relates these facts.

In the presence of the electric fields applied, the electrons can be shown to attain velocities approaching that of sound. Such motion gives rise to a rather large Doppler shift in the phonon spectrum as seen by the speeding electrons. Frequency shifts sufficient to account for the 0.01 eV "line" spacings are easily obtained for electrons moving at 0.05 to 0.3 of the velocity of sound. And this hypothesis yields several other qualitative features of the experimentally observed results. Further investigations, both theoretical and experimental, are in progress to test the Doppler-shift conjecture quantitatively.

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RADIO-FREQUENCY EFFECTS IN SUPERCONDUCTING THIN FILM BRIDGES

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Parks and Mochel[1] have observed structure in the resistive behavior of a very fine bridge of a thin superconducting film as a function of an applied magnetic field. They interpreted this structure in terms of the effect of quantized "vortex" formation on the transition point of the film, as well as of longitudinal vortex motion along the bridge. We felt that this effect could be interpreted more directly, still in terms of vortices but in a manner more nearly allied to the magnetic field variation of the Josephson effect,[2] as a resistive effect in a fully superconducting sample.

To confirm this interpretation we tried an experiment analogous to Shapiro's experiment[3] on the ac Josephson effect. We shone radio-frequency power in the range $200-10^4$ Mc/sec on a number of thin film bridges and observed the resulting modifications in the direct current I-V characteristic. On the bridge shown in Fig. 1 we observed the I-V characteristic shown in Fig. 2(a) with rf frequency as a parameter, and in Fig. 2(b) with the rf power at a frequency of 810 Mc/sec as a parameter.

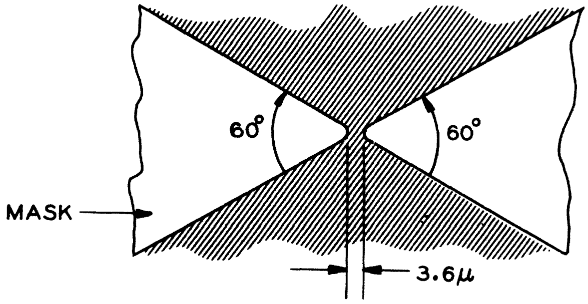

FIG. 1. Sample configuration.

We list a number of our results, some of which are evident in these curves.

(a) The steps in the I-V characteristic occur at voltages related to the frequency by the Josephson condition $mh\nu = 2eV$. A number of weaker subharmonics $mh\nu = 2neV$ occur also. Under less favorable conditions, we observe only smooth bumps at these voltages.

(b) There is a tendency to instability at low voltages which often hides the first steps.

(c) The whole phenomenon is strikingly sensitive to magnetic field, often disappearing when as little as 0.1 gauss is applied.

(d) It is noteworthy that the upper regions of the curves of Fig. 2 are linear, roughly, and parallel to the curve with very high rf power which has, as far as we can tell, been driven normal. Nonetheless they show Josephson steps at the correct voltages. This cannot occur unless there is a superconducting path (i.e., $\Delta \neq 0$)

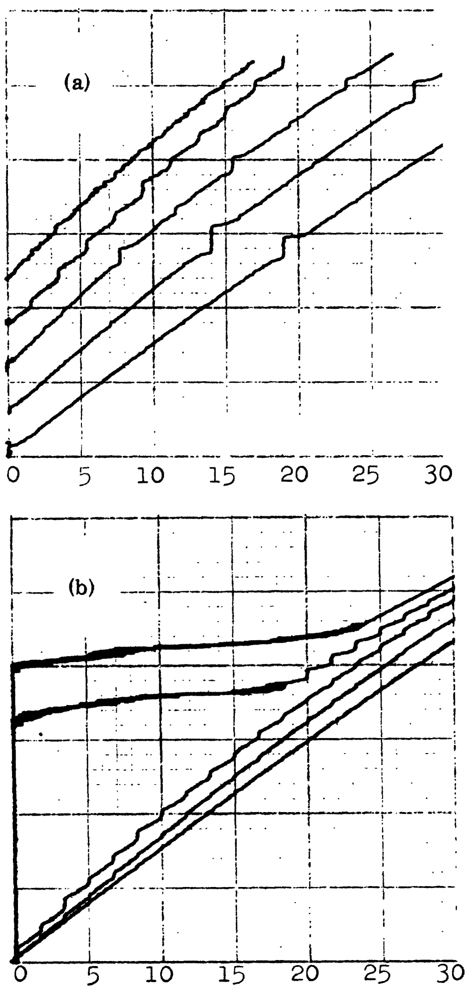

FIG. 2. (a) I-V characteristic at different frequencies: 0.28, 0.94, 3.8, 6.8, and 9.25 kMc/sec, respectively, starting from the top. Voltage scale is 5 μV/div and current scale is 133 μA/div. (b) dc I-V characteristics with rf power at a frequency of 810 Mc/sec as a parameter. From top curve down, curves are at the following relative power levels: no rf, -6, -4, -2, and 0 dBm. Voltage scale 5 μV/div, current 133 μA/div.

from one side across the bridge to the other; thus we conclude that nearly normal differential conductivity is present in a fully superconducting sample and that the normal current simply adds to the supercurrent.

(e) This form of bridge showed very little structure in its magnetic field characteristic. Bridges of the form used by Parks and Mochel, some 60 μ long and 4 μ wide in the constriction, on the other hand, tended to show more pronounced structure than Parks reports, and to behave in a rather spectacular but irregular fashion when both magnetic fields and radio-frequency currents are applied. Because of its irregularity we attempt no interpretation of these data.

Our interpretation of the data rests on an equation which, we propose, is necessary to complete the set of Ginzburg-Landau equations in samples which show finite voltages and resistive phenomena. We call it the "other Ginzburg-Landau equation":

$$\lambda_D^{\,2}\rho \propto i\hbar(\Psi^*\partial\Psi/\partial t-\Psi\partial\Psi^*/\partial t)-4eV|\Psi|^2 \simeq 0. \quad (1)$$

This equation is analagous to a fourth component of the current equation, giving the charge. Since the space charge is certainly very small, and the coefficient is the Debye rather than the London length, this combination is very nearly zero. Here Ψ is the Ginzburg-Landau order parameter proportional to the energy-gap function Δ, V is the electrostatic potential in the superconductor, and the other symbols have their standard meanings. When the charge is zero, this leads to an equation for the phase φ of the order parameter,

$$\hbar d\varphi/dt = 2eV, \quad (2)$$

which is closely related to the Josephson frequency condition.

These equations are, in many circumstances, obvious consequences of Maxwell's equations, flux quantization, and gauge invariance, but in others must be derived a priori.[4,5] They imply that a fully superconducting sample can sustain a dc voltage if the relative phases of the energy gaps in its separate parts vary in time. The simplest means by which the phase can vary in our sample is for nodes of the wave function —"vortices"—to pass across the bridge between the two superconducting regions in the transverse direction, from vacuum to vacuum. In the presence of a current this transverse motion is driven by the Lorentz force between the magnetic flux of the vortex and the current; the driving energy[4,5] is

$$\Delta E = J_s h/2e. \quad (3)$$

The formation of the nodal vortex line in the superconducting bridge is opposed by an energy barrier because its core has a reduced superconducting condensation energy.

The role of the radio-frequency current superposed on the dc current is to modulate the driving energy as a function of time. When the modulation is sufficient, it is clear that the passage of nodes can be synchronized to the driving frequency so that one, two, or more nodes are driven across the barrier at each cycle. This will occur when the driving current is such that the normal rate of passage of nodes is close to the appropriate harmonic. Such synchronization leads, by Eq. (2), to a fixed voltage related to the frequency by harmonics of the Josephson relation. Subharmonics can be generated also by an obvious generalization of the synchronization mechanism. At low currents vortex motion will be thermally activated and therefore irregular and unstable.

Even a small magnetic field will force quantized vortices irregularly into the wide regions of the film away from the constriction. This will remove the energy barrier for motion of vortices in the wide regions and allow them to pass at all currents; synchronization to an rf current will be more difficult. This explains effect (c).

The fact that the driving force in Eq. (3) is proportional only to the supercurrent tells one the manner in which normal and supercurrents add [see effect (d)]. This will be the subject of a later communication.

In the long, narrow bridges the quasiperiodic structure of voltage vs magnetic field no doubt reflects a variation in the energy barrier for vortices presented by the bridge. Where the width of the bridge contains precisely one node, this barrier will be minimized and the voltage a maximum. This interpretation is precisely analogous to that of the Josephson magnetic field effect[5] and was the point of departure of our investigations. The interaction of these relatively complicated vortex structures with rf currents can lead to complicated effects.

The thin films were prepared by vacuum evaporation of high-purity tin on a sapphire substrate through a mask formed by two pointed V-shaped strips. The geometry of the evaporated film is shown by the shaded area of Fig. 1. The width of the bridge was adjustable and the experimental results are shown for a gap 3.6 μ wide. Four leads are connected to the film, two leads to supply the dc current while the other two are used for measuring the dc voltage across the bridge. The radio-frequency power is applied by a loop terminating a coaxial line. The loop is placed adjacent to the film and is oriented so that the rf currents induced in the film flow in the same direction as the dc current. Both loop and sample are enclosed in a copper can soldered to the coaxial-line outer conductor. The data were plotted by a Mosely X-Y recorder preceeded by a Beckman FITGO amplifier. When necessary the current in sample was held constant using a specially designed servo-loop. The magnetic field perpendicular to the film was measured by a Bell "240" incremental gaussmeter. This instrument provides an accurately field-proportional output voltage which was used to drive the X-Y recorder. The helium cryostat temperature was stabilized by a device described by Adkins.[6]

Since the completion of the above work, similar experiments have been described by Lambe et al.[7] Those authors, however, did not observe Josephson jumps in the I-V characteristic nor provide any theoretical interpretation of their results.

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