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OFF-SHELL CORRECTION IN PION PHOTOPRODUCTION

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In this note we present some preliminary results of analyzing pion photoproduction data^{1,2} by taking into account the virtuality of the exchanged pion in the Drell model.^{3,4}

Ferrari and Selleri⁵ derived an approximate expression for the off-shell pion-nucleon 3, 3 scattering amplitude containing an unknown pion form factor $K(\Delta^2)$ depending only on Δ^2 , the square of the four-momentum of the virtual pion. They applied their result to an analysis,⁶ on the basis of the one-pion exchange (OPE) mechanism, of single-pion production data from nucleon-nucleon collisions. It is shown that in the calculation of the amplitude for this process there occurs the function

$$\Phi(\Delta^2) = K^2(\Delta^2)K'(\Delta^2)\psi(\Delta^2) \quad (1)$$

where $K(\Delta^2)$ appears twice since it is associated with each pion-nucleon vertex, $K'(\Delta^2)$ contains

all the higher order corrections to the pion propagator, and $\psi(\Delta^2)$ is a known function depending on the parameters of the 3, 3 resonance. The moderate success they met in substantiating the OPE by fitting this data with an empirical function $\Phi(\Delta^2)$ suggests the importance of a similar calculation in the case of photoproduction.

Drell's expression³ for photoproduction of negative pions from a heavy target nucleus A is

$$\frac{d^2\sigma}{dpd\Omega} = \frac{\alpha}{8\pi^2} \left(\frac{\sin\theta}{1 - \beta \cos\theta} \right)^2 \frac{p(k - \omega)}{k^3} \sigma_A(T) \quad (2)$$

where the photon has energy k , the pion of mass μ is observed in solid angle $d\Omega$ about θ and in momentum interval dp about p . The corresponding energy is ω , the velocity is β , and total π^+ - A cross section at kinetic energy $T = k - \omega - \mu$ is $\sigma_A(T)$. In Drell's terminology, the photon pro-

duces a $\pi^+ - \pi^-$ pair in the Coulomb field of the target A , the π^- is observed, while the virtual π^+ interacts strongly with the target, and initiates any undetected final states.

In case A is a proton, we assume that the $\pi^+ - p$ interaction occurs predominantly in the 3, 3 state so that the Ferrari-Selleri result may be applied unambiguously. If A is a complex nucleus, the situation is somewhat ambiguous, but in what follows we assume that the strong interaction within A is purely quasi-elastic (i. e., a single nucleon, treated as an unbound particle, participates in the interaction) and also occurs in the 3, 3 state. In this way, both cases are treated in a unified manner, although admittedly we may be drastically oversimplifying the treatment of complex nuclei.

The finite mass of the nucleon and, following Ferrari and Selleri,^{5,7} the off-shell nature of the exchanged pion can thus be incorporated in Eq. (2), in case A is a proton, by replacing $\sigma_A(T)$ by $\sigma_{33}(T, \Delta^2)$, where

$$\sigma_{33}(T, \Delta^2) = \sigma_{33}(T, \mu^2)C(T, \Delta^2). \quad (3)$$

$\sigma_{33}(T, \mu^2)$ is the on-shell 3, 3 pion-nucleon cross section, and⁸

$$C(T, \Delta^2) \equiv \frac{q_L}{k - \omega} \left(\frac{p}{\omega} \right)^3 \left[1 + \frac{\mu^2 - \Delta^2}{4m^2} \right] \times \left[\frac{1 + 3\alpha_r}{(1 + \alpha_r)^3} \right]^2 \frac{q_i}{q_f} K^2(\Delta^2). \quad (4)$$

In Eq. (4), m = nucleon mass, $\alpha_r = (\mu^2 - \Delta^2)/2m \times (E_r - m)$, E_r is the 3, 3 resonance energy (1.238 BeV), q_L is the virtual pion momentum in the laboratory system, and q_i and q_f are, respectively, the initial and final momenta in the barycentric system of the virtual pion and target nucleon. Also T must be redefined in (2), (3), and (4) to be the kinetic energy of the real pion emerging from the strong interaction in the rest frame of the recoil nucleon.

Since higher order corrections to the pion propagator and the $\gamma - \pi^+ + \pi^-$ vertex precisely cancel⁹ by Ward's identity, only $K^2(\Delta^2)$ appears in (4). It is interesting to observe that knowledge of $K^2(\Delta^2)$ from analysis of photoproduction data could, upon comparison with the empirical function $\Phi(\Delta^2)$ in (1), lead to determination of the higher order corrections $K'(\Delta^2)$ to the pion propagator. Such a determination seems unwarranted at this preliminary stage of our analysis since it

would be undesirable to depend heavily on photoproduction data from a complex nucleus as we are doing in this note.

For complex nuclei, a plausible (but speculative) way to make the extensions corresponding to Eq. (3) is to replace $\sigma_A(T)$ by

$$\sigma_A(T, \Delta^2) = \sigma_A(T, \mu^2)C(T, \Delta^2) \quad (5)$$

where $\sigma_A(T, \mu^2)$ is that part of the pion-nucleus cross section attributed to quasi-elastic scattering in the 3, 3 state.¹⁰

Equations (3) and (5) were inserted in (2) in our analysis for values of T below the 3, 3 resonance. Above the resonance the cross sections on the right-hand side of (3) and (5) were replaced by total cross sections since the off-shell corrections for the other states now contributing substantially to the cross sections are hopefully not considerably different from Eq. (4). This procedure is sensible for experiments where T is restricted to the region of the 3, 3 resonance. We should point out, however, that in the experiments considered here the range of T is ≈ 0.0 -1.0 BeV where other states are indeed important.

Since $q_i > q_f$, the factor (q_i/q_f) in Eq. (4) when considered alone causes a possibly significant enhancement of the cross section. If we fold in a bremsstrahlung spectrum $\varphi(k)dk$, the region of integration may include $q_f = 0$, where $q_i \neq 0$. A singularity is prevented in (3), as pointed out by Ferrari and Selleri,⁶ since $\sigma_{33}(T, \mu^2)$ vanishes as q_f^4 .

It is possible to unfold approximately the integration over $\varphi(k)$. We define

$$N(p, \theta) \equiv \int dk \varphi(k) C_1(T, \Delta^2) d^2\sigma / dp d\Omega \quad (6)$$

where the subscript 1 means that we have set $K^2(\Delta^2) = 1$ in Eq. (4), and we define $E(p, \theta)$ to be the corresponding experimental value. Then $K^2(\Delta_0^2) = E(p, \theta)/N(p, \theta)$ where Δ_0^2 , the average value of Δ^2 in the region of integration, is given by

$$\Delta_0^2 = [1/N(p, \theta)] \int dk \varphi(k) C_1(T, \Delta^2) \Delta^2 d^2\sigma / dp d\Omega. \quad (7)$$

This procedure correctly establishes a value of $K^2(\Delta^2)$ if $K^2(\Delta^2)$ is linear in Δ^2 over the region of integration, a justifiable assumption since the range of values of Δ^2 in the integral is small for the experiments considered here.

Figure 1 shows values of $K^2(\Delta^2)$ determined in this manner using data from pion production in hydrogen and beryllium. The curves in Fig. 1

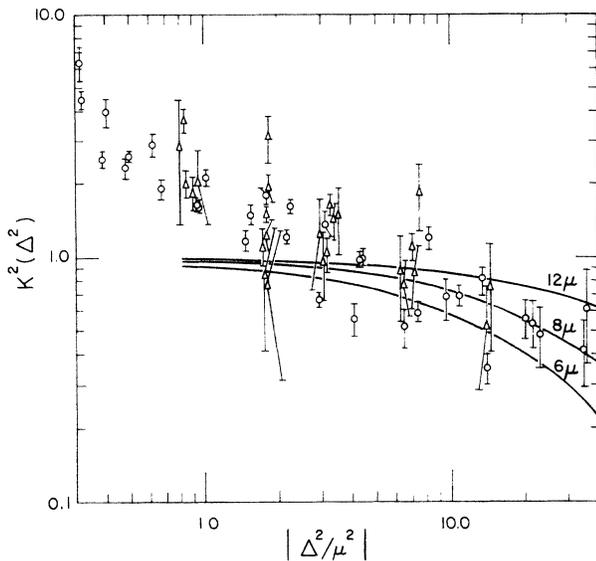


FIG. 1. Pionic form factor squared vs mass squared of virtual pion. Circles indicate experimental points from reference 2, triangles from reference 1. A few data from reference 1 are omitted for clarity. The curves are plots of Eq. (8). Values of a are indicated above the curves.

are crude fits to the data of the form

$$K^2(\Delta^2) = [1 + (\mu^2 - \Delta^2)/a^2]^{-2} \quad (8)$$

with $a = 6\mu$, 8μ , and 12μ .

Figure 2 illustrates the importance of including the term $C(T, \Delta^2)$ in the calculation. The Drell angular distribution is plotted for photoproduction in beryllium from a photon spectrum with maximum $k = 5.82$ BeV and $p = 5.0$ BeV/c. Four cases are considered: (a) $C(T, \Delta^2)$ given by Eq. (4) is used in (5) with $K^2(\Delta^2)$ given by (8) and $a = 8\mu$; (b) same as (a) but with $K^2(\Delta^2) = 1$; (c) $C(T, \Delta^2)$ is replaced by 1; (d) same as (c) but with $m = \infty$ (static approximation).

We list some conclusions, evident from the figures:

(i) Although there is quite a scattering of values of $K^2(\Delta^2)$ over the range of Δ^2 , there is a general trend for $K^2(\Delta^2)$ to decrease with $|\Delta^2|$ as in the corresponding analysis of reference 2. The main difference is that our analysis has pulled down $K^2(\Delta^2)$ by a factor of almost 2 for the larger momentum transfers.

(ii) There is no clear separation in Fig. 1 between the data for production in hydrogen (reference 1) and that for production in beryllium (reference 2). This suggests that the quasi-elastic treatment of complex nuclei is perhaps justified.

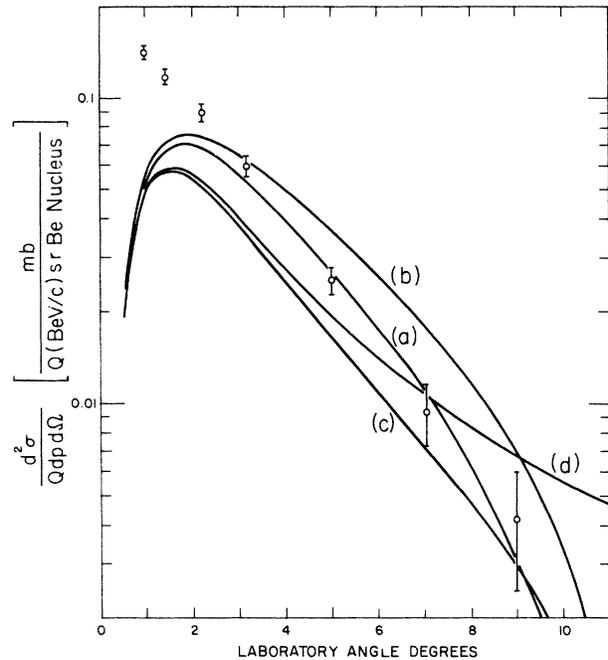


FIG. 2. Plot of photoproduction cross section per equivalent quantum for observing a 5.0-BeV/c π^- produced in Be by bremsstrahlung of peak energy 5.82 BeV. See text for explanation of the curves. The experimental points are from reference 2.

(iii) The data are not grossly incompatible with Eq. (8) for $|\Delta^2| > 3\mu^2$.

(iv) The OPE represents only a small part of the physics going on for $|\Delta^2| < 3\mu^2$.

(v) The maximum possible effect of the off-shell correction is quite significant since curve b of Fig. 2 is about 2.5 times larger than the uncorrected curve c over a substantial part of the phase space of the observed pion. Also note that the static curve d deviates sharply from the other curves at sufficiently large angles.

It should be emphasized that these conclusions are for the most part tentative, since it is possible that they depend crucially on the assumptions stated in this note. Further investigations of these assumptions and analysis of more recent pion photoproduction data are in preparation and will be contained in a forthcoming paper.

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SCALING LAW FOR HIGH-ENERGY ELASTIC SCATTERING

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Foley *et al.*¹ have made extensive measurements of elastic p - p , \bar{p} - p , π^\pm - p , and K^\pm - p scattering in the momentum range 7 to 20 BeV/c. In order to compare the results for different incident particles, one may try a scaling law of the type suggested by an optical model²,

$$d\sigma/dt = \pi R^4 f(R^2 t). \quad (1)$$

Here t is the square of the momentum transfer, in units of cm^{-2} , and to make the definition of the scaling length R explicit we take $f(0) = 1$, so that

$$R^2 = (d\sigma/dt|_{t=0}/\pi)^{1/2}. \quad (2)$$

According to this scaling law, we are to compare values of $(d\sigma/dt)/(d\sigma/dt|_{t=0})$ as functions of the square of the momentum transfer measured in dimensionless units,

$$t' = (d\sigma/dt|_{t=0}/\pi)^{1/2} t. \quad (3)$$

If t is given in $(\text{BeV}/c)^2$, rather than cm^{-2} ,

$$t' = 0.90416(d\sigma/dt|_{t=0})^{1/2} t, \quad (4)$$

with $(d\sigma/dt|_{t=0})$ in $\text{mb}/(\text{BeV}/c)^2$.

Foley *et al.* have expressed the experimental results in the form

$$(d\sigma/dt)/(d\sigma/dt|_{t=0}) = \exp(-bt + ct^2), \quad (5)$$

with t in $(\text{BeV}/c)^2$. In terms of the dimensionless variable, (5) takes the form

$$(d\sigma/dt)/(d\sigma/dt|_{t=0}) = \exp(-b't' + c't'^2), \quad (6)$$

with

$$b' = b/[0.90416(d\sigma/dt|_{t=0})^{1/2}],$$

$$c' = c/[0.81751(d\sigma/dt|_{t=0})]. \quad (7)$$

The upper limits of momentum transfers reached in the Brookhaven experiments were $t_{\text{max}} \sim 0.8-1.1(\text{BeV}/c)^2$, except for \bar{p} - p scattering where $t_{\text{max}} \sim 0.5(\text{BeV}/c)^2$. The corresponding values for t' range from $t'_{\text{max}} = 4$ for K^+ - p , to $t'_{\text{max}} = 8$ for p - p scattering.

The values of the experimentally determined parameters are listed in Table I, and shown graphically in Figs. 1 and 2 as functions of p_0 , the momentum of the bombarding particle.

Figure 1 shows that after the rescaling, the value of b' , which describes the initial rate of decrease of the cross section, is virtually identical for bombarding protons or antiprotons, and also identical within the errors, but with a 50% larger value, for all the pseudoscalar mesons. The error on the K^- - p scattering is so large that the result in this case is not compelling. The large errors for K^- - p scattering are a reflection not merely of larger statistical errors of the