the K_1 and K_2 amplitudes.

Therefore, the interference term is of the form

$$2\,\mathrm{Re}\{\xi \exp(i\Delta m t)\exp[-\tfrac{1}{2}(\lambda_1+\lambda_2)t]\},$$

where Δm is the K_1-K_2 mass difference while λ_1 and λ_2 are the decay rates of the K_1 and K_2 mesons. Thus, on the basis of Eq. (1) the interference term would have a magnitude of about 5×10^{-3} relative to the K_1 decay term and it would decay at half the rate. For $\lambda_1 t\approx 4$, the correction to the exponential decay curve would therefore be of the order of 5% and its relative importance would increase with increasing *it*.

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NEUTRINO PRODUCTION OF MUON PAIRS THROUGH A DIRECT FOUR-FERMION COUPLING*

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The production of muon pairs by muon neutrinos has been observed at CERN.[1] If one assumes that intermediate bosons have been produced, then the detection of muon pairs corresponds to observing the final-state products of the process

$$\nu_\mu + Z \to W^+ + \mu^- + Z \to \nu_\mu + \mu^+ + \mu^- + Z. \qquad (1)$$

The observation of muon pairs in itself does not imply that an intermediate boson exists, because of the possibility of the existence of the direct four-fermion coupling

$$(G/\sqrt{2})[\bar{\psi}_\mu \gamma_\lambda (1+\gamma_5)\psi_{\nu_\mu}][\bar{\psi}_{\nu_\mu}\gamma_\lambda(1+\gamma_5)\psi_\mu].$$

The weak coupling constant G, in units where $\hbar=c=1$ and M_p is the proton mass, is $1.01\times10^{-5}/M_p^2$. Such a direct coupling is possible in a current-current formulation of weak interactions.[2] If this coupling exists the process

$$\nu_\mu + Z \to \nu_\mu + \mu^+ + \mu^- + Z \qquad (2)$$

is possible and has the same final-state products as process (1). For incident neutrino energies far above the threshold for intermediate boson production, one expects process (1) to be dominant, because it is proportional to G, while process (2) is proportional to G^2. However, it is known experimentally that the majority of the neutrinos in the CERN experiment cannot be far above threshold, because the "elastic" process

$$\nu_\mu + n \to \mu^- + p,$$

which is proportional to G^2, is dominant in the CERN experiment.[1] Consequently, it is crucial to have an accurate evaluation of process (2), before attributing the presence of muon pairs to the production of intermediate bosons.

In this note the results of exact calculations of the coherent production,

$$\nu_\mu + Z \to \nu_\mu + \mu^+ + \mu^- + Z(\mathrm{coh}), \qquad (3)$$

and the production from free protons,

$$\nu_\mu + p \to \nu_\mu + \mu^+ + \mu^- + p, \qquad (4)$$

are reported.

To lowest order in G and the fine structure constant α, the matrix elements for production from a spin-$\frac{1}{2}$ target particle whose mass is M are[3]

$$\mathfrak{M}_{\text{I}}=(G_e J_\rho/\sqrt{2}q^2)[\bar{u}(P_-)\gamma_\rho S(P_- - q)\gamma_\lambda(1+\gamma_5)u(k_i)]$$
$$\times[\bar{u}(k_f)\gamma_\lambda(1+\gamma_5)v(P_+)],$$

$$\mathfrak{M}_{\text{II}}=(G_e J_\rho/\sqrt{2}q^2)[\bar{u}(P_-)\gamma_\lambda(1+\gamma_5)u(k_i)]$$
$$\times[\bar{u}(k_f)\gamma_\lambda(1+\gamma_5)S(q-P_+)\gamma_\rho v(P_+)],$$

where

$$J_\rho=ie\bar{u}(P_f)\left[\gamma_\rho G_m+\frac{(P_i+P_f)_\rho(G_e-G_m)}{2M(1+q^2/4M^2)}\right]u(P_i).$$

The expression for the square of the matrix elements, after summing over the final spin states and averaging over the initial spin states of the target particle, can be written as

$$\sum_{\text{spins}}|\mathfrak{M}_{\text{I}}+\mathfrak{M}_{\text{II}}|^2=A_{\lambda\rho}M_{\lambda\rho}.$$

The lepton parts of the matrix elements contribute the symmetric tensor $M_{\lambda\rho}$. The electromagnetic vertex at the target particle contributes the symmetric tensor $A_{\lambda\rho}$. The algebra of the spin sums is greatly reduced by use of a Fierz transformation.[4]

In terms of the Chew-Low variables,[5] $w^2=-(k_i+q)^2$ and q^2, and the invariants

$$I_1(w^2,q^2)=\int\cdots\int\text{tr}(M_{\lambda\rho})\delta^{(4)}(k_i+q-P_- -P_+ -k_f)$$
$$\times\frac{d^3P_-}{2E_-}\frac{d^3P_+}{2E_+}\frac{d^3k_f}{2\omega_f},$$

$$I_2(w^2,q^2)=\int\cdots\int k_{i\lambda}k_{i\rho}M_{\lambda\rho}\delta^{(4)}(k_i+q-P_- -P_+ -k_f)$$
$$\times\frac{d^3P_-}{2E_-}\frac{d^3P_+}{2E_+}\frac{d^3k_f}{2\omega_f},$$

the total cross section can be written in the form[6]

$$\sigma=\frac{1}{(2\pi)^8}\int_{w_{\min}^2}^{w_{\max}^2}\int_{w_{\min}^2}^{q_{\max}^2}\frac{\pi dw^2dq^2}{16(P_i\cdot k_i)^2}A_{\lambda\rho}T_{\lambda\rho}$$

where

$$T_{\lambda\rho}=\left[\frac{I_1}{(w^2+q^2)}+\frac{12q^2I_2}{(w^2+q^2)^3}\right]\left[\frac{2q^2}{(w^2+q^2)}k_{i\lambda}k_{i\rho}+\frac{(w^2+q^2)}{2}\delta_{\lambda\rho}+k_{i\lambda}q_\rho+k_{i\rho}q_\lambda\right]-\frac{4I_2}{(w^2+q^2)^2}[q^2\delta_{\lambda\rho}-q_\lambda q_\rho].$$

By use of covariant integration techniques, I_1 and I_2 were written as double integrals over Lorentz invariants. In order to represent I_1 and I_2 as double integrals, it was necessary to do one nontrivial integration which produced logarithms. The resulting expressions are much too lengthy to be given here.[7]

The remaining four integrations were done with the aid of an IBM-7090 computer. The numerical integration was done by using the University of Michigan Computing Center's subroutine for multiple integrals. The subroutine was checked by numerically integrating several four-dimensional integrals which could be done analytically. The numerical and exact integrations always agreed within 0.1%.

In the calculation of process (3), the target mass was set equal to infinity and G_e replaced by $ZF(q^2)$, where $F(q^2)$ is the nuclear form factor. The total cross section for process (3) has been calculated in a Weizsäcker-Williams-like approximation by Shabalin.[8] For purposes of comparison, the total cross section for process (3) was calculated using the same form factor that Shabalin used. The results are given in Table I. Shabalin's results are from 2 to 3 times smaller than the present calculation. Process (3) with Shabalin's form factor has also been numerically calculated recently by Czyz, Sheppey, and Walecka.[9] The results of these authors are generally 5-6% higher than the present calculation.

The total cross section for process (3) was calculated with the more realistic Fermi form

Table I. A comparison of the numerical and approximate values of the total coherent cross section for the process $\nu_\mu+{}_{26}\text{Fe}^{56}\rightarrow\nu_\mu+\mu^++\mu^-+{}_{26}\text{Fe}^{56}$. The form factor used is given in reference 8.

Neutrino energy (BeV)	Shabalin (cm^2)	Present calculation (cm^2)
0.5	1.3×10^{-45}	2.36×10^{-45}
1.0	2.6×10^{-44}	5.42×10^{-44}
2.0	2.0×10^{-43}	4.86×10^{-43}
20.0	1.8×10^{-41}	5.53×10^{-41}

Table II. The numerical values of the total coherent cross section for the process $\nu_\mu + Z \to \nu_\mu + \mu^+ + \mu^- + Z$ for two different target nuclei, ${}_{29}Cu^{63.5}$ and ${}_{13}Al^{27}$. The form factor used is given in reference 10.

Neutrino energy (BeV)	σ(coh) for ${}_{29}Cu^{63.5}$ (cm^2)	σ(coh) for ${}_{13}Al^{27}$ (cm^2)
0.5	1.34×10^{-45}	0.697×10^{-45}
1.0	5.08×10^{-44}	1.96×10^{-44}
2.0	5.53×10^{-43}	1.83×10^{-43}
3.0	1.66×10^{-42}	5.20×10^{-43}
4.0	3.33×10^{-42}	1.01×10^{-42}
6.0	8.04×10^{-42}	2.37×10^{-42}
10.0	2.19×10^{-41}	6.21×10^{-42}

factor as given by Hofstadter and Blankenbecler.[10] The results for two different target nuclei, ${}_{29}Cu^{63.5}$ and ${}_{13}Al^{27}$, are given in Table II.

The total cross section for process (4) was calculated using the proton form factors of Hand, Miller, and Wilson.[11] The results are given in Table III. The range of q^2 in the numerical cases considered here is such that the proton form factors are needed beyond their presently measured values. However, an integration with constant form factors showed that for the energies considered here the total cross section for process (4) is quite insensitive to the presently unknown high momentum-transfer region. The total cross section for process (4) has been numerically calculated, by Czyz, Sheppey, and Walecka,[9] with the older proton form factors of Hofstadter, Bumiller, and Yearian.[12] This calculation differs from the present one by at most 10%.

In the quasielastic scattering approximation, the total cross section for production from a

Table III. The numerical values of the total cross section for the process $\nu_\mu + p \to \nu_\mu + \mu^+ + \mu^- + p$. The proton form factors used are given in reference 11.

Neutrino energy (BeV)	σ_p (cm^2)
0.5	5.77×10^{-47}
1.0	1.37×10^{-45}
2.0	1.05×10^{-44}
3.0	2.71×10^{-44}
4.0	4.93×10^{-44}
6.0	1.07×10^{-43}
10.0	2.60×10^{-43}

complex nucleus, i.e., process (2), can be roughly approximated by[13]

$$\sigma(\text{tot}) = \sigma(\text{coh}) + Z\sigma_p.$$

The values of σ(tot) obtained in this manner have not been folded into the CERN neutrino spectrum, because of some uncertainty in the presently available estimate of the neutrino spectrum. However, our results indicate that it is very unlikely that the number of muon pair events explainable by a direct four-fermion coupling acting with universal strength would be more than 0.05% of the observed "elastic" events.

In conclusion we wish to point out the following effects which have been neglected: (1) higher order weak terms; (2) the contribution from the neutrons in the quasielastic scattering; (3) the Pauli exclusion principle in the quasielastic scattering. The first effect is not understood at all now and is impossible even to estimate. The last two effects tend to compensate one another.[14] They could be calculated without too much additional labor, but this seems unwarranted at this time.

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SIGMA LEPTONIC DECAYS AND CABIBBO'S THEORY OF LEPTONIC DECAY*

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The purpose of this Letter is to present the final results on Σ^+ and Σ^- leptonic decays from the CERN stopping-K^- experiment and to compare these results with Cabibbo's theory of weak interactions.

A sample of about 400 000 Σ^+ and Σ^- hyperons were studied for leptonic decays of the following six types:

$$\left.\begin{array}{l}\Sigma^- \to n + e^- + \nu \quad (1a)\\ \Sigma^- \to n + \mu^- + \nu \quad (1b)\end{array}\right\} \frac{\Delta S}{\Delta Q} = +1,$$

$$\left.\begin{array}{l}\Sigma^- \to \Lambda + e^- + \nu \quad (1c)\\ \Sigma^+ \to \Lambda + e^- + \nu \quad (1d)\end{array}\right\} \Delta S = 0,$$

$$\left.\begin{array}{l}\Sigma^+ \to n + e^+ + \nu \quad (1e)\\ \Sigma^+ \to n + \mu^+ + \nu \quad (1f)\end{array}\right\} \frac{\Delta S}{\Delta Q} = -1.$$

This report is based on the observation of 130 $\Sigma^\pm$ leptonic decays. The Σ hyperons were produced by K^- mesons, from the CERN proton synchrotron, coming to rest in the Saclay 81-cm hydrogen bubble chamber. The details of the experimental method will be published in an extended paper elsewhere.[1]

(A) Relative strength of $\Delta S = +\Delta Q$ and $\Delta S = -\Delta Q$ transitions.—No definite event of the type $\Delta S = -\Delta Q$ has been seen.

We have found 52 unambiguous $\Sigma^- \to n + e^- + \nu$ events and 22 unambiguous $\Sigma^- \to n + \mu^- + \nu$ events versus zero $\Sigma^+ \to n + (e^+, \mu^+) + \nu$ events. Given the differences in production ratios of Σ^- and Σ^+ hyperons from (K^-, p) reactions at rest[2] and the criteria[1] imposed on our events to eliminate background, the ratio of $\Sigma^+ \to n + \pi^+$ decays to $\Sigma^- \to n + \pi^-$ decays is calculated to be 1/3.8. If we define

$$\rho = \frac{\text{rate of } \Delta Q = -\Delta S \text{ transitions}}{\text{rate of } \Delta Q = +\Delta S \text{ transitions}},$$

we find that the upper limit with 90% confidence for ρ is 12%. The Columbia-Rutgers-Princeton collaboration independently obtains a similar upper limit of 15%.[3] It is quite certain that these decays are at least considerably suppressed in rate, and perhaps absent altogether.

(B) $\Delta S = 0$ and $\Delta S = +\Delta Q$ hyperon decay rates; and tests of $\Delta I = 1$ rule and (μ, e) universality.—The rates, or branching ratios, of the decay modes that we have observed are summarized in Table I, together with the predictions of the original universal Fermi interaction (UFI),[4] conserved vector current (CVC) theory.[5] Our experimental branching ratio $R_{e^-} = (\Sigma^- \to n + e^- + \nu)/(\Sigma^- \to n + \pi^-) = (1.4 \pm 0.3) \times 10^{-3}$ is in excellent agreement with two other recent measurements that yielded[3]