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# LEPTONIC DECAY OF THE $\Omega^-$ PARTICLE*

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Recently a particle of strangeness -3 and of mass about 1680 MeV has been found[1] which fits the prediction[2-5] for the tenth member (the isosinglet) of the spin-parity $\frac{3}{2}^+$ baryon decuplet. Because such a particle can decay only via the weak interactions, it is interesting to study its leptonic decays, since they may appear with a rate large enough to be observed in the near future.

Assuming $V$-$A$ leptonic interaction, the most general interaction for the process

$$\Omega^- \rightarrow \Xi^0 + l + \bar{\nu}, \quad l = \mu, e, \tag{1}$$

can be written in the form

$$H' = [\bar{\Xi}(G_A{}'O^{\nu\rho}/\sqrt{2} + i\gamma_5 G_V{}'Q^{\nu\rho}/\sqrt{2})\Omega_\nu] \times [\bar{l}\gamma_\rho(1-i\gamma_5)\nu], \tag{2}$$

where

$$O^{\nu\rho} = (f_1 + f_2 P^\lambda p_\lambda/Mm)g^{\nu\rho} + (f_3 P^\rho/M + f_4 p^\rho/m + f_5\gamma^\rho)p^\nu/m,$$

$$Q^{\nu\rho} = (f_1{}' + f_2{}' P^\lambda p_\lambda/Mm)g^{\nu\rho} + (f_3{}'P^\rho/M + f'p^\rho/m + f_5{}'\gamma^\rho)p^\nu/m, \tag{3}$$

$P, M$ and $p, m$ being the four-momentum and the mass, respectively, of the $\Omega^-$ and of the $\Xi^0$ particle; $f_1$, $f_2$, $f_3$, $f_4$, and $f_5$ are form factors ($f_1$ which is equal to unity has been put in for convenience only). $\Xi$, $l$, and $\nu$ are the usual spinors, whereas $\Omega_\nu$ is a $\frac{3}{2}$ spin operator. We adopt for it the Rarita-Schwinger representation[6] and, following Kusaka[7] and Brown and Telegdi[8] we will use the set of orthonormal positive-energy spin states defined as

$$\Omega_\nu{}^\xi = H_\nu{}^\xi\Omega^+ + F_\nu{}^\xi\Omega^-, \tag{4}$$

where $\nu$ and $\xi$ are vector and polarization indices, respectively, $\Omega^\pm$ are spin-up and -down positive-energy $\frac{1}{2}$-spinors, and $H_\nu{}^\xi, F_\nu{}^\xi$ are the following four-vectors:

$$\begin{aligned}
&H_\nu{}^{(1)} \equiv 2^{-1/2}(1,i,0,0), && F_\nu{}^{(1)} \equiv 0,\\
&H_\nu{}^{(2)} \equiv 0, && F_\nu{}^{(2)} \equiv 2^{-1/2}(1,-i,0,0),\\
&H_\nu{}^{(3)} \equiv (1/\sqrt{6})(1,-i,0,0), && F_\nu{}^{(3)} \equiv (\sqrt{\tfrac{2}{3}})(0,0,1,0),\\
&H_\nu{}^{(4)} \equiv (\sqrt{\tfrac{2}{3}})(0,0,1,0), && F_\nu{}^{(4)} \equiv -(1/\sqrt{6})(1,i,0,0).
\end{aligned} \tag{5}$$

In the square matrix element we need, the leptonic term is straightforward. In the baryonic term, instead, we have the products $\Omega_\nu{}^\xi\bar{\Omega}_\mu{}^\xi$ which are linear combinations of spin-up and -down Dirac spinors. Thus we need two step operators such that (lower indices refer to energy and upper indices refer to spin)

$$O_-\Omega_+{}^+ = \Omega_+{}^-,$$

$$O_+\Omega_+{}^- = \Omega_+{}^+, \tag{6}$$

and the projection operators

$$S_+\Omega_+=\Omega_+{}^+, \quad S_-\Omega_+=\Omega_+{}^-. \tag{7}$$

With their help we find

$$\Omega_\nu{}^\xi\bar{\Omega}_\mu{}^\xi=\sum_{i=1}^{4}[(H_\nu{}^\xi H_\mu{}^{\xi*}+F_\nu{}^\xi H_\mu{}^{\xi*}O_-)S_+ +(F_\nu{}^\xi F_\mu{}^{\xi*}+H_\nu{}^\xi F_\mu{}^{\xi*}O_+)S_-]\Lambda_+(P)\Omega^i\bar{\Omega}^i, \tag{8}$$

where $\Lambda_+(P)$ is the positive energy projection operator.

The step operators in our representation are found to be

$$O_\mp=[\gamma_5\gamma_0/(\pm n_1+in_2)](n_1\gamma_2-n_2\gamma_1),$$
$$\vec{n}=\vec{P}/|\vec{P}|, \tag{9}$$

and the spin projection operators are

$$S_\pm=\tfrac{1}{2}(1\pm i\gamma_5\gamma_0\vec{\gamma}\cdot\vec{n}). \tag{10}$$

Using these operators in the coordinate frame where $\vec{n}\equiv(0,0,\vec{P}/|\vec{P}|)$, then going to the rest system of the $\Omega$, we obtain

$$\Omega_\nu{}^\xi\bar{\Omega}_\mu{}^\xi=\tfrac{1}{4}R_{\nu\mu}{}^\xi(1+\gamma_0)\sum_{i=1}^{4}\Omega^i\bar{\Omega}^i, \tag{11}$$

with

$$R_{\nu\mu}{}^\xi=L_{\nu\mu}{}^\xi-iM_{\nu\mu}{}^\xi\gamma_1\gamma_2+iN_{\nu\mu}{}^\xi\gamma_2\gamma_3+O_{\nu\mu}{}^\xi\gamma_3\gamma_1,$$

$$L_{\nu\mu}{}^\xi=H_\nu{}^\xi H_\mu{}^{\xi*}+F_\nu{}^\xi F_\mu{}^{\xi*}, \qquad M_{\nu\mu}{}^\xi=F_\nu{}^\xi F_\mu{}^{\xi*}-H_\nu{}^\xi H_\mu{}^{\xi*},$$

$$N_{\nu\mu}{}^\xi=H_\nu{}^\xi F_\mu{}^{\xi*}+F_\nu{}^\xi H_\mu{}^{\xi*}, \qquad O_{\nu\mu}{}^\xi=F_\nu{}^\xi H_\mu{}^{\xi*}-H_\nu{}^\xi F_\mu{}^{\xi*}. \tag{12}$$

From this point on, the usual spin-$\frac{1}{2}$ projection and trace technique can be used to calculate $|M_{fi}|^2$.

We will treat first the case of a completely unpolarized $\Omega$. Averaging on the four polarization states we find

$$\langle L_{\nu\mu}\rangle_{\rm av}=\tfrac{1}{3}\delta_{\nu\mu}(1-\delta_{\nu0}), \quad \langle M_{\nu\mu}\rangle_{\rm av}=\tfrac{i}{6}\epsilon_{0\nu\mu3},$$

$$\langle N_{\nu\mu}\rangle_{\rm av}=-\tfrac{i}{6}\epsilon_{01\nu\mu}, \qquad \langle O_{\nu\mu}\rangle_{\rm av}=\tfrac{1}{6}\epsilon_{0\mu2\nu}. \tag{13}$$

The result is a complicated expression, but a number of terms are smaller than the main ones by a factor of $(\vec{p}/m)^2$. To a first approximation, those terms can be dropped out, leading to the final expression ($w$ is the energy of the $\Xi$)

$$(m\mu/\omega w)|M_{fi}|^2=(2\pi)^5K(w)[F_1(p,k,k')+(2R/m)F_2(p,k,k')]$$
$$+(2\pi)^5K'(w)[F_1(p,k,k')+(2R'/m)F_2(p,k,k')], \tag{14}$$

where

$$(2\pi)^5K'(w)=\tfrac{1}{3}G_V{}'^2(1-w/m)(f_1{}'+f_2{}'w/m)m/w, \quad (2\pi)^5K(w)=\tfrac{1}{3}G_A{}'^2(1+w/m)(f_1+f_2w/m)m/w, \tag{15}$$

$$F_1=\frac{3\omega\omega'-\vec{k}\cdot\vec{k}'}{\omega\omega'},\quad F_2=\frac{\omega\vec{p}\cdot\vec{k}'+\omega'\vec{p}\cdot\vec{k}}{\omega\omega'},$$

and

$$R=\frac{f_3+f_4w/m+f_5}{f_1+f_2w/m},\quad R'=\frac{f_3'+f_4'w/m+f_5'}{f_1'+f_2'w/m};\tag{16}$$

$\omega, \vec{k}$ being the energy and momentum of the lepton, $\omega', \vec{k}'$ that of the neutrino. Two main features emerge at this point. First, owing to the small amount of the phase space available for the $\Xi$, the ratio $w/m$ is strictly close to one for every value of $w$ so that, from (15),

$$(2\pi)^5K(w)\approx\tfrac{2}{3}G_A{}'^2(f_1+f_2)^2,\quad K'(w)\approx 0,\tag{17}$$

and the entire result, in this approximation, depends only on $G_A'^2$ and not on $G_V'^2$. Second, in the same approximation we have

$$R\approx(f_3+f_4+f_5)/(f_1+f_2),\tag{18}$$

and (14) appears to be a one-parameter formula and opens the possibility that the experimental data, when available, will test what is the best value of $R$, i.e., of the ratio among the indirect terms and the direct ones.

The decay rate, given by

$$\Gamma=4\pi\int dp\,p^2K(w)[I_1+2RI_2/m],\tag{19}$$

where

$$I_i=\int\delta^4(P-p-k-k')F_id^3kd^3k',\tag{20}$$

has been calculated for the electron decay

$$\Omega^-\to\Xi^0+e+\bar{\nu}\tag{21}$$

neglecting the electron mass and choosing the angle $\theta$ between the cascade and the electron as angular parameter. The total rate is found to be[9]

$$\Gamma\approx F_A{}'^2(9.29-1.32R)10^{18}\ \mathrm{sec}^{-1},\tag{22}$$

where $F_A'$ is a dimensionless constant defined as $F_A'=G_A'm_p^2/\sqrt{2}$, and in the static limit, we put $f_1+f_2\approx 1$.

For sake of comparison, we may see what the order of magnitude would be putting for $F_A'$ the value of the $\beta$ decay coupling constant $F_A$,

$$F_A=G_Am_p^2/\sqrt{2}=(1.25/\sqrt{2})G_Vm_p^2$$
$$=(1.25/\sqrt{2})1.01\times10^{-5}.\tag{23}$$

We find

$$(\Gamma)_{F'_A=F_A}=(7.4-1.05R)\times10^8\ \mathrm{sec}^{-1},\tag{24}$$

and using the value of the lifetime given in reference 1,

$$\frac{\Gamma(\Omega\to\Xi+e+\nu)}{\Gamma(\Omega\to\Xi+\pi)}\lesssim 5\%.\tag{25}$$

However, this being a strangeness-changing decay, this rate should be further reduced by an order of magnitude.

In addition, several distributions and rates have been obtained. Among those, the ratio of the total forward to the total backward rate has been computed after integrating separately for $0°\leq\theta\leq 90°$ and for $90°\leq\theta\leq 180°$ and in Fig. 1 a plot of this ratio as a function of $R$ is shown. This is the most promising distribution to be compared with the experiment and even with relatively few events available, it could help in drawing some conclusions about whether or not the indirect terms are important in the matrix element. More detailed information on this subject can probably be gotten from the angular correlations in the case of a polarized decay, and a calculation of this is in progress, but those correlations are more difficult to detect experimentally and will require more events to be useful.

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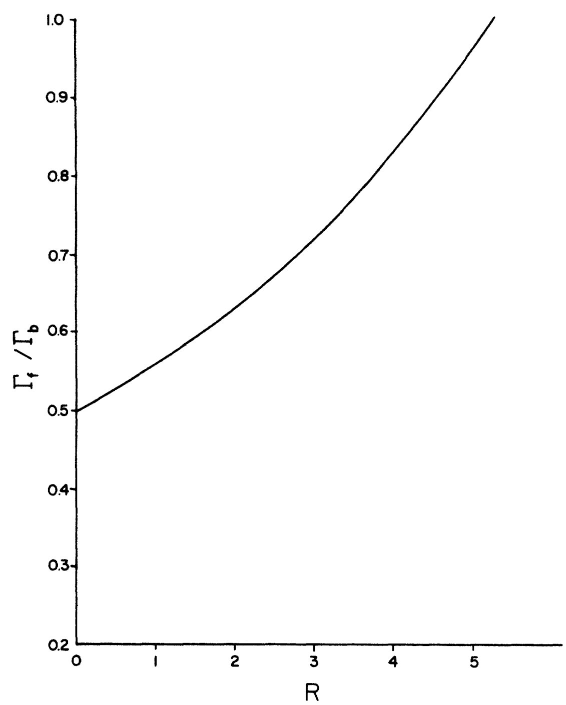


FIG. 1. Forward/backward ratio, $\Gamma_f/\Gamma_b$, as a function of $R$.

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# FOUR-DIMENSIONAL BARYON SPACE AND QUARKS

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The purpose of this note is to indicate a possible four-dimensional interpretation of the empirically determined symmetries which seem to be emerging from the study of the physics of elementary particles. Previous authors[1,2] who have sought to understand the phenomena of elementary particles from a space-time viewpoint have assumed a preferred symmetry structure for the "internal" space-time, e.g., the homogeneous Lorentz group, and thereby attempted to deduce the properties to be expected for the baryons. However, the semiempirical, semiformal approach, which to date has proven far more successful, seems to lean more toward a group of rank 2, and very possibly of dimension 8 [that is, SU(3)]. Rather than abandon all hope of a four-dimensional interpretation we preferred to examine the possibilities which remain available assuming that a group such as SU(3) proved to be correct. A study of some known theorems on groups of motions of Riemannian manifolds has led to a remarkably unique result which we wish to report here.

It is well known that the maximum group of motions which a Riemannian $V_4$ can admit is a 10-parameter group, and that this can occur only for a space of constant curvature,[3] that is for a space of rather trivial geometric structure. A somewhat less known theorem, due to Fubini,[4] states that if a $V_4$ admits a 9-parameter group it necessarily admits a 10-parameter group and is therefore of constant curvature. The maximal symmetry that a $V_4$ can have without having a trivial geometric structure is therefore an 8-parameter group of motions. Egoroff[5] has shown that if a $V_4$ admits a $G_8$ it must necessarily be an Einstein space, that is, a solution of the Einstein vacuum field equations (with cosmological constant)!

More recently, Vranceanu[6] and Egoroff[7] have determined the necessary structure of those $G_8$'s which can be permitted as motion groups of a $V_4$, and have explicitly determined all (positive-definite) $V_4$'s which can admit an 8-dimensional maximal group of motions. That a $V_4$ with Minkowskian signature cannot admit a maximal $G_8$ may be seen as follows: Since a $G_8$ necessarily acts transitively on a $V_4$ the stability subgroup (i.e., the subgroup which leaves some one point of $V_4$ fixed) must have 4 dimensions. Furthermore, if we choose Riemann normal coordinates in the neighborhood of the fixed point, it is evident that due to the Minkowskian signature the stability subgroup must also be a subgroup of the homogeneous Lorentz group. However, it can be shown that the homogeneous Lorentz group, unlike the orthogonal group in 4 dimensions, does not have a 4-dimensional subgroup which can be embedded into a maximal transitive $G_8$. Thus, the maximal group that a Riemann space of nonconstant curvature and Minkowski signature can admit is a $G_7$. (That a $G_7$ can, in fact, be attained may easily be seen by the metric $ds^2 = dt^2 - dl^2$, where $dl^2$ is the metric of a positive-definite $V_3$ of constant curvature.)

An inspection of the solutions of Vranceanu and Egoroff reveals that there are only two $G_8$'s which can be represented as motions of a $V_4$, only one of the $G_8$'s being compact. The noncompact $G_8$ is obtainable from the compact group by analytic