for suggesting this investigation and for much advice and encouragement during the course of the analysis. The author wishes in addition to acknowledge useful discussions with Professor L. W. Jones.

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[8]The analysis of the experimental data excluded only those events with values of $\omega$ in the $N^*$ mass band.

## SPIN AND UNITARY SPIN INDEPENDENCE OF STRONG INTERACTIONS*

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The purpose of this Letter is twofold. We want first to point out that the group SU(4) introduced by Wigner[1] to classify nuclear states can be extended to the relativistic domain and it is, therefore, relevant for particle physics. We will next show that when strangeness is taken into account the group SU(4) becomes enlarged to[2] SU(6) which contains, as a subgroup, $SU(3)\otimes[SU(2)]_q$. $[SU(2)]_q$ is the unitary subgroup (little group) of the Lorentz group that leaves invariant the momentum four-vector $q$.

The group we consider here embodies SU(3) and the ordinary spin in the same way as Wigner's SU(4) embodies isotopic spin and ordinary spin. Preliminary results on the classification of particles based on SU(6) seem encouraging enough to motivate a study of this group.[3]

We begin by discussing the first point. Let us assume that the $\rho$, $\omega$, and $\pi$ mesons are coupled to the nuclear field through a symmetrical Lagrangian of the form

$$L_{NM} = g\{\bar\psi\gamma_\mu\psi\omega_\mu + \bar\psi\gamma_\mu\tau^a\psi\rho_\mu{}^a + i\psi\gamma_5\gamma_\mu\tau^a\psi\varphi_\mu{}^a\}, \quad (1)$$

where $a$ denotes the isotopic spin index. Let us further impose the subsidiary conditions

$$\partial_\mu\omega_\mu = 0, \quad \partial_\mu\rho_\mu{}^a = 0,$$

$$\partial_\lambda\varphi_\mu{}^a - \partial_\mu\varphi_\lambda{}^a = 0, \quad (2)$$

which insure that $\omega_\mu, \rho_\mu{}^a, \varphi_\mu{}^a$ describe, respectively, particles with $(J=1^-, T=0)$, $(J=1^-, T=1)$, and $(J=0^-, T=1)$. The pion field $\pi^a$ is related to the axial vector field $\varphi_\mu{}^a$ through $\varphi_\lambda{}^a = (1/\mu)\partial_\lambda\pi^a$, $\mu$ being the mass common to all mesons.

The conditions (2) are compatible with the equations of motion only if $L$ includes, besides $L_{NM}$ [Eq. (2)], additional terms such that the mesons $(\rho, \omega, \pi)$ are coupled to conserved currents. Thus $\omega$ and $\rho$ are coupled to the conserved baryon and isotopic-spin currents, respectively, while the pion is coupled to a conserved axial-vector current.

It can now be shown[4] that $L$ is invariant under a group[5] $\mathcal{G}_4$ which induces for each momentum $q$ of the mesons a unitary unimodular transformation among the 15 degenerate states $\omega$, $\rho$, and $\pi$. In counting the multiplicity we include, for a given momentum, the spin states just as for Wigner's supermultiplets. Under this transformation the nucleon $(S=\frac{1}{2}, T=\frac{1}{2})$ transforms like the four-dimensional representation of the group.

In the nonrelativistic limit, $L_{NM}$ gives rise to a potential which describes spin- and isospin-independent exchange forces (Majorana forces) between nucleons. This potential is, therefore, invariant under Wigner's group SU(4). If now a purely spin-dependent perturbation is introduced, $\omega$ and $\rho$ remain degenerate whereas the pion splits from them within the supermultiplet. We note that $\omega$, $\rho$, and $\pi$ are associated with the adjoint representation of SU(4). When this representation is reduced under the subgroup $SU(2)\otimes[SU(2)]_q$ it splits into states with $(J=1^-, T=0)$, $(J=1^-, T=1)$, and $(J=0^-, T=1)$.

These considerations are readily extended to include strange particles. In this case the SU(2) isotopic-spin group is replaced by SU(3) so that $\mathcal{G}_4$ goes over into a group $\mathcal{G}_6$ whose little group is $[SU(6)]_q$ which admits $SU(3)\otimes[SU(2)]_q$ as a subgroup.

The representation of SU(6) can be characterized by five integers $(\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5)$ where the $\lambda_i$'s are functions of the five Casimir operators. Table I shows some of the representations of SU(6) together with their SU(3) and spin structure. The symbols $(m,n)$ in the third column refer to the SU(3) and spin multiplicity, respectively.

The lowest nontrivial representation (10000) has six dimensions. It represents a fundamental SU(3) triplet with (ordinary) spin $\frac{1}{2}$. Its SU(3) $\otimes$SU(2) content is (3, 2). The conjugate representation (00001) describes the antiparticle and its content is (3*, 2).

A Lagrangian similar to (1) can be written which couples invariantly the fundamental triplet to mesons corresponding to the 35-dimensional adjoint representation. When a spin-dependent perturbation is introduced the 35 states split into a pseudoscalar octet and a degenerate vector nonet[6] with negative parity. These can be identified with the observed $(\pi, K, \eta)$ and $(\rho, \omega, K^*, \varphi)$ multiplets. SU(6) provides, therefore, a natural explanation of the degeneracy of the vector octet and the vector singlet in the nonet. All other meson-meson resonances must belong to self-conjugate representations of SU(6). Possible candidates are (00000) with even or odd parity, (10001) with even parity, (11011) with even or odd parity, etc.

The baryon octet and the $J=\frac{3}{2}^+$ decuplet can be grouped as a 56-dimensional representation obtained from the symmetrical combination of three fundamental triplets. The reduction of the direct product of $\underline{6}\otimes\underline{6}\otimes\underline{6}$ gives rise to three representations with 20, 70, and 56 dimensions. The fact that the ground state of the three-body configuration is symmetrical (56-dimensional representation) in the spin and unitary-spin variables implies that the two-body forces between them are repulsive. This seems to exclude a scheme based on only three fundamental quarks[7] whereas it is consistent with model II discussed in Appendix IV of reference 6. The connection of higher representations with possible baryon resonances is discussed by Pais.[3]

Table I. Some representations of SU(6) and their unitary spin and spin content.

| Labeling $(\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5)$ | Dimensions $D(\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5)$ | Unitary spin and spin multiplicities $(n, m)$ |
|---|---|---|
| (00000) | 1 | (1, 1) |
| (10000) | 6 | (3, 2) |
| (00001) | 6* | (3*, 2) |
| (01000) | 15 | (3*, 3), (6, 1) |
| (00100) | 20 | (8, 2), (1, 4) |
| (20000) | 21 | (6, 3), (3*, 1) |
| (10001) | 35 | (8, 3), (8, 1), (1, 3) |
| (30000) | 56 | (10, 4), (8, 2) |
| (11000) | 70 | (10, 2), (8, 4), (8, 2), (1, 2) |

The splitting between the $J=0^-$ octet and the $J=1^-$ nonet suggests that the mass operator contains a spin-dependent term which can only be a function of $J(J+1)$. A simple mass formula for an SU(6) supermultiplet is the mass squared[6] formula

$$\mu^2 = \mu_0{}^2 + \alpha J(J+1) + \gamma[T(T+1) - \tfrac{1}{4}Y^2]$$

for mesons and

$$M = M_0 + aJ(J+1) + bY + c[T(T+1) - \tfrac{1}{4}Y^2]$$

for baryons.

These are by no means the most general mass formulas that can be written on the basis of a broken SU(6) symmetry. The mass formula problem is further discussed by Pais.[3]

The interaction Lagrangian with conserved currents is generated from the free Lagrangian through a gauge transformation[4] associated with the group $\mathcal{G}_6$.[8] As in the case of the electromagnetic interaction this implies parity conservation for the strong interactions invariant under $\mathcal{G}_6$. Hence all the states of an SU(6) supermultiplet must have the same parity. Our scheme is, therefore, different from others that have been discussed recently[2,9,10]; in particular it does not predict $0^+$ and $1^+$ mesons degenerate with the existing $0^-$ and $1^-$ mesons. The degenerate states associated with the meson states for given momentum $\vec{q}$ and given SU(6) quantum numbers are simply the states corresponding to the opposite momentum and the same SU(6) quantum numbers.

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†On leave from the Middle East Technical University, Ankara, Turkey.

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[1]E. P. Wigner, Phys. Rev. 51, 105 (1937). For recent evidence on the validity of the supermultiplet model,

see P. Franzini and L. A. Radicati, Phys. Letters 6, 322 (1963).

[2]The group SU(6) has been suggested in a somewhat different context by M. Gell-Mann, to be published. Gell-Mann's point of view is, however, different from the one discussed here, being based on the algebra of the conserved and quasiconserved currents.

[3]For a more detailed analysis of the applications, see A. Pais, following Letter [Phys. Rev. Letters 13, (1964)].

[4]F. Gürsey and L. A. Radicati, to be published.

[5]The group $\mathcal{G}_4$ is noncompact and may be regarded as an extension of the Lorentz group by means of the isotopic spin group. The generators of $\mathcal{G}_4$ are the covariant spin operators, the isotopic spin operators and their products. The little group of $\mathcal{G}_4$ for fixed momentum $q$ is $[\mathrm{SU}(4)]_q$.

[6]F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. 135, B467 (1964).

[7]M. Gell-Mann, Phys. Letters 8, 214 (1964).

[8]It is clear that the fundamental triplets will be coupled to the mesons through $F$-type coupling only. Since the baryons do not belong to the lowest representation of SU(6), the gauge operators generate a larger algebra which produces $F$-type couplings with the vector mesons and $F$- and $D$-type couplings with the pseudoscalar mesons.

[9]P. G. O. Freund and Y. Nambu, Phys. Rev. Letters 12, 714 (1964).

[10]A. Salam and J. C. Ward, unpublished.

## IMPLICATIONS OF SPIN-UNITARY SPIN INDEPENDENCE*

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It is the purpose of this note to discuss further the possibility[1] that a broken $[\mathrm{SU}(6)]_{\vec{q}}$ is a useful symmetry in strong interactions.

To introduce some questions which arise, consider Wigner's nuclear SU(4)-multiplet theory.[2] Representations of this group label multinucleon states in a given nuclear $l$ shell. This is useful largely because spin-orbit coupling can be neglected to a good approximation for low-lying states. Spin-orbit forces will lead to some recoupling and accordingly the classification under SU(4) gets less good for higher excitations, as emphasized by Wigner.

Likewise for SU(6). Call $(M)_{\vec{q}}$ and $(B)_{\vec{q}}$ the respective meson and baryon representations. For $M$-$B$ scattering one must reduce out $\{(B)\otimes(M)\}_\alpha$ where $\alpha$ represents the orbital variables. After taking out the center of mass, one can choose $\alpha = (k, l, l_z)$, $l$ = orbital angular momentum. For each partial wave there may be recoupling between $l$ and the $(B, M)$ spins. Where this is unimportant, we can just reduce out $(B)\otimes(M)$.

This leads to a maximum possible spin for the baryon resonances, namely $\frac{5}{2}$ with the proposed choice of representations.[1] Higher spins are a sure sign of $(l, s)$ coupling. In the region where this starts to happen (it appears[3] to be ~2 BeV), the assignment of resonances to "new" SU(6) multiplets becomes considerably more complicated.

In view of this complexity, it may be asked whether it is necessary to put (8, 2) and (10, 4) in $\underline{56}$, as proposed,[1] because the breakdown SU(6) → factorized [SU(3)⊗SU(2)] (first stage) → broken SU(3) (second stage) has a first stage of which the scale is not known beforehand. However, the choice $\underline{56}$ becomes more suggestive through mass considerations. The success of the Gell-Mann–Okubo formula as an effective first-order perturbation leads one to try the assumption that SU(6) → broken SU(3) is additive in the first- and second-stage breakdowns with coefficients that depend on the (five) Casimir operators $C_i$ of SU(6) only. This is achieved by $M = M_0 + a_0(C_i)F_8 + b_0(C_i) \times d_{8jk}F_jF_k$, or[4]

$$M = M_0 + a(C_i)Y + b(C_i)[I(I+1) - \tfrac{1}{4}Y^2 - \tfrac{1}{3}F^2] \qquad (1)$$

$(F^2 = F_iF_i)$. $M_0$ is the central mass of an SU(3) multiplet,

$$M_0 = M_{00}(C_i) + m(C_i, F^2, d_{ijk}F_iF_jF_k, J(J+1)). \qquad (2)$$

$M_{00}$ is the central mass of the SU(6) multiplet. We shall see shortly that the dependence of the SU(6)-breaking term $m$ on both spin and unitary-spin invariants is essential, and the same is true for the $C_i$ dependence of the quantities $a, b$, etc.

Application of Eq. (1) to the meson $\underline{35}$ yields (using the quadratic mass relation) $\rho^2 - \pi^2 = K^{*2} - K^2$, known[5] to be true within the $\rho$-mass accuracy. Equation (1) as a linear mass formula gives for the $\underline{56}$ a calculated (10, 4) equidistance ≃ 130 MeV, derived from the (8, 2), close enough to the experimental value ≈145 MeV to make the choice $\underline{56}$ quite attractive.[6] The first-stage split be-