (7) with the field equations, and more explicit consequences of Eq. (7) is in preparation.

The theories considered so far are characterized by working in the free-field limit. On the other hand, broken symmetry theories of primary interest at present are such that the free field cannot support the broken symmetry. It is of great concern, then, to know if any of these theories are of types that can be characterized by time-independent transformations L_η and hence certainly have zero-mass states. This characterization is almost certainly not possible for the existing theories and hence we have not constructed a general proof of the Goldstone theorem. However, the above emphasizes the fact that all general arguments presently available to insure the presence of zero-mass particles are equivalent to the broken-symmetry procedure. Therefore, it is not desirable to dismiss lightly the power of the Goldstone theorem. Work which will be published elsewhere suggests that in a fully quantized relativistic theory it is highly probable, where spurious states $|0'\rangle$ occur to guarantee the consistency of a theory, that they are linked to the presence of a zero-mass particle.

I am indebted to W. Gilbert for many discussions on broken symmetries and the suggestion that the photon mass might be explained through a broken symmetry. I enjoyed the benefit of many very enlightening conversations with R. F. Streater on the topics discussed in this paper. I would like to thank John M. Charap and P. T. Matthews for useful comments and for reading the manuscript, and Professor Abdus Salam for hospitality.

---

*The research reported in this document has been sponsored in part by the U. S. Air Force Office of Scientific Research Office of Aerospace Research, through the European Office of Aerospace Research, U. S. Air Force.

†National Science Foundation Postdoctoral Fellow.

[1]A. Klein and B. W. Lee, Phys. Rev. Letters 12, 266 (1964).

[2]J. Goldstone, Nuovo Cimento 19, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962); S. A. Bludman and A. Klein, Phys. Rev. 131, 2364 (1963).

[3]G. S. Guralnik, thesis, Harvard University (unpublished).

[4]J. Schwinger, Phys. Rev. 125, 397 (1962).

[5]J. Schwinger, Phys. Rev. 128, 2425 (1962); L. S. Brown, Nuovo Cimento 29, 617 (1963); B. Zumino, to be published.