continuation. The compact G_8 is manifestly of rank two and semisimple. From this it follows immediately that the group is, in fact, simple and is locally isomorphic to SU(3)! For completeness we exhibit the metrics which admit a G_8:

$$ds^2 = \epsilon\left(\epsilon + \sum_{i=1}^{4} y^{i2}\right)^{-2}\left[\left(\epsilon + \sum_{i=1}^{4} y^{i2}\right)\left(\sum_{i=1}^{4} dy^{i2}\right) - \left(\sum_{i=1}^{4} y^i dy^i\right)^2 - (y^1 dy^2 - y^2 dy^1 + y^3 dy^4 - y^4 dy^3)^2\right],$$

where $\epsilon = \pm 1$, the upper sign referring to the space which admits the compact group.[7]

We thus observe that the requirement that the physics of baryons be represented on the most symmetric V_4 which is not geometrically trivial leads uniquely to a consideration of a group locally isomorphic to SU(3), and to a space which is necessarily of positive definite metric and a solution of the Einstein field equations with cosmological constant. In addition we are now in a position to state the following result: If we require that the wave functions for the nucleons be single-valued functions on the V_4, we do not obtain all the irreducible representations of SU(3), but only all those representations which are obtained by repeated formation of direct products and reduction starting with the 8-dimensional representation. Thus the nonexistence of "quarks",[8] or equivalently, the integrity of the fundamental electric charge, which appeared so puzzling from the point of view that the relevant group be precisely SU(3), can be understood simply as a single-valuedness requirement on the wave functions in V_4. (The analogy of integral orbital angular momentum for the single-valued representations of the rotation group is obvious.) The wave function for the eightfold representation is readily seen to be a symmetric second-order tensor which has zero trace and unit determinant.

Deviations from strict SU(3) symmetry can be obtained by analytically continuing the metric to the Minkowski signature, but in view of the fact that the relationship between the positive-definite internal space and external Minkowski space is obscure at this time, there does not seem to be much motivation for such a procedure.

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MASS AND COUPLING CONSTANT FORMULAS IN BROKEN SYMMETRY SCHEMES*

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The action of higher symmetries in the realm of strongly interacting particles is, as is well-known, to build up conserved currents. However, these symmetries[1,2] are not exact and therefore the currents are also only "partially" conserved. The most suitable method of expressing "partial" conservation of currents is based on the Goldberger-Treiman[3] relations. In the context of symmetries of the type[4,5] $[\mathrm{SU}(3)]^n$, we shall explore the consequences of partial current conservation and show that relations among masses and coupling constants usually derived on the basis of lowest order perturbation theory can be easily obtained by the Goldberger-Treiman method and that this method also leads to new relations not obtainable by perturbative techniques.

We shall start with the usual SU(3) invariance and derive the Gell-Mann–Okubo[1,6] mass formula with this method. Let us consider the matrix elements of the conserved vector currents of unitary spin between eightfold-baryon states

$$\langle B'|v_\mu^S|B\rangle = \bar{u}_{B'} F_{B'B}^S A(q^2)\gamma_\mu u_B, \tag{1}$$

where q_μ is the four-momentum transfer and $F_{B'B}{}^S$ are well-known coefficients. Formula (1) as it stands is valid only in the limiting case of exact SU(3) symmetry (i.e., degenerate baryon masses). Suppose now that a breakdown of SU(3) occurs and the baryon masses are split. Formula (1) is no more valid as it stands, for new "induced" terms will appear on its right-hand side. Assuming that the breakdown of SU(3) is "spontaneous,"[7] the symmetry will be preserved in the global system of mutually orthogonal Hilbert spaces, built on the continuous manifold of mutually orthogonal vacua.[8] In the case of a spontaneous breakdown of SU(3) symmetry the baryon field operators have exact octet transformation properties under SU(3), but the vacuum state now transforms according to a continuum-dimensional representation of SU(3). Therefore an SU(3) transformation that leads from $v_\mu{}^S$ to $v_\mu{}^{S'}$ will at the same time change the vacuum Ω to an orthogonal vacuum Ω'. We will therefore have a relation among the matrix elements of $v_\mu{}^S$ between eightfold baryon states in a world built on the vacuum Ω and those of $v_\mu{}^{S'}$ between the eightfold baryon states of an orthogonal world built on Ω'. There will, however, be no relation among the matrix elements of $v_\mu{}^S$ and $v_\mu{}^{S'}$ between the baryon states of the same world. All this results in the presence of induced terms in the matrix elements of $v_\mu{}^S$ between the baryon states of the physical world which now has the form

$$\langle B' | v_\mu{}^S | B\rangle = \bar{u}_{B'}\{F_{B'B}{}^S[A_S(q^2)\gamma_\mu + C_S(q^2)q_\mu] + D_{B'B}{}^S[G_S(q^2)\gamma_\mu + H_S(q^2)q_\mu] + \cdots\}u_B, \tag{1a}$$

where the form factors now explicitly depend on the index S and the dots stand for divergenceless ($\sim\sigma_{\mu\nu}q_\nu$) terms.[9] Spontaneous breakdown of SU(3) distinguishes itself by the fact that the current $v_\mu{}^S$ is conserved:

$$\langle B' | \partial_\mu v_\mu{}^S | B\rangle = (B-B')(F_{B'B}{}^S A_S + D_{B'B}{}^S G_S) + q^2(F_{B'B}{}^S C_S + D_{B'B}{}^S H_S) = 0, \tag{2}$$

B and B' being the baryon masses. This relation has to be obeyed for all q^2 and this leads in the well-known fashion to the existence of an octet of massless scalar mesons ("zerons").[10] In reality, however, the zerons are not massless, but this is a problem that we do not wish to discuss here. If we keep S in (2) fixed but vary B and B' then except for the so-far arbitrary mass differences (B-B') in the first term, (2) has definite transformation properties. Therefore, in order that the relations (2) be mutually compatible, certain relations must be satisfied among the baryon masses and among the form factors. Ignoring electromagnetic mass splittings, these relations turn out to be[11]

$$3\Lambda + \Sigma = 2N + 2\Xi, \tag{3a}$$

$$G_\pi = G_\eta = G_K = G_{\bar{K}} = 0, \tag{3b}$$

$$C_\pi = H_\pi = C_\eta = H_\eta = 0. \tag{3c}$$

(3a) is just the Gell-Mann–Okubo mass formula, (3b) shows that the Dirac form factor of baryons must be of the F-type. (3c) offers a realization of an incomplete octet[12] κ of zerons. A last requirement derived from (2) is that the F/D ratio in the κ-baryon coupling be the same as the F/D ratio in the baryon mass formula $(F/D)_B \approx -3.2$. This derivation of the mass formula can be readily extended to boson octets where automatically masses squared appear instead of masses. It can also be extended to higher multiplets (10, 27, etc.).

With this result in mind we can now consider the case of an SU(3)⊗SU(3) scheme where eight axial vector currents share with the eight vector currents the fate of partial conservation. Such would be the case if the baryons were massless. By requiring the baryons to be massive but degenerate we automatically break the SU(3)⊗ SU(3) symmetry but maintain the usual SU(3) symmetry and therefore the exact (without induced scalar terms) conservation of vector currents. If we view this breakdown of SU(3)⊗ SU(3) as spontaneous, then an octet P of massless pseudoscalar mesons (pzerons) will exist so that axial-vector-current conservation should be retained. At this intermediate stage we can write the matrix elements of the divergence of the axial-vector current between single baryon states in the form

$$\langle B' | f\partial_\mu a_\mu{}^P | B\rangle = (D_{BB'}{}^P + xF_{BB'}{}^P)2BA + q^2 C(D_{BB'}{}^P + x'F_{BB'}{}^P) \tag{4}$$

(f^{-1} being the pseudoscalar meson decay amplitude) and the requirement that a_μ be conserved yields the usual Goldberger-Treiman relations

$$x = x',\tag{5a}$$

$$q^2 C = 2BA.\tag{5b}$$

Again the left-hand side of (5b) can be interpreted as the pzeron-baryon coupling constant (it multiplies a $D+xF$ type coupling) which is strong [but the coupling is SU(3) invariant]. Equation (4) exhibits the spontaneous breakdown of SU(3) ⊗ SU(3). The final stage in which even the so-far untouched SU(3) invariance breaks down (the baryon masses split) affects axial-vector-current conservation like a small perturbation and in relations (5) C will now acquire a dependence on B and B' and we shall have

$$g_{BB'P} = (D_{BB'}{}^P + xF_{BB'}{}^P)[q^2 C_{BB'}(q^2)]_{q^2=0}$$

$$= (D_{BB'}{}^P + xF_{BB'}{}^P) A(B+B').\tag{5c}$$

It can be seen that the departures from the exact SU(3) coupling scheme are small $[\sim(B+B)/(B+B')_{\rm av} - 1 \lesssim 15\,\%]$ and are completely determined by the baryon mass spectrum. It is also easily verified that the coupling constants given by (5c) satisfy the relations given by perturbation theory,[13] but (5c) contains only two free parameters x and A as opposed to the seven parameters left free by perturbation theory.

This is to be compared with the anomalous coupling (3c) of zerons that does not obey the perturbation-theoretic relations. The difference between the two cases is easily understood once one realizes that in the limit of exact SU(3) symmetry (degenerate baryon masses) the zerons do not couple to baryons whereas even in this limit the pzerons are strongly coupled to baryons due to the spontaneous breakdown of SU(3)⊗SU(3) symmetry related to the mass of the fermions.

A much more dramatic deviation from SU(3) invariance is found by this method for the B^*BP couplings (B^* being the $J^P = \frac{3}{2}^+$ baryon decuplet). We can write

$$\langle B|f\partial_\mu a_\mu|B^*\rangle$$

$$= C_{B^*B}{}^P[A + C'(B^* - B) + D'(B^{*2} - B^2) + Eq^2],\tag{6}$$

where the $C_{B^*B}{}^P$ are the well-known Clebsch-Gordan coefficients.[14,15] We then identify

$$C_{B^*B}{}^P Eq^2|_{q^2=0} = g_{B^*BP}$$

and find

$$g_{B^*BP} = C_{B^*B}{}^P[A + C(B^* - B)],\tag{7}$$

$$C = C' + D'(B^* + B) \approx C' + D'(B^* + B)_{\rm av}.\tag{8}$$

Because of the second term on the right-hand side of (7), which varies by a whole order of magnitude as B and B^* change, an appreciable departure of g_{BB^*P} from the unitary-symmetric situation is expected. That this appears indeed to be the case has been pointed out recently on the basis of an N/D calculation by Wali and Warnock.[14] Our relations (5c) and (7) also clarify the reason why with completely SU(3)-symmetric BBP couplings, highly asymmetric BB^*P couplings have been found in reference 14. Again (7) satisfies the perturbation-theoretic sum rules.[15]

The predictions of (7) are compared with experiment[16] in Table I. Already for the four measurable coupling constants our two-parameter formula (7) makes two predictions (as compared to a single sum rule obtained in the context of perturbation theory[15]). A comparison of (7) with the dynamical results of reference 14 shows that we obtain a good agreement for $g_{BB^*\pi}$ and $g_{BB^*\eta}$ but not for the K and $\bar{K}$ coupling constants. This can be remedied if we let A and C in (7) depend on the index P. The perturbation-theory sum rules[15] then require

$$C_\eta = C_\pi = C_K = C_{\bar{K}} = C,\tag{9a}$$

$$3A_\eta + A_\pi = A_K + A_{\bar{K}}.\tag{9b}$$

The fact that $\partial_\mu a_\mu{}^\pi$ and $\partial_\mu a^\eta$ have matrix elements between the same B and B^* states can be used to show that

$$A_\eta = A_\pi.\tag{9c}$$

Table I. Calculated and experimental $B^*B\pi$ coupling constants.[a]

B^*BP	$X_{B^*BP}{}^{\rm theor}$	$X_{B^*BP}{}^{\rm exp}$	$X_{B^*BP}{}^{\rm exact\ SU(3)}$
$N^*N\pi$	1[b]	1.0	1.0
$Y^*\Lambda\pi$	0.88[b]	1.0 ± 0.2	1.0
$Y^*\Sigma\pi$	0.58	0.3 ± 0.3	1.0
$\Xi^*\Xi\pi$	0.66	0.71 ± 0.15	1.0

[a]The normalized constants X_{B^*BP} are defined in terms of the coupling constants in Formula (10).
[b]Input.

(9b) and (9c) lead to

$$A_P = A_0 + A_1 Y_P, \tag{9d}$$

where Y_P is the hypercharge of the meson P. Inserting (9) in (7) we find

$$X_{B^*BP} = \frac{g_{B^*BP}/C_{B^*B}{}^P}{g_{N^*N\pi}/C_{N^*N}{}^\pi} = a + bY_P + c\frac{B^*-B}{N^*-N}. \tag{10}$$

Here $g_{N^*N\pi}/C_{N^*N}{}^\pi$ and N^*-N have been introduced simply for normalization purposes. A comparison of (10) with the results of reference 14, exhibited in Table II, leads to a surprisingly good agreement (discrepancy ≲ 15 %).

We have thus seen that the Goldberger-Treiman relations can be used as an efficient tool for the study of mass and coupling-constant sum rules. The general feature of the couplings of pseudoscalar mesons is that they depart strongly (weakly) from symmetry if the parities of the two particles to which the mesons couple are opposite (identical). We have restricted ourselves to the consideration of eight conserved vector and eight conserved axial-vector currents. Of course, the number of conserved currents can be increased up to 32 as we suggested in a previous paper.[5]

Finally, a word is in order about conserved tensors of higher order. We can construct, e.g., a conserved spin vector in the theory of a massless fermion field interacting with a vector field with the following field equation:

$$\partial_\mu(\gamma_\mu + ieA_\mu)\psi = 0. \tag{11}$$

The group that leaves this equation invariant is

$$\psi \rightarrow \psi + \chi, \tag{12}$$

where χ is a constant spinor, whereas the conserved spin vectors can be directly read off Eq. (11) to be

$$u_\mu = (\gamma_\mu + ieA_\mu)\psi, \tag{13a}$$

$$v_\mu = (\gamma_\mu - ieA_\mu)\gamma_5\psi. \tag{13b}$$

The fermion mass term will break the invariance under the group (12). However, Goldberger-Treiman relations can be written for the matrix elements of $\partial_\mu u_\mu$ and $\partial_\mu v_\mu$ which will involve poles corresponding to particles of spin parity $\frac{1}{2}^+$ and $\frac{1}{2}^-$. It is, however, completely unclear at the present moment whether any "weak" interactions have u_μ and/or v_μ as their source. If

Table II. Predictions of B^*BP coupling constants.

B^*BP	X_{B^*BP} given by Formula (10) with $a=-0.2$, $b=0.6$, $c=1.2$	X_{B^*BP} calculated in reference 14.	X_{B^*BP} for exact SU(3) symmetry.
$N^*N\pi$	1.00	1.00	1.0
$Y^*\Lambda\pi$	0.88	0.78	1.0
$Y^*\Sigma\pi$	0.58	0.66	1.0
$\Xi^*\Xi\pi$	0.66	0.64	1.0
$Y^*\Sigma\eta$	0.58	0.58	1.0
$\Xi^*\Xi\eta$	0.66	0.58	1.0
$N^*\Sigma K$	0.59	0.63	1.0
$Y^*\Xi K$	0.67	0.56	1.0
$Y^*N\bar{K}$	0.98	0.87	1.0
$\Xi^*\Lambda\bar{K}$	0.86	0.75	1.0
$\Xi^*\Sigma\bar{K}$	0.57	0.63	1.0
$\Omega\Xi\bar{K}$	0.64	0.64	1.0

one goes further to tensors of second order $T_{\mu\nu}$, then the symmetric energy-momentum tensor is known to be the source of the gravitational field. The existence of an octet of conserved antisymmetric tensors $A_{\mu\nu}{}^i$ would set up a pattern for the anomalous magnetic moments of baryons. It turns out that this pattern would follow the exact SU(3)-symmetric pattern even in the presence of baryon mass splittings. This possibility has been suggested recently by Sakurai[17] on the basis of ω-φ mixing.

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symmetry-breaking interaction. In this latter case Formula (1a) would contain additional terms because the baryon states would not be pure octet states but would contain admixtures of 10, 27, etc. Equation (1a) is essentially the generalization of the Wigner-Eckart theorem to the case of a degenerate vacuum.

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A remark was made that the method of integration of references 3 and 4 was incorrect. This remark is not true. We wish to apologize to the authors of references 3, 4, and 6 for having made this incorrect statement. The results of the paper are unaffected.