ground and excited-state vibrational wave functions. For the thermal activated decay, the reduced lateral displacement would increase the effective barrier height. The decrease in the nonradiative de-excitation processes results in an increased fluorescence lifetime and quantum yield. The expected correlation of reduced Stokes shift and increased fluorescence lifetime are found in the observed data. The present observation of fluorescence changes near the Néel temperature supports the earlier hypotheses that the origin of the lower temperature fluorescent anomalies in MnF_2 and $KMnF_3$ was due to magnetic ordering of the excited state Mn^{2+} spins. A more complete report of other examples of correlation of magnetic and fluorescence effects in manganese salts will be published in the near future.

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ANISOTROPIC SPIN-WAVE THERMAL CONDUCTIVITY IN FERROMAGNETS

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Sato[1] has suggested that transport of energy by magnons might be detected at sufficiently low temperatures. Early observations of thermal conductivity in magnetic insulators[2,3] were interpreted in terms of phonon transport limited by phonon-magnon interactions. Magnon transport has been detected in yttrium iron garnet[4-6] and in EuS.[7,8] In principle, if the mean free path is determined by boundary scattering, then magnon transport should be identified by its characteristic temperature dependence, i. e., T^2 in zero internal field. It has been observed[3] that the field dependence is a more reliable indicator: The thermal conductivity decreases with increasing field for magnon transport; it increases with increasing field for phonon transport influenced by phonon-magnon scattering. In this note the theory of magnon thermal conduction of Douthett and Friedberg[3] is modified to include the effects of the ubiquitous long-range magnetic dipolar coupling. The major effect is seen to be a change in the magnon group velocities, which results in a dependence of the magnon conductivity upon the direction of measurement relative to the magnetization direction. The ferromagnetic europium chalcogenides[9] are favorable materials in which to look for the phenomenon because of their large magnetization (measure of the strength of the dipolar coupling) and low Curie point relative to the Debye temperature (indicating a large magnon population relative to that of the phonons).[10]

The flow of heat in the direction $\vec{m}$ ($\vec{m}$ is a unit vector) is related to the thermal gradient in the same direction by the thermal conductivity[11]

$$K_m = \frac{1}{\Omega} \sum_{\text{modes}} (\vec{v}\cdot\vec{m})^2 \tau S. \tag{1}$$

Here Ω is the sample volume and for each mode, $\vec{v}$ is the group velocity, τ the relaxation time, and S the specific heat capacity. For simplicity we assume the scattering to be due to boundaries and define one effective mean free path l for all modes. Then $\tau = l/|\vec{v}|$. The spin-wave energies are given by[12]

$$e_{\vec{k}} = (\epsilon_{\vec{k}} + g\beta H)\{1 + \varphi_{\vec{k}} \sin^2\theta_{\vec{k}}\}^{1/2}, \tag{2}$$

where $\epsilon_{\vec{k}}$ is the exchange energy, $\theta_{\vec{k}}$ is the angle between $\vec{k}$ and $\vec{M}$, and

$$\varphi_{\vec{k}} = g\beta 4\pi M/(\epsilon_{\vec{k}} + g\beta H). \quad (3)$$

The group velocity is

$$\vec{v} = \hbar^{-1}\nabla_{\vec{k}} e_{\vec{k}}$$
$$= \frac{(1+\frac{1}{2}\varphi_{\vec{k}}\sin^2\theta_{\vec{k}})\nabla_{\vec{k}}\epsilon_{\vec{k}} + g\beta 2\pi M \nabla_{\vec{k}}\sin^2\theta_{\vec{k}}}{\hbar(1+\varphi_{\vec{k}}\sin^2\theta_{\vec{k}})^{1/2}}. \quad (4)$$

The magnon conductivity is calculated by summing over only the spin-wave modes in Eq. (1). An approximate calculation has been made under the conditions that

$$2SJ \gg k_B T \gg g\beta 4\pi M. \quad (5)$$

This means that $\epsilon_{\vec{k}} \approx 2SJk^2a^2$ and an expansion in powers of $\alpha = g\beta 4\pi M/k_B T$ may be sought. The magnon conductivity parallel to $\vec{M}$ is

$$K_{\parallel} \approx K_0 - 2K_v - K_p; \quad (6)$$

and perpendicular to $\vec{M}$,

$$K_{\perp} \approx K_0 + K_v - 2K_p. \quad (7)$$

Here K_0 is that conductivity calculated by Douthett and Friedberg,[3]

$$K_0 = \frac{lk_B{}^3T^2}{3\pi\hbar(2SJa^2)} \times \sum_{n=1}^{\infty} \exp[-n\gamma][6n^{-3} + 4\gamma n^{-2} + \gamma^2 n^{-1}], \quad (8)$$

with

$$\gamma = g\beta H/k_B T. \quad (9)$$

The dipolar modification of the group velocity has its expression in

$$K_v = \tfrac{1}{5}\alpha\frac{lk_B{}^3T^2}{3\pi\hbar(2SJa^2)} \times \sum_{n=1}^{\infty} \exp[-n\gamma][2n^{-2} + 2\gamma n^{-1} + \gamma^2], \quad (10)$$

and K_p represents the effect of the modification of the specific heat. The calculation gives $K_p = K_v$. In zero field, $\gamma = 0$, and

$$\frac{K_{\perp}}{K_{\parallel}} \approx 1 + \frac{2\zeta(2)}{15\zeta(3)}\alpha = 1 + 0.182\frac{g\beta 4\pi M}{k_B T}. \quad (11)$$

Numerical calculations have been carried out for a face-centered cubic ferromagnet with

$$\epsilon_{\vec{k}} = 2SJ\sum_{\vec{l}}(1-\cos\vec{k}\cdot\vec{l}), \quad (12)$$

$\vec{l}$ being the nearest-neighbor vectors of the lattice. Parameters have been chosen to match the properties of the ferromagnetic europium chalcogenides. These have appreciable second-neighbor exchange[9] and we choose $J = J_1 + J_2$ which is correct for the long-wavelength spin waves. The values used are given in the following table:

	$2SJ/k_B$ (°K)	$g\beta 4\pi M/k_B$ (°K)
EuSe (I)	0.43	1.6
EuS (II)	0.84	2.2
EuO (III)	5.4	2.7

The results obtained are contained in Figs. 1, 2, and 3. We note that the rapid temperature dependence of K is damped as spin-wave excitation near the Brillouin-zone boundary becomes appreciable, that at such a temperature $K_{\perp}/K_{\parallel}$ levels out at a value greater than unity, and that $K_{\perp}/K_{\parallel}$ is quite insensitive to the magnitude of the applied internal field.

The anisotropic magnon conductivity may be

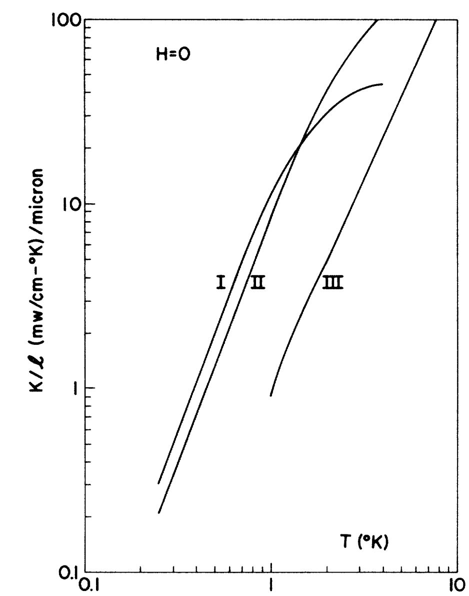

FIG. 1. Magnon thermal conductivities parallel to the domain magnetization calculated for ferromagnetic europium chalcogenides (the free path l is measured in microns).

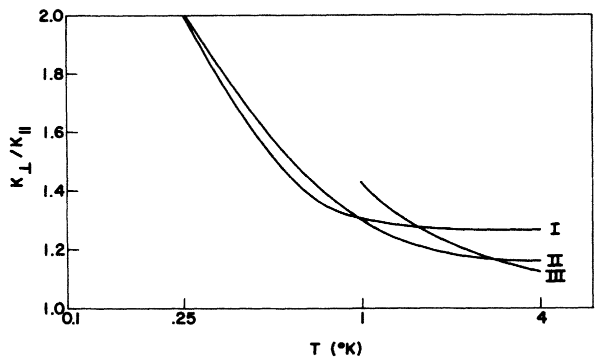

FIG. 2. Calculated ratio of magnon thermal conductivities perpendicular and parallel to the domain magnetization.

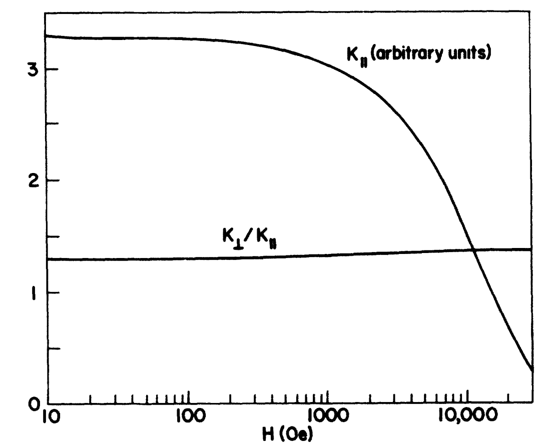

FIG. 3. Magnon thermal conductivity calculated as a function of internal field for EuS at 1°K. The ratio $K_\perp K_{||}$ is nearly constant at a value ≈1.3 for all fields.

observed in a magnetically saturated specimen by simply rotating the applied field (i.e., magnetization) relative to the direction of measurement.

On the other hand, the internal field vanishes below saturation. Then, in the region of technical magnetization by rotation of domains, it may be expected that an applied field dependence (i.e., magnetization dependence) of the thermal conductivity, similar to that of ordinary magnetostriction, may result from the anisotropic magnon conductivity.

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