

see P. Franzini and L. A. Radicati, Phys. Letters **6**, 322 (1963).

²The group SU(6) has been suggested in a somewhat different context by M. Gell-Mann, to be published. Gell-Mann's point of view is, however, different from the one discussed here, being based on the algebra of the conserved and quasiconserved currents.

³For a more detailed analysis of the applications, see A. Pais, following Letter [Phys. Rev. Letters **13**, (1964)].

⁴F. Gürsey and L. A. Radicati, to be published.

⁵The group G_4 is noncompact and may be regarded as an extension of the Lorentz group by means of the isotopic spin group. The generators of G_4 are the covariant spin operators, the isotopic spin operators and their

products. The little group of G_4 for fixed momentum q is $[SU(4)]_q$.

⁶F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, B467 (1964).

⁷M. Gell-Mann, Phys. Letters **8**, 214 (1964).

⁸It is clear that the fundamental triplets will be coupled to the mesons through F -type coupling only. Since the baryons do not belong to the lowest representation of SU(6), the gauge operators generate a larger algebra which produces F -type couplings with the vector mesons and F - and D -type couplings with the pseudoscalar mesons.

⁹P. G. O. Freund and Y. Nambu, Phys. Rev. Letters **12**, 714 (1964).

¹⁰A. Salam and J. C. Ward, unpublished.

IMPLICATIONS OF SPIN-UNITARY SPIN INDEPENDENCE*

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It is the purpose of this note to discuss further the possibility¹ that a broken $[SU(6)]_{\bar{q}}$ is a useful symmetry in strong interactions.

To introduce some questions which arise, consider Wigner's nuclear SU(4)-multiplet theory.² Representations of this group label multinucleon states in a given nuclear l shell. This is useful largely because spin-orbit coupling can be neglected to a good approximation for low-lying states. Spin-orbit forces will lead to some recoupling and accordingly the classification under SU(4) gets less good for higher excitations, as emphasized by Wigner.

Likewise for SU(6). Call $(M)_{\bar{q}}$ and $(B)_{\bar{q}}$ the respective meson and baryon representations. For M - B scattering one must reduce out $\{(B) \otimes (M)\}_{\alpha}$ where α represents the orbital variables. After taking out the center of mass, one can choose $\alpha = (k, l, l_z)$, l = orbital angular momentum. For each partial wave there may be recoupling between l and the (B, M) spins. Where this is unimportant, we can just reduce out $(B) \otimes (M)$.

This leads to a maximum possible spin for the baryon resonances, namely $\frac{3}{2}$ with the proposed choice of representations.¹ Higher spins are a sure sign of (l, s) coupling. In the region where this starts to happen (it appears³ to be ~ 2 BeV), the assignment of resonances to "new" SU(6) multiplets becomes considerably more complicated.

In view of this complexity, it may be asked whether it is necessary to put (8, 2) and (10, 4) in 56, as proposed,¹ because the breakdown SU(6)

– factorized $[SU(3) \otimes SU(2)]$ (first stage) – broken SU(3) (second stage) has a first stage of which the scale is not known beforehand. However, the choice 56 becomes more suggestive through mass considerations. The success of the Gell-Mann–Okubo formula as an effective first-order perturbation leads one to try the assumption that SU(6) – broken SU(3) is additive in the first- and second-stage breakdowns with coefficients that depend on the (five) Casimir operators C_i of SU(6) only. This is achieved by $M = M_0 + a_0(C_i)F_8 + b_0(C_i) \times d_{8jk}F_j F_k$, or⁴

$$M = M_0 + a(C_i)Y + b(C_i)[I(I+1) - \frac{1}{4}Y^2 - \frac{1}{3}F^2] \quad (1)$$

($F^2 = F_i F_i$). M_0 is the central mass of an SU(3) multiplet,

$$M_0 = M_{00}(C_i) + m(C_i, F^2, d_{ijk}F_i F_j F_k, J(J+1)). \quad (2)$$

M_{00} is the central mass of the SU(6) multiplet. We shall see shortly that the dependence of the SU(6)-breaking term m on both spin and unitary-spin invariants is essential, and the same is true for the C_i dependence of the quantities a, b , etc.

Application of Eq. (1) to the meson 35 yields (using the quadratic mass relation) $\rho^2 - \pi^2 = K^{*2} - K^2$, known⁵ to be true within the ρ -mass accuracy. Equation (1) as a linear mass formula gives for the 56 a calculated (10, 4) equidistance ≈ 130 MeV, derived from the (8, 2), close enough to the experimental value ≈ 145 MeV to make the choice 56 quite attractive.⁶ The first-stage split be-

tween (10, 4) and (8, 2) is ≈ 235 MeV, comparable in magnitude to a , the F -type octet split.

There is an important new aspect to the (effective) \overline{BBM} coupling in this theory. It follows from

$$\underline{56} \otimes \underline{56} = \underline{1} + \underline{35} + \underline{405} + \underline{2695} \quad (3)$$

that this coupling is, in fact, unique, because of the single occurrence of $\underline{35}$. Hence, the F/D ratio is determined by $SU(6)$. The fact that both F and D must occur in this coupling was noted by Gürsey and Radicati.⁷ Hence, there is no R invariance (unless one "doubles" the theory which is unattractive).

Let us next consider a few consequences based on the additional assumption that the (spin, F -spin) multiplets need not be strongly recoupled to l . As (10, 4) decays into baryon and meson (where energetically possible) one should at least know whether $\underline{56}$ is in $\underline{35} \otimes \underline{56}$. It is, as

$$\{\underline{35} \otimes \underline{56}\}_{\alpha} = \underline{1134} + \underline{700} + \underline{70} + \underline{56}. \quad (4)$$

For the decay of the (10, 4) the label α now specifically refers to $l=1$. [For the one-particle states on the right-hand side of Eq. (4) we may imagine to be in their rest frame.] Equation (3) also indicates which other $SU(6)$ representations are possible candidates for resonances which can decay into (octet + meson) or (decuplet + meson).

It is natural to consider next the other "small" representation, $\underline{70}$, of Eq. (3) with content

$$\underline{70} = (1, 2) + (8, 4) + (10, 2) + (8, 2).$$

It is tempting to fill (1, 2) with $Y_0^*(1405)$. For this to work, one needs $\text{spin}(Y_0^*) = \frac{1}{2}$. It is furthermore desirable for Y_0^* to have odd parity, in order that it can be the resonant state sought for in the interpretation of (K^-, p) data.⁸ This would fix the parity of the other terms in Eq. (3) to be negative. Thus, the incomplete γ octet⁹ becomes a possible candidate for $(8, 4)^-$ in $\underline{70}^-$. There would then be harmony between the spin-parity of this last multiplet and the desirable properties of $Y_0^*(1405)$.

Concerning the status of the γ octet, for both $Y_0^*(1520)$ and $N^{**}(1512)$ the evidence for $\frac{3}{2}^-$ is good.¹⁰ The assignment $\frac{3}{2}^-$ to $Y_1^{**}(1660)$ seems dubious.¹¹ However, according to Willis¹² this possibility cannot be excluded. In connection with the $SU(3)$ mass formula this assignment for Y_1^{**} would imply a $\Xi^*(1600)$ with $\frac{3}{2}^-$. If this at all exists,¹³ its production seems to be at most

$\sim 1-2\%$ of $\Xi^*(1530)$. [This would mean a first-stage split $(8, 4)-(1, 2)$ of ≈ 185 MeV, comparable to the one for $(10, 4)-(8, 2)$.] It seems that a $\frac{3}{2}^-$ octet could well be there, even though not all the correct ingredients may be at hand as yet.

If these assignments within the representation $\underline{70}^-$ are correct, there is a prediction of the existence of an $\frac{1}{2}^-$ octet and decuplet. In the spirit of Eq. (1), one may anticipate that there should be octet-decuplet relations also within the $\underline{70}$. If this is so and if we assume, to give an example, that the γ octet is fixed by the masses 1512, 1520, 1660, and (1600?) then the equidistance in $(10, 2)^-$ should be ~ 60 MeV, i.e., it is a $\underline{10}$ with its " Ω " as lowest state. This, in turn, would imply a sum rule for $(8, 2)^-$, namely, $\Sigma^* \sim \Xi^* + 60$ MeV. These assignments to $\underline{70}$ can only possibly work if the first-stage split m of Eq. (2) depends on unitary spin as well as on spin. The simplest possibility of a dependence of m on F is $\alpha(C_i)F^2$ with $\alpha(C_i) > 0$ which would give equidistant central masses for the sequence (1, 2), (8, 2), and (10, 2) with (1, 2) lowest. Note that a and b in Eq. (1) are generally different for the $\underline{56}$ and the $\underline{70}$, due to their C_i dependence.¹⁴

The content of $\underline{1134}$ and $\underline{700}$ is, of course, very complex. In particular one (1, 2) and one (1, 4) are herein contained. The assignment of Y_0^* to $\underline{70}$ is therefore not unambiguous; one must hope that some simplicity prevails.

Finally, note that the small baryon representation¹ $\underline{20}$ is a baryon-two-meson state (for example, in $\underline{70} \otimes \underline{35}$). One can also discuss two-meson states, using $\underline{35} \otimes \underline{35} = \underline{1} + \underline{35} + \underline{35} + \underline{189} + \underline{280} + \underline{280} + \underline{405}$.

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¹F. Gürsey and L. Radicati, preceding Letter [Phys. Rev. Letters **13**, 173 (1964)]. Notations used here are the same as in this Letter.

²E. Wigner, Phys. Rev. **51**, 105 (1937).

³J. Helland et al., Phys. Rev. Letters **10**, 27 (1963).

⁴The formula of R. J. Oakes and C. N. Yang, Phys. Rev. Letters **11**, 174 (1963), gives (in our notation) an a which does depend on the $SU(3)$ representation.

⁵S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

⁶This closeness has been noted by many people. The fact that the (10, 2) is stable in the central $SU(3)$ -mass

limit is amusing in that it closely resembles the old strong-coupling treatment for the (3,3) resonance which has, in fact, SU(4) characteristics in its algebra [W. Pauli and S. Dancoff, Phys. Rev. 62, 85 (1942)].

⁷Reference 1, footnote 8.

⁸R. H. Dalitz, Ann. Rev. Nucl. Sci. 13, 338 (1963).

⁹S. Glashow and A. Rosenfeld, Phys. Rev. Letters 10, 192 (1963).

¹⁰For Y_0^* , see R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters 9, 66 (1962). For N^{**} , see P. Auvil and C. Lovelace, Imperial College Report No. ICT P/64/37 (unpublished); M. Olsson and G. B. Yodh, University of Maryland Technical Report No. 358 (unpublished).

¹¹M. Taher-Zadeh et al., Phys. Rev. Letters 11, 470 (1963).

¹²W. J. Willis, private communication.

¹³P. L. Connolly et al., Proceedings of the Sienna International Conference on Elementary Particles (Società Italiana di Fisica, Bologna, Italy, 1963), Vol. I, p. 125.

¹⁴One should also consider the (l,s) coupling as a "third stage" which may lead to recurrences of SU(6) multiplets with higher J values. This coupling is not the same as the spin-orbit coupling of T. Kycia and K. Riley, Phys. Rev. Letters 10, 266 (1963) (K-R). An effective (l,s) coupling in the present meaning may be responsible for the $\Delta I = 2$ recurrences noted by K-R. If this picture makes sense, then the K-R mass plot indicates that the third stage is (once again) linearly independent of the first one and that both 56 and 70 recur with J raised by 2.