CP VIOLATION IN K^0 DECAYS*

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Christenson, Cronin, Fitch, and Turlay[1] have reported an apparent violation of CP invariance which manifests itself in the decay mode $K_2^0 \to 2\pi$. The amplitude of this mode relative to the normal $K_1^0 \to 2\pi$ amplitude is quoted as approximately

$$|\xi| = 2.3 \times 10^{-3}, \quad (1)$$

a result which, if verified by further experiments, is somewhat surprising because of the small size of the effect. The purpose of this Letter is to suggest that the effect may be an indirect manifestation of a "maximal" CP violation in the leptonic interactions of the K^0 meson. Such a leptonic mechanism requires that the $\Delta S = \Delta Q$ rule be strongly violated. "Maximal violation" is then defined by the statement that the $\Delta S = -\Delta Q$ interaction is out of phase with the $\Delta S = \Delta Q$ interaction by $\frac{1}{2}\pi$. If this description is correct, the interpretation of most of the experiments designed to test the $\Delta S = \Delta Q$ rule in K^0 decay is probably not valid.

The states of the K_1^0 and K_2^0 mesons may be written in the form[2]

$$|K_1^0\rangle = N(|K^0\rangle + r|\bar{K}^0\rangle),$$
$$|K_2^0\rangle = N(|K^0\rangle - r|\bar{K}^0\rangle),$$

where N is a normalization constant and r is related to the off-diagonal elements of the "mass-squared matrix," W, by

$$r = [\langle \bar{K}^0|W|K^0\rangle / \langle K^0|W|\bar{K}^0\rangle]^{1/2}. \quad (3)$$

If the ratio of the 2π-decay amplitudes of the K^0 and $\bar{K}^0$ is denoted by

$$\zeta = A(\bar{K}^0 \to 2\pi)/A(K^0 \to 2\pi), \quad (4)$$

then the ratio of the K_2^0 and K_1^0 amplitudes is clearly given (CPT invariance is assumed) by

$$\xi = (1 - \zeta r)/(1 + \zeta r). \quad (5)$$

The apparent experimental magnitude of ξ is given by Eq. (1). We note that two (nearly) independent parameters are involved in ξ, one of them (ζ) measuring the CP violation in the 2π-decay amplitude only and the other (r) measuring the violation in all possible virtual processes. CP invariance implies both

$$\zeta = 1 \quad (6)$$

and

$$r = 1, \quad (7)$$

from which the usual result

$$\xi = 0 \quad (8)$$

follows.

The value given in Eq. (1) would indicate that ζr differs from one by only a very small amount,

$$\zeta r = 1 + \epsilon, \quad (9)$$

with

$$|\epsilon| \approx 5 \times 10^{-3}. \quad (10)$$

This may imply that $\zeta \neq 1$, or $r \neq 1$, or both. If $\zeta \neq 1$ it follows, in general, that $r \neq 1$, since the violation of CP at the $K^0 \to 2\pi$ vertex indicated by $\zeta \neq 1$ implies that the contribution of virtual pairs of pions to the matrix elements of W (which is the self-energy matrix) will also violate CP, and r is determined by these matrix elements as indicated in Eq. (3).

It may seem that, by virtue of this relationship, a maximal CP violation in the nonleptonic amplitude, expressed by a large deviation of ζ from unity, could be compensated by an almost equal and opposite deviation in r to account for the small 2π rate. Although a model satisfying this condition can be constructed, it requires that the phase of the $K\pi\pi$ vertex be constant for all values of the momenta off the mass shell and independent of the isotopic spin of the $\pi\pi$ state.[3] Furthermore, the model requires that contributions to the K^0-meson self-energy matrix from 3π and 4π states are negligible. Finally, the model would be expected to lead to a large violation of T invariance in the decay $\Lambda^0 \to p + \pi$.

Since no such violation has been observed[4] and the other assumptions do not seem reasonable, we turn to the possibility that CP is strictly valid in the nonleptonic interactions so that $\zeta = 1$. Then

$$r = 1 + \epsilon, \quad (11)$$

where the experimental estimate of $|\epsilon|$ is given by Eq. (10). The deviation ϵ must now be due entirely to the contribution of leptonic interactions to the self-energy matrix. Let us assume that the violation of CP for these interactions is as large as possible. A theoretical estimate of a rough upper limit on the resulting value of ϵ may be made by noting that the leptonic coupling enters both the mass matrix and the partial decay rate quadratically. Hence the ratio of the leptonic to the nonleptonic contribution to $\langle \bar{K}^0 | W | K^0 \rangle$ and to $\langle K^0 | W | \bar{K}^0 \rangle$ may be of the order of the branching ratio for leptonic decays, that is, about 1/600. A maximum violation of CP due to this contribution would result if the contribution had opposite signs for the two matrix elements. In that case, Eqs. (3) and (11) yield[5]

$$|\epsilon| \approx 1/300, \tag{12}$$

which is somewhat smaller than the value, Eq. (10), given by the experiment. However, in view of the uncertainty associated with the connection between the self-energy matrix, which involves a divergent integral, and the lifetime, this small discrepancy does not seem significant, but it does emphasize that from the point of view of a leptonic violation, the observed effect is a very large one.

The leptonic contribution to the off-diagonal elements of W required for our purpose can only occur if the $\Delta S = \Delta Q$ rule is violated. The states $|K^0\rangle$ and $|\bar{K}^0\rangle$ which differ by $\Delta S = 2$ and $\Delta Q = 0$ are connected by this matrix element and the connection must be due to the emission and reabsorption of leptons along with one or more pions; therefore, $\Delta S = -\Delta Q$ for either the emission or absorption vertex.

If the amplitude for $K^0 \to \pi^- + e^+ + \nu$ ($\Delta S = \Delta Q$) is denoted by f and that for $\bar{K}^0 \to \pi^- + e^+ + \nu$ ($\Delta S = -\Delta Q$) is denoted by g, the leptonic contributions to the self-energy depend on these amplitudes as follows:

$$\langle \bar{K}^0 | W | K^0 \rangle_{\text{lept}} \sim f g^* \tag{13}$$

and

$$\langle K^0 | W | \bar{K}^0 \rangle_{\text{lept}} \sim f^* g. \tag{14}$$

In order that these matrix elements yield a result of the order of Eq. (12), g must be of the same order as f, <u>hence the violation of $\Delta S = \Delta Q$ must be strong</u>. Furthermore, to obtain a CP violation, fg^* must not be real; in fact, for the violation to be of the order suggested above, it must be almost purely imaginary. We are thereby led to the following definition of maximal CP violation: <u>The violation of CP in the leptonic decays is maximal if the $\Delta S = -\Delta Q$ interaction is out of phase with the $\Delta S = \Delta Q$ interaction by $\frac{1}{2}\pi$.</u>

The experimental result, Eq. (1), indicates that such a maximal violation does indeed occur. This may be tested by other experiments; in particular, the test for CP suggested by Sachs and Treiman,[6] based on the time dependence of the total leptonic decay rate, will show a very large effect if this suggestion is correct. This experiment requires a statistically much more reliable curve than has been obtained to date in connection with the $\Delta S = \Delta Q$ experiments.[7]

These tests of the $\Delta S = \Delta Q$ rule have leaned very heavily on the presumption of CP invariance. Conclusions have been reached by fitting the total leptonic decay-rate curve to the sum of two exponentials, but the CP violation suggested here would lead to large deviations from such a time dependence, so that averages over broad time intervals, such as those that have been taken, would be most misleading.

The only reliable test of $\Delta S = \Delta Q$ in the K_{l3}^0 decays would appear to be the time dependence of the charge asymmetry, but presently available data do not suffice to lead to any conclusion. Of course, the CP violation itself leads to a charge asymmetry, but if $\Delta S = \Delta Q$ this is an effect of order ϵ, appearing as a time-independent shift of the magnitude of the $\pi^+ l^- \bar{\nu}$ decay curve relative to the $\pi^- l^+ \nu$ curve. It would be most evident as a charge asymmetry in the K_2^0 decay, which must be expected to be of the order of the $K_2 \to 2\pi$ rate in any case.

Tests of T invariance involving the polarization of the muons or electrons in the leptonic decay of both hyperons and K mesons are also essential for clarification of the situation. For a maximal violation as defined here, the polarization must be due to interference between the $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ amplitudes. Therefore, no T-violating polarization effect would be expected either in hyperon decays or in $K^\pm$ decays since the $\Delta S = \pm\Delta Q$ transitions do not interfere in those cases. However, a strong polarization effect might be expected in the case of K_2^0 decay, for example.

One final remark concerning the $K^0 \to 2\pi$ mode should be made. The existence of a $K_2^0 \to 2\pi$ mode leads immediately, without reference to any particular theory, to a small deviation from the exponential form for the apparent decay of the K_1^0 meson. For a beam consisting only of K^0 mesons at $t = 0$, the time dependence of the 2π amplitude is given by

$$A_{2\pi}(t) \sim e^{-i\omega_1 t} + \xi e^{-i\omega_2 t},$$

where ω_1 and ω_2 are the complex frequencies of the K_1 and K_2 amplitudes.

Therefore, the interference term is of the form

$$2\,\mathrm{Re}\{\xi \exp(i\Delta m t)\exp[-\tfrac{1}{2}(\lambda_1+\lambda_2)t]\},$$

where Δm is the K_1 - K_2 mass difference while λ_1 and λ_2 are the decay rates of the K_1 and K_2 mesons. Thus, on the basis of Eq. (1) the interference term would have a magnitude of about 5×10^{-3} relative to the K_1 decay term and it would decay at half the rate. For $\lambda_1 t \approx 4$, the correction to the exponential decay curve would therefore be of the order of 5% and its relative importance would increase with increasing *it*.

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[5]This estimate of a leptonic effect on the 2π mode has already been presented by Sachs and Treiman, reference 3, footnote 8.

[6]Sachs and Treiman, reference 3.

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NEUTRINO PRODUCTION OF MUON PAIRS THROUGH A DIRECT FOUR-FERMION COUPLING*

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The production of muon pairs by muon neutrinos has been observed at CERN.[1] If one assumes that intermediate bosons have been produced, then the detection of muon pairs corresponds to observing the final-state products of the process

$$\nu_\mu + Z \to W^+ + \mu^- + Z \to \nu_\mu + \mu^+ + \mu^- + Z. \qquad (1)$$

The observation of muon pairs in itself does not imply that an intermediate boson exists, because of the possibility of the existence of the direct four-fermion coupling

$$(G/\sqrt{2})[\bar{\psi}_\mu \gamma_\lambda (1+\gamma_5)\psi_{\nu_\mu}][\bar{\psi}_{\nu_\mu}\gamma_\lambda(1+\gamma_5)\psi_\mu].$$

The weak coupling constant G, in units where $\hbar = c = 1$ and M_p is the proton mass, is $1.01\times10^{-5}/M_p^2$. Such a direct coupling is possible in a current-current formulation of weak interactions.[2] If this coupling exists the process

$$\nu_\mu + Z \to \nu_\mu + \mu^+ + \mu^- + Z \qquad (2)$$

is possible and has the same final-state products as process (1). For incident neutrino energies far above the threshold for intermediate boson production, one expects process (1) to be dominant, because it is proportional to G, while process (2) is proportional to G^2. However, it is known experimentally that the majority of the neutrinos in the CERN experiment cannot be far above threshold, because the "elastic" process

$$\nu_\mu + n \to \mu^- + p,$$

which is proportional to G^2, is dominant in the CERN experiment.[1] Consequently, it is crucial to have an accurate evaluation of process (2), before attributing the presence of muon pairs to the production of intermediate bosons.

In this note the results of exact calculations of the coherent production,

$$\nu_\mu + Z \to \nu_\mu + \mu^+ + \mu^- + Z(\mathrm{coh}), \qquad (3)$$

and the production from free protons,

$$\nu_\mu + p \to \nu_\mu + \mu^+ + \mu^- + p, \qquad (4)$$

are reported.

To lowest order in G and the fine structure