earlier work[2] on the scattering of a laser beam from a plasma of lower density it has been observed that the scattered radiation is broadened by the thermal motion of the electrons. This cannot be so in the present case, since the upper limit of 0.4 Å on the width of the scattered line corresponds to an electron temperature $T_e \leq 4°K$. A possible explanation of this anomaly is that the linewidth is governed, through cooperative effects[3] within the plasma, by the motion of the ions. This is so if the parameter $\alpha = 1/KD > 1$, where $K = 4\pi\lambda_0^{-1}\sin\frac{1}{2}\theta$ and $D = (kT_e/4\pi n_e e^2)^{1/2}$, the Debye length in the plasma. Here θ is the angle of observation of the scattered radiation with respect to the incident beam, λ_0 the wavelength of the laser beam, and T_e the electron temperature. For $\lambda_0 = 6943$ Å, $\theta = \frac{1}{2}\pi$, $n_e = 5\times10^{19}$ cm^{-3}, $\alpha > 1$ if $T_e < 10^8$ °K. The upper limit of 0.4 Å on the linewidth of the scattered radiation would then correspond to an ion temperature $T_i \leq 10$ eV.

The wavelength shift of the scattered radiation was dependent upon the point of observation of the spark along the axis of the lens L_1. The shift was almost always towards shorter wavelengths, although on occasion a weak line has also been observed shifted slightly towards longer wavelengths. When a Dove prism was used to image the region of the spark lying along the axis of the lens onto the slit of the spectrograph, an inclined image was obtained, showing that the shift was least for points along the axis nearest the lens L_1.

We interpret the shift in wavelength of the scattered radiation as a Doppler shift due to a motion of the plasma as a whole towards the lens L_1 during the initial stage of the spark when scattering takes place. Such a motion is clearly seen on a streak photograph of the spark taken with an image converter camera [Fig. 1(b)]. The initial velocity of luminous front towards the lens L_1 is ~10^7 cm/sec, in good agreement with the maximum observed shift of 3 Å towards the blue. The velocity decreases as the plasma moves towards the lens L_1, in agreement with the observation that the wavelength shift is least for points nearest the lens L_1. The presence of a second, less luminous, front moving in the opposite direction explains the occasional observation of a weaker line shifted towards the red. A more detailed study of the expansion of the spark is under way and will be the subject of a further communication.

We would like to acknowledge that our interest in this problem arose initially out of a discussion with Dr. R. W. Minck of the Ford Scientific Laboratory, Dearborn, Michigan.

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EXCITATION OF CYCLOTRON HARMONIC RESONANCES IN A MERCURY-VAPOR DISCHARGE*

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Measurements of the topside profile of the ionosphere have been made recently with the Canadian sounder satellite "Alouette." In these, the time delay of radar echoes from the ionosphere following the pulsed emission of signals of slowly varying frequency was studied. In addition to the expected results, a series of sharp resonances at integral multiples of the local electron cyclotron frequency, ω_c, with ringing continuing for many rf periods after the transmitter pulse had ceased, was observed.[1] The experiments to be described were carried out to determine whether these resonances could be excited in the laboratory, and to shed some light on their origin. There is already evidence that such effects occur in laboratory plasmas. Not only have many peaks in noise emission been observed at these frequencies,[2] but in addition, such noise output can be greatly enhanced by passing an electron beam from a shielded gun through the plasma.[3,4] Weak absorption of rf signals at these frequencies has also been seen.[5] We shall give results showing that the transmission between two probes immersed in a plasma exhibits strong resonances in a situation in which absorption effects are hardly measurable.

Our transmission measurements were made in the positive column of a mercury-vapor discharge, as shown in Fig. 1(a). Variation of magnetic field strength at a fixed working frequency, ω, resulted

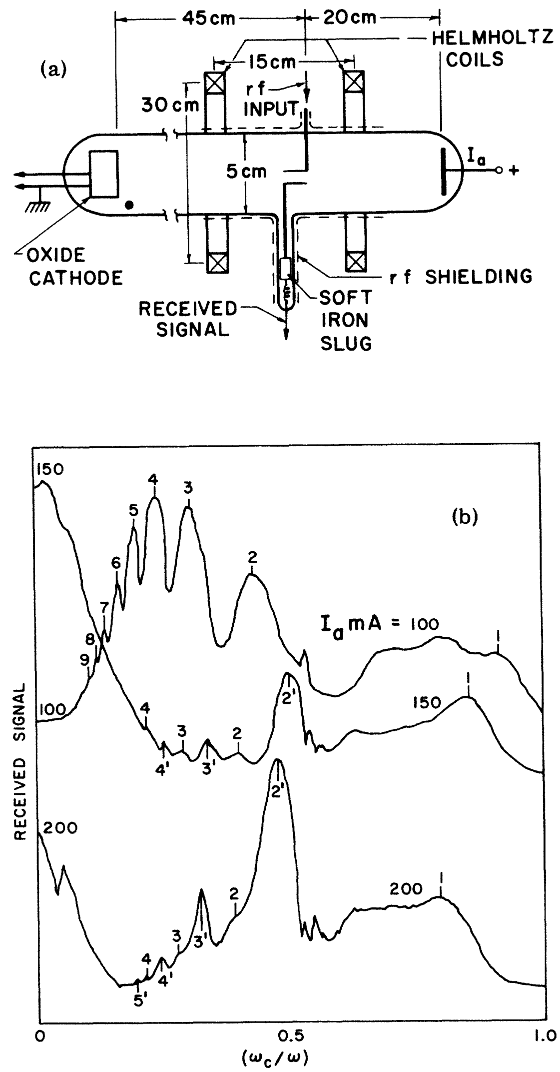

FIG. 1. Experimental setup, and typical transmission records (separated vertically for clarity). (Experimental details: Working frequency 400 Mc/sec; probes 1 cm long × 0.25 mm diam, 3 mm apart; neutral pressure ≈ 2×10^{-3} mm Hg).

in the pronounced resonance effects recorded in Fig. 1(b). Up to 10 resonances could be observed at about 2 μ pressure, and up to 17 at 0.4 μ. The amplitudes of the peaks varied considerably with discharge current and two distinct sets were observable. One of these dominated at low current (Type I). The other (Type II), designated by a prime in Fig. 1(b), became important as the current increased. This is illustrated in Fig. 1(b) by the records chosen, and in Fig. 2(a) where the resonant frequencies are plotted against current. The cyclotron frequency was determined

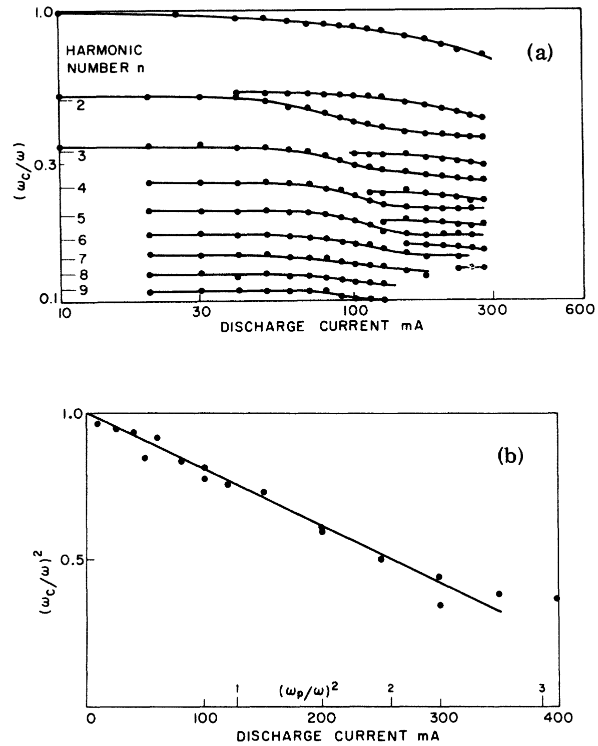

FIG. 2. Experimental data derived from transmission records [conditions as given on Fig. 1(b)].

from a calibration of the Helmholtz coils accurate to within one percent, and has been normalized to ω. At low discharge currents, the simple integral relation $(\omega_c/\omega)=1/n$ is closely approached. As the discharge current increases, (ω_c/ω) and the amplitudes of the first set of peaks decrease, and the second set appears.

A plot of the variation of $(\omega_c/\omega)^2$ with discharge current is given in Fig. 2(b) for the fundamental mode ($n=1$). If we assume that at the modest magnetic fields employed in the experiment (<150 gauss) ω_p^2 is proportional to the discharge current, and that the mean electron density across the tube varies only slowly with the magnetic field, it is possible to calibrate the current axis in terms of $(\omega_p/\omega)^2$. The electron density was known to within 30% from measurements made with the dipole resonance technique.[6] Figure 2(b) shows that the following empirical relation is satisfied:

$$\omega^2=\omega_c^2+0.25\omega_p^2. \qquad (1)$$

Experiments were carried out to determine whether the effects observed are due to nonlinearities, and whether the resonances occur only in the region close to the probes. With small inputs, the received signal amplitude was found to be directly proportional to the input. Varying the probe potentials up to the point of electron saturation current did not affect the shape of the resonance curve, although there was some change in amplitude. Increasing probe separation changed the relative amplitudes of the different resonances but not the value of magnetic field at resonance. Only very weak resonances at the fundamental and second harmonic could be observed when exciting and detecting probes outside the discharge column were used. We conclude from the previously published work, and the foregoing results, that the effect is a linear volume phenomenon, probably due to propagation of waves of extremely short wavelength across the column.[7]

Bernstein[8] has derived the following dispersion relation for propagation of electrostatic waves across a magnetic field:

$$1 = \left(\frac{\omega_p}{\omega_c}\right)^2 \sum_{n=1}^{\infty} \frac{S_n}{[(\omega/\omega_c)^2 - n^2]}, \qquad (2)$$

where S_n is written for $[2n^2 \exp(-\lambda) I_n(\lambda)/\lambda]$; I_n is a modified Bessel function of the first kind and nth order; $\lambda = (k^2 e V_e/m\omega_c^2) = (kr_c)^2$; k is the propagation constant; V_e is the electron temperature in volts; and r_c is the gyroradius of the average thermal electron. These waves are not subject to Landau damping, and may have very short wavelengths.

Equation (2) yields a series of pass bands with lower cutoffs ($\lambda = 0$) at $\omega^2 = \omega_c^2 + \omega^2$ and $\omega = n\omega_c$ ($n > 1$). When $\omega_p^2 \leq \omega^2$, the pass bands are given approximately by

$$\omega^2 = (n\omega_c)^2 + \omega_p^2 S_n \qquad (3)$$

and have upper cutoffs where S_n is maximum. We have $(S_1)_{\max} = 1$, and $(S_n)_{\max} \approx 0.9/n$ ($n > 1$). For $\omega_p^2 > \omega^2$, the approximation of taking only one term of the series is not valid. We can say, however, that the upper cutoff of the nth band must always lie below $(n+1)\omega_c$, and that there is a short-wavelength cutoff which occurs at frequencies slightly higher than $n\omega_c$. It should be noted that the experimental curves of Fig. 2(a) tend to the limit $(n+1)\omega_c$ for high (ω_p/ω). The subsidiary resonances arise in the region $(\omega_p/\omega) \approx 1$, where we expect the short-wavelength cutoff to be displaced.

Resonances such as those observed in satellite and laboratory noise studies should probably occur where the group velocity, v_g, is zero, i.e., at the upper and lower cutoff frequencies. In the case of transmission measurements this is not necessarily so. If there are appreciable collision losses, and in our experiments $\nu \approx 50 \times 10^6$ coll/sec, compared to a working frequency of 400 Mc/sec, then the propagation constant becomes $(k_r - jk_i)$, where k_i is given closely by $[\nu(\partial k/\partial\omega)]$, and the transmission loss is consequently a minimum where $v_g (= \partial\omega/\partial k)$ is a maximum. Experimental values of (ω/ω_c) taken from Fig. 2(a) are compared in Table I with predicted values, based on Bernstein's theory, and obtained by use of Eq. (3). Except for $n = 1$, there are two points at which the group velocity has a maximum value. For $n > 1$, one of these, or the higher frequency cutoff point ($v_g = 0$), agrees closely with the experimental results for Type I modes. However, this may be fortuitous for the accuracy of the data is not sufficient to allow us to decide whether the criterion of minimum or maximum group velocity is effective. The $n = 1$ resonance behavior with variation of pressure and other parameters is different from the others, and may well have a different origin.

It has been suggested by Canobbio and Croci[7] that the anomalously high noise emission observed in the laboratory[2] near the cyclotron harmonic frequencies is related to the Bernstein modes. Now that both transmission and growth[4] have been observed at these frequencies it seems that the phenomenon can be explained in terms of a beam/plasma collective interaction with the Bernstein modes. Such a hypothesis would appear to be more likely than that of single-particle radiation from the orbiting electrons,[9,10] or even a single-particle interaction

Table I. Comparison of experimental values of (ω/ω_c) with zero and maximum group velocity predictions ($I_a = 100$ mA; $\omega_p^2 \approx 0.8\ \omega^2$; $V_e = 2.0$ eV measured by probe).

n	Expt.	$v_g = 0$	$v_{g\max}(1)$	$v_{g\max}(2)$
1	1.15	···	1.41	···
2	2.40	2.42	2.16	2.28
3	3.45	3.40	3.16	3.32
4	4.40	4.41	4.18	4.31
5	5.35	5.39	5.17	(not computed)
6	6.45	6.40	6.18	(not computed)
7	7.55	7.42	7.18	(not computed)

with the Bernstein modes as has been suggested by Canobbio and Crocci.[7]

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ELECTRON SCATTERING BY NEUTRALIZED ACCEPTORS IN GERMANIUM

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Based upon the hydrogenic model suggested by Pearson and Bardeen,[1] Erginsoy[2] has developed a formula for the electron scattering by neutralized impurities:

$$1/\tau_I = 20\hbar a^* N_I / m^*. \tag{1}$$

Here a^* is the effective Bohr radius for the impurity electron, N_I is the impurity concentration, and $m^{*-1} = \frac{1}{3}(m_l^{-1} + 2m_t^{-1})$. Though this formula has been widely used for many kinds of neutral impurity in semiconductors, its validity should in reality be confined to the pentavalent donors in Ge or Si. Through the cyclotron resonance work by Fukai et al.,[3] it has been verified experimentally that electron scattering by neutralized group-V impurities in Ge or Si is surprisingly well explained by the Erginsoy's formula. So in this Letter we shall focus our attention on the next typical impurities—the group-III elements.

Neutralized trivalent acceptors in Ge or Si should be compared to antihydrogen in the framework of the effective-mass approximation. Electron scattering by antihydrogen, however, is equivalent to positron scattering by a hydrogen atom apart from the charge relation which does not affect the cross-section calculation. Theoretical treatments of positron scattering by hydrogen atoms have been developed by several authors,[4-7] and the phase-shift calculations for various energies of the incident positron are available. Combining the most recent results by Schwartz[6] and Rotenberg,[7] which are very close to each other (Fig. 1), one obtains the inverse collision time

$$1/\tau_A = 3.5\hbar a_A^* N_A / m^*, \tag{2}$$

where a_A^* is the effective Bohr radius for the acceptor hole, N_A is the acceptor concentration, and m^* is the same as that in (1). This is only

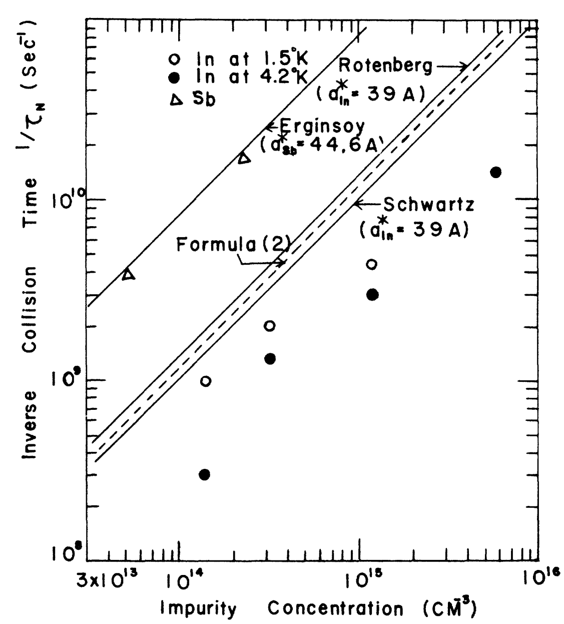

FIG. 1. Inverse collision time due to neutralized impurities is plotted against the impurity concentration. The straight lines give theoretically expected values: "Erginsoy" for Sb with the e^-H scattering model and others for In with the e^+H scattering model. The effective Bohr radii have been adjusted by the method of quantum defect.[3]