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MAGNITUDE OF THE K_1^0-K_2^0 MASS DIFFERENCE*

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Since Gell-Mann and Pais first pointed out the necessity of a minute mass difference between the K_1^0 and K_2^0 mesons in 1955,[1] there have been six measurements of this quantity reported in the literature.[2-7] These measurements have yielded values ranging from 0.50 ± 0.15 to 1.9 ± 0.3 (in units of $\hbar/\tau_1$ where τ_1 is the mean lifetime of the K_1^0 meson). Although no two experiments were truly identical, they can all be put into two general classes; viz., those which study the development of the $\bar{K}^0$ component in an initially pure K^0 beam[2-5] and those which study coherent regeneration of K_1^0 mesons by a K_2^0 beam.[6,7] With the exception of reference 5, there is a marked tendency for experiments of the first type to give significantly higher values than those of the second type (see Table I). Because most of the experiments are complicated and require elaborate analyses, it is not yet clear whether this discrepancy is due to some new, but unrecognized, phenomenon or is simply a result of the experimental difficulties. In an effort to clarify the experimental side of this problem, we report here a new measurement of the mass difference obtained from a spark chamber study of coherent regeneration as a function of regenerator thickness.[8] This method is relatively simple, involves no subtle corrections, and can be shown to be insensitive to small violations of CP invariance in neutral K decays.

The principle of our method is as follows. When a K_2^0 beam passes through an iron slab of thickness x, the K_1^0 intensity builds up behind the slab due to the difference in strong interactions between the K^0 and $\bar{K}^0$ components with the iron nuclei. In the forward direction, the K_1^0 amplitudes regenerated from different parts of the slab add up coherently and the intensity depends on the mass difference δ as given first by Good,[9]

$$I(x,\delta) = \frac{|ƛ N f_{12}(0,p)\Lambda|^2}{\delta^2+\frac{1}{4}} e^{-x/\mu} \times [1-2e^{-x/2\Lambda}\cos(x\delta/\Lambda)+e^{-x/\Lambda}], \quad (1)$$

where $ƛ(=h/p)$ is the wavelength of the incoming K_2^0 of momentum p, N the number of nuclei per cm^3, $f_{12}(0)$ the amplitude for forward regeneration by a single nucleus, $\Lambda(=c\tau_1 p/mc)$ the mean decay length of the K_1^0, and μ the nuclear mean free path in iron. We have determined the value of δ by measuring the relative magnitude of $I(x,\delta)$ as a function of x. For this purpose, it was necessary to know the momentum of each event, to measure the nuclear mean free path μ, and to separate the coherent regeneration from the diffraction regeneration. It should be emphasized that this method does not require a knowledge of the regeneration amplitude $f_{12}(0,p)$ and is free from any multiple-scattering correction to the diffraction regeneration.[4]

Table I. Summary of measurements of $K_1{}^0$-$K_2{}^0$ mass difference.

Method	Apparatus	δ	Reference
Ratio of coherent to diffraction regeneration in Fe and Pb	Propane bubble chamber	$0.84^{+0.29}_{-0.22}$	a
Charge-exchange scattering	Counter	1.9 ± 0.3	b
Hyperon production	Propane bubble chamber	1.5 ± 0.2	c
Hyperon production	Hydrogen bubble chamber	$0.6^{+0.4}_{-0.6}$	d
Coherent regeneration as a function of gap between two Cu plates	Spark chamber with magnets	0.50 ± 0.15	e
K^0 β decay	Heavy liquid bubble chamber	0.78 ± 0.20	f
Coherent regeneration as a function of thickness of Fe	Spark chamber	0.82 ± 0.12	This experiment

[a] See reference 9.
[b] See reference 2.
[c] See reference 3.
[d] See reference 4.
[e] See reference 10.
[f] See reference 13.

The experiment was performed in a 30° neutral beam at the Brookhaven AGS operating at a proton energy of 28 BeV. The charged particles and gamma rays were removed from the beam by passing them through 4 in. of lead and a sweeping magnet. A 3-in.×3-in. collimator was placed about two meters upstream from the detector system. The regenerated $K_1{}^0$'s were detected by observing their charged decay products ($K_1{}^0 \to \pi^+ + \pi^-$) in a counter-controlled spark-chamber system as shown in Fig. 1. The momentum of each event was determined by measuring the direction of the charged particles of the V decay and assuming that the direction of the $K_1{}^0$ particle is known. In the case of $K_1{}^0$'s produced by coherent regeneration, the last assumption was well satisfied because the incident beam direction was defined within 2 milliradians.

The main source of background comes from $K_2{}^0$ mesons which decay in the fiducial volume of the spark chamber with the two charged particles of their three-body decay simulating a $K_1{}^0$ event. In order to take into account this $K_2{}^0$ background, a separate run was made with the

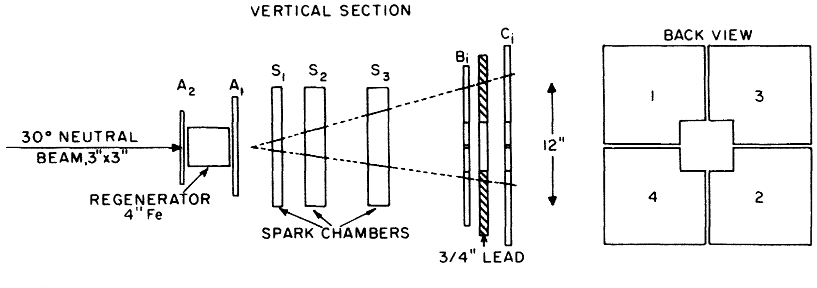

FIG. 1. Geometry of triggering counters (A_1, A_2, B_i, C_i), spark chambers, and regenerator in normal position. The spark chambers were triggered by $\bar{A}_1\bar{A}_2B_1C_1B_2C_2$ or $\bar{A}_1\bar{A}_2B_3C_3B_4C_4$.

iron regenerator moved upstream by 38 cm from its normal position. Using a normalization factor obtained from neutral beam monitors, the K_2^0 background was statistically subtracted from the data of each normal run. Another type of background, that of Λ^0 and K_1^0 produced in inelastic interactions, has been investigated and found not to be significant.

Since the measurement of the mass difference was performed simultaneously with that of the K_2^0 lifetime, most of the data were taken with the detector placed at the two different positions along the beam. Four thicknesses of iron, 10, 15, 20, and 25 cm, were used at the position close to the target (Run I), while only two, 10, and 20 cm, at the far position (Run II). In addition, data were also taken with 3.8, 6.3, and 10 cm iron at an intermediate position (Run III). At present, out of a total of 60 000 pictures, 45 000 pictures have been analyzed. The scanning and measuring were performed on a projected image of the 90° stereo photographs of each event using a digitized measuring device. The space reconstruction and kinematical analysis were made using an IBM-7090 computer.

The evidence for coherent regeneration of K_1^0's is best given by the distribution of angles between the normal to the decay plane and the incident beam direction. Figure 2(a) shows such a distribution for both normal and background runs with the 20-cm iron regenerator. Events were selected with the following requirements: (1) the momentum between 0.9 and 1.9 BeV/c, (2) vertex between 0.5 cm and 16.5 cm downstream from veto counter A_1 (corresponding to about two mean decay lengths of a K_1^0 of 1.4 BeV/c),[10] and (3) vertex within the cross-sectional area of 10 cm ×10 cm centered around the beam. The effect of the iron regenerator is clearly demonstrated by the narrow peak in the forward direction for the normal run in comparison with the rather flat distribution obtained from the background run. Figure 2(b) represents all K_1^0 events with the K_2^0 background subtracted. It consists of the transmission peak with a full width at half-maximum of 12 milliradians and an underlying wider distribution which is consistent with the existence of diffraction-regenerated K_1^0's. Assuming that both distributions can be represented by Gaussians of unknown widths and amplitudes, a least-squares fit was made to the observed decay-plane distribution as indicated by solid curves in Fig. 2(b).[11] The area of the narrower distribution was taken to be the number of coherently regenerated K_1^0's.

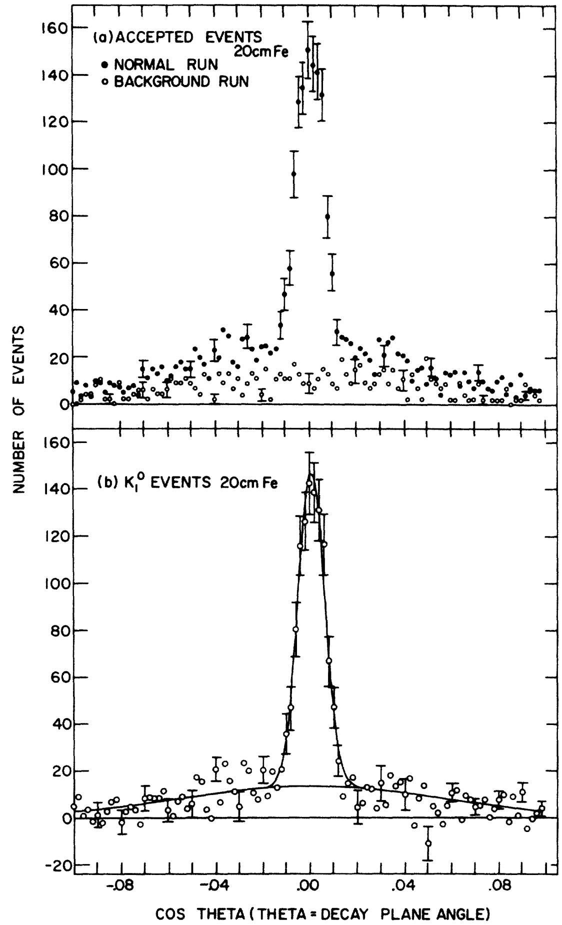

FIG. 2. Decay-plane distribution of events for 20 cm Fe. In (a), closed circles correspond to normal run, and open circles to background run. (b) shows the net effect of K_1^0 with K_2^0 background subtracted. Two solid curves represent the best-fitted Gaussian distributions to the experimental points.

Since the incident K_2^0 beam was not monoenergetic,[12] it was necessary to integrate Eq. (1) over momentum weighting it with the incident K_2^0 momentum spectrum $W(p)$ and the triggering efficiency $E(p)$. The product of $W(p)E(p) \times |f_{12}(0,p)|^2$ was obtained from the effective momentum spectra measured with the 10-cm-thick regenerator. The average value of the nuclear mean free path μ was measured in a separate attenuation experiment[13] and was found to be 14.1 ± 1.3 cm in iron. Its momentum dependence was computed on the basis of an optical-model calculation.

The three sets of experimental data are shown in Fig. 3 along with theoretical curves computed to fit the data of Run I with three different values

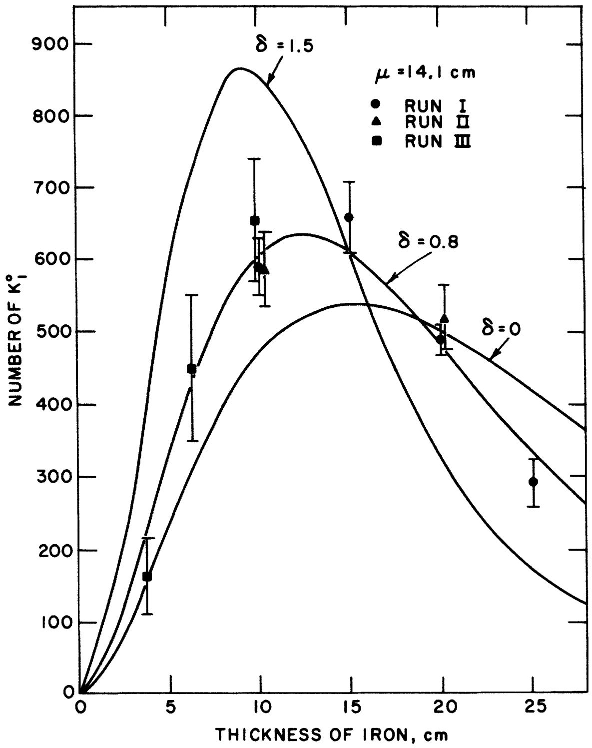

FIG. 3. Computed regeneration curves for $\delta = 0$, 0.8, and 1.5, with experimental points. The mean free path used in this calculation was 14.1 cm.

of δ. For the purpose of graphical presentation, normalization among the three sets of data was made at the 10-cm point using the ratio of theoretical values calculated for each set with $\delta = 0.8$. To determine the best value of δ, least-squares fits were made independently to the three sets of data, the χ^2-vs-δ curves for the three sets were then added; this procedure was repeated for values of μ varying by plus or minus one standard deviation from the central value. From this calculation, we find $\delta = 0.82 \pm 0.12 \times (\hbar/\tau_1)$, where we use $\tau_1 = 0.89 \times 10^{-10}$ sec; the over-all χ^2 for this fit is 2.8. The error quoted reflects the uncertainty in the determination of μ in addition to the statistical deviation given by the χ^2 variation. The error due to systematic effects is estimated to be less than ±0.03.

Table I summarizes the measurements of δ which have been published to date. Our value is in general agreement with the previous results of the regeneration type experiments,[6,7] and hence reinforces the discrepancy between the two classes of experiments mentioned earlier. The effect of a small violation of *CP* invariance on coherent regeneration has been discussed by Whatley[14] and by Sachs[15]; Whatley estimated that if $\frac{1}{3}\%$ of K_2^0's decay via the $\pi^+\pi^-$ mode, the δ determination of Good et al.[6] would be changed by at most 12%. Similarly, we estimate an upper limit of 9% on our determination of δ for this degree of violation. It is interesting to note that the most recent measurement of δ by an experiment of the other class, that of Aubert et al.[5] who study $K^0\,\beta$ decay, yields a result in agreement with the regeneration experiments.

We would like to express our gratitude and appreciation to many people who contributed to this experiment: to Dr. G. B. Collins for his support and interest; to Dr. M. H. Blewett and the AGS staff for setting up the experiment; to Dr. J. Fischer for his contributions to the initial phases of the experiment; to Mr. E. P. Bihn and Mr. R. W. Rothe for their indispensible assistance during the run; to Mr. B. Arbeit for writing computer programs; to Mr. R. W. Burris, Mr. D. S. Loebbaka, and Mr. L. C. Wang at the University of Maryland, for aid in the analysis, and to Mrs. I. Pagnamenta and Mr. H. Lang for the scanning and measuring of events.

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[8] This was done in conjunction with a measurement of the K_2^0 lifetime. For the details of the experimental arrangement and a preliminary value of the K_2^0 life-

time, see J. V. Jovanovich, J. Fischer, T. Fujii, F. Turkot, R. W. Burris, D. S. Loebbaka, and G. T. Zorn, Proceedings of the International Conference on Fundamental Aspects of Weak Interactions, Brookhaven National Laboratory Report No. BNL-837, 1964 (unpublished), p. 42.

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[10] From the vertex distributions in this range, we have obtained a preliminary value for the $K_1{}^0$ lifetime $\tau_1 = (0.89 \pm 0.07) \times 10^{-10}$ sec.

[11] To check on the reliability of this procedure, the fit was also made using a maximum-likelihood method. The results were consistent with those obtained by the least-squares method.

[12] The momentum spectrum of detected $K_1{}^0$ has a maximum at ~1.4 BeV/c and falls off to zero at ~0.8 and at ~2.5 BeV/c.

[13] An extension of this experiment was done with the objective of studying coherent regeneration in more detail and measuring total cross sections of $K_2{}^0$'s in C, Cu, and U. The results of these measurements will be reported at a later date.

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ASYMMETRIC μ-PAIR PHOTOPRODUCTION AT SMALL ANGLES*

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Photoproduction of μ pairs has been measured recently[1] with the aim of further probing the theory of quantum electrodynamics as applied to muons. A symmetric experimental arrangement as used in these measurements offers two advantages[2]:

(1) In both Bethe-Heitler diagrams, Figs. 1(a) and 1(b), the virtual muon is equally removed from its mass shell by a spacelike amount $(k-p_\pm)^2 - m_\mu{}^2 \approx -kE\theta^2$ corresponding to $\approx -(450\text{ MeV})^2$ for the extreme energies and angles of the recently reported observations. These diagrams can be computed to order Ze^3 and expressed in terms of measurable structure factors for scattering, elastic, or total inelastic, from nucleons or nuclei.[3]

(2) Interference of the Bethe-Heitler with the virtual Compton diagrams, Fig. 1(c), vanishes identically for the symmetric arrangement, and the squares of the Compton amplitudes are estimated to be negligibly small for this condition in which momentum transfer to the nucleus is very low.

In this Letter an extremely asymmetric kinematic condition for photoproducing μ pairs is discussed for probing electrodynamics, as well as, possibly, for studying the photoproduction of vector resonances. This arrangement is designed to be of maximum advantage to electron linacs which provide beams of electrons and photons with very high currents but in short pulses that hamper multiparticle relative to single-particle counting experiments.

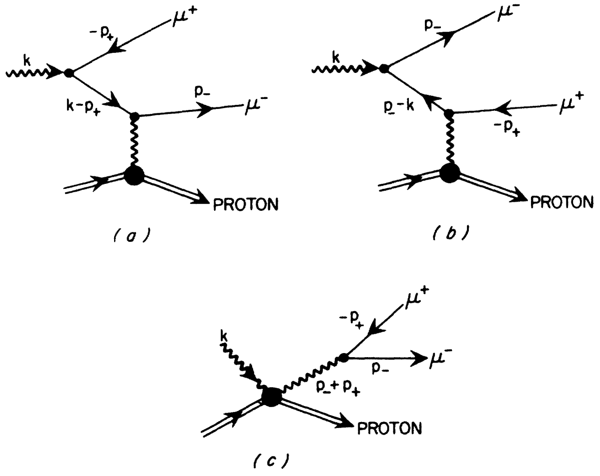

FIG. 1. Diagrams for photoproduction of μ pairs from protons.

We consider photoproduction of μ pairs with all of the energy concentrated on one member of the pair which is detected while the other one emerges almost at rest—i.e., with nonrelativistic energy. We have in mind specifically the photoproduction from hydrogen, detecting the μ^- with an energy E_- within $\approx$40 MeV of the maximum possible energy. Since photoproduction of only a single π^- from hydrogen is excluded by charge conservation, this region is not kinematically available to a decay μ^- from a π^- which at lower energies swamps the events of interest.

In the limit of low momentum p_+ for the undetected μ^+—i.e., to leading order in $p_+/m_\mu \to 0$—and in the high-energy approximation, $m_\mu/k \to 0$, the Bethe-Heitler formula becomes

$$\frac{d\sigma}{dE_- \, d\Omega_-} = \frac{\alpha^3}{2\pi} \frac{p_+ k}{m_\mu{}^5} \left\{ \frac{\theta_-{}^2 k^2/m_\mu{}^2}{[\frac{1}{2} + \frac{1}{2}(\theta_-{}^2 k^2/m_\mu{}^2)]^3} \right\}. \tag{1}$$

This cross section is approximately equal to