Two crucial tests of the reality of the effect can be made. The first is that the precession angle must increase linearly with t. We calculate the average over the data of $\langle Ha\xi \cos\varphi \rangle$, which is proportional to the precession angle. According to Eq. (4) the result should be $-MH^2 \times \langle a^2\rangle\langle\cos^2\varphi\rangle\langle\xi^2\rangle t$, a linear function of t with slope proportional to $-M$. Figure 2 shows our result. The second test is that the precession-induced polarization must have the proper correlation with the magnetic-field direction. We calculate $\langle at\xi\rangle$. The result according to Eq. (4) should be $\langle a\rangle\langle t\rangle\langle\xi\rangle - M\langle a^2\rangle\langle t^2\rangle H\cos\varphi$. We subtract the (known) first term and plot against φ, folding $\varphi = 0$ to -180 deg with 0 to +180 deg. The result is shown in Fig. 3. Within the limited statistical accuracy, the data satisfy the tests.

We wish to thank Professor Luis W. Alvarez for his encouragement and support. We also thank Lester John Lloyd for his many contributions to this experiment.

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[4]According to M. Nauenberg (Phys. Rev., to be published), if the baryons are composite states of triplets then $\mu_\Lambda = (1+\frac{1}{6}q)\mu_n$. For quark triplets, $q = -\frac{1}{3}$ and $\mu_\Lambda = \frac{1}{2}\mu_n = -0.95$. For singly charged triplets, $q = -1$ and $\mu_n = \frac{5}{6}\mu_n = -1.59$. Nicola Cabibbo has pointed out to us that present calculations based on SU(3) must take $m_\Lambda = m_n$ and thus have uncertainties of the same order as the fractional mass difference, which is 17%.

[5]Terms in t^2 do not contribute to our final result because they are odd in $\cos\varphi$. Terms in t^3 would lead to a correction of about 1%.

[6]Our final result ($\mu_\Lambda = -1.39$) is based on an analysis that takes into account both the inhomogeneity of H and the small components not along $\hat{x}$. The simpler analysis described here gives $\mu_\Lambda = -1.33$.

[7]For a forward Λ, $\hat{n}(0)\times\hat{H}$ is along $\hat{\Lambda}$. For our sample, the laboratory angular distribution of the Λ's is strongly peaked forwards (maximum angle ≈30 deg), and the average component of $\hat{n}(0)\times\hat{H}$ along $\hat{n}(0)\times\hat{\Lambda}$ is about a fifth of that along $\hat{\Lambda}$.

[8]Even if there were a parity-nonconserving polarization component along $\hat{\Lambda}$ at $t=0$, i.e., an additional term in Eq. (3), it would not affect our result for μ_Λ because such a term has no correlation with the magnetic field and gives no contribution after the average over φ.

[9]We expect $\langle\cos\varphi\rangle = 0$ and find 0.0036. We expect $\langle\xi\rangle = 0.05$ and find 0.0617. Their product makes the first term negligible.

[10]The error in $\langle at\xi\cos\varphi\rangle$ is $[\langle(at\xi\cos\varphi)^2\rangle/N]^{1/2}$. Equation (5), which expresses M in terms of the moments of the distribution, is completely equivalent to the method of maximum likelihood, for our case. For a distribution of the form $1+ax$, the likelihood function is $\Pi_i(1+ax_i)$ and the most likely value of a is given by the solution of the equation $\Sigma_i x_i/(1+ax_i) = 0$. For $|a| \ll 1$, we expand the denominator and obtain $a = \Sigma x_i/\Sigma x_i^2$, which is equivalent to Eq. (5). We are indebted to Frank T. Solmitz for this observation.

[11]Average values over the 8553 events (with the brackets omitted) are $t = 2.748\times10^{-10}$ sec, $t^2 = 14.07 \times 10^{-20}$ sec^2, $\cos\varphi = 0.0036$, $\cos^2\varphi = 0.496$, $\xi = 0.0617$, $\xi^2 = 0.295$.

KINEMATICAL INTERPRETATION OF THE FIRST π-p RESONANCE*

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Evidence for a pi-rho resonance has recently been pointed out on the basis of π-p reaction experiments involving four final-state particles.[1-3] It is the purpose of this note to indicate, by means of a simple model for the dynamics of the production process in such reactions, a possible explanation for an observed peak in the final state π-ρ mass spectrum. The calculation reported here presumes only to be an improvement over the phase-space calculation with which the π-ρ production data were compared. The complexity of the kinematics in a many-particle final state and the uncertainties in the behaviors of off-mass-shell matrix elements make a more elaborate calculation at present unfeasible.

The reaction which is analyzed corresponds to the process

$$\pi^+ + p \to \pi^+ + p + \rho \to \pi^+ + p + \pi^- + \pi^+. \qquad (1)$$

It is assumed that the reaction associated with the observed peak in the π-ρ spectrum proceeds principally via peripheral collisions which are dominated by the one-pion-exchange diagram of Fig. 1(a). The cross section associated with this diagram is written

$$d\sigma=\frac{1}{(2\pi)^4}\left(\frac{g^2}{4\pi}\right)\frac{1}{16F_I}\frac{[\Delta^2-(m_2-m)^2][\Delta^2-(m_2+m)^2]}{m_2^{\,2}}\frac{F(\Delta^2)}{(\Delta^2-m^2)^2}|M_{\pi N}'|^2\delta^4(p_f-p_i)\frac{d\bar{q}_1 d\bar{q}_2 d\bar{q}}{q_{10}q_{20}q_0}. \quad (2)$$

Here $F(\Delta^2)$ is the form factor associated with the pion propagator normalized to unity at $\Delta^2=m^2$, and F_I is the invariant flux equal to the product of the nucleon mass M_1 and the incident pion momentum in the laboratory. The quantity $g^2/4\pi \cong 1.8$ is the effective $\pi\pi\rho$ coupling constant, and m and m_2 are, respectively, the masses of π and ρ.

The matrix element $M_{\pi N}'$ corresponds to the off-mass-shell π^+-p scattering amplitude. Neglecting spin dependence in the π-N interaction, the on-mass-shell amplitude $M_{\pi N}$ is related to the pion-nucleon differential cross section $d\sigma_{\pi N}/d\Omega$ by the relation

$$|M_{\pi N}|^2=(8\pi\omega)^2 d\sigma_{\pi N}/d\Omega, \quad (3)$$

where ω is the total center-of-mass energy of the π-N system. The cross section $d\sigma_{\pi N}/d\Omega$ is characterized at high energy by a pronounced diffraction peak at small four-momentum transfer t^2, which can be expressed by the equation[4]

$$d\sigma_{\pi N}/d\Omega=(d\sigma/d\Omega)_0 e^{\lambda t^2}. \quad (4)$$

It is the contention of this note that a similar diffraction character exhibited by the squared amplitude $|M_{\pi N}'|^2$ over a significant range of its variables accounts for a pronounced peak observed in the cross section $d\sigma$ as a function of the π-ρ center-of-mass energy, and $|M_{\pi N}'|^2$ is therefore replaced here by the two-parameter expession given by Eqs. (3) and (4). The use of this model is consistent with the experimental observation that the production process occurs with small four-momentum transfer to the nucleon. The form (4) is expected to be valid provided the center-of-mass energy ω of the outgoing pi-nucleon system is sufficiently high. The calculation of this note is restricted to such values of ω.

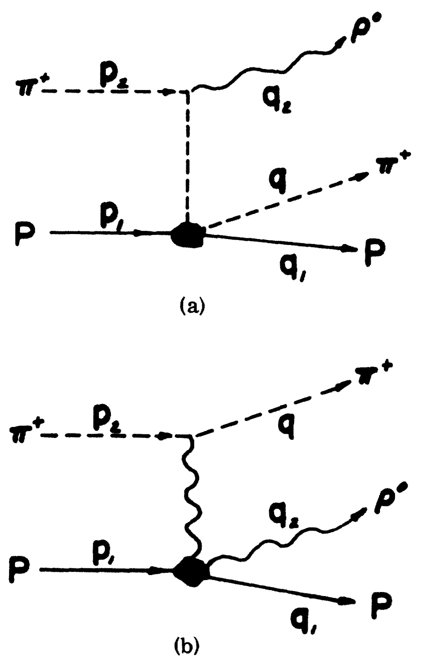

FIG. 1. Single-particle exchange diagrams giving rise to a kinematical peak in the π-ρ mass spectrum.

In calculation of (2) we choose to evaluate both the form factor $F(\Delta^2)$ and the spin factors $[\Delta^2-(m_2-m)^2][\Delta^2-(m_2+m)^2]$ at the point $\Delta^2=m^2$ so that the momentum-transfer dependence of the cross section is contained solely in the propagator factor $1/(\Delta^2-m^2)^2$. This manner of treating the Δ^2 dependence has lead to qualitative agreement with experiment in a calculation similar to that discussed here.[5] Making use of this simplification, and approximating $M_{\pi N}'$ by the mass-shell amplitude $M_{\pi N}$ given by Eqs. (3) and (4), the cross section is obtained in the form

$$d\sigma=\frac{1}{(2\pi)^2}\left(\frac{g^2}{4\pi}\right)\frac{1}{F_I}\left(\frac{d\sigma}{d\Omega}\right)_0\frac{(m_2^{\,2}-4m^2)}{(\Delta^2-m^2)^2}$$
$$\times\omega^2 e^{\lambda t^2}\delta^4(p_f-p_i)\frac{d\bar{q}_1 d\bar{q}_2 d\bar{q}}{q_{10}q_{20}q_0}. \quad (5)$$

Equation (5) is a function of five independent invariant variables,[6]

$$W^2=(p_1+p_2)^2,$$
$$\Delta^2=(q_2-p_2)^2,$$
$$t^2=(q_1-p_1)^2,$$
$$u^2=(q+q_2)^2,$$
$$\omega^2=(q+q_1)^2. \quad (6)$$

The value of the total center-of-mass energy W determines the range of values of the remaining variables. By use of the relations

$$d\bar{q}_1/q_{10} \equiv (2\pi/2F_I)du^2dt^2, \quad \delta^4(p_f-p_i)d\bar{q}_2d\bar{q}/q_{20}q_0 = [(q/u)d(\cos\theta_2)d\varphi_2]_{q,q_2}, \tag{7}$$

the invariant phase-space factor can be re-expressed in terms of u^2, t^2, and the spherical angles of $\bar{q}_2$ relative to $\bar{p}_2$ evaluated in the center of mass of the q, q_2 system [indicated by the subscript in (7)]. Substituting (7) in Eq. (5) one obtains for the cross-section differential in the variable u^2 the result

$$\frac{d\sigma}{du^2} = \left(\frac{g^2}{4\pi}\right)\frac{(m_2^2-4m^2)}{4F_I^2}\left(\frac{d\sigma}{d\Omega}\right)_0 \left\{\int_{(-t^2)_{\min}}^{(-t^2)_{\max}} d(-t^2)\int d(\cos\theta_2)\frac{d\varphi_2}{2\pi}\frac{q}{u}\frac{\omega^2 e^{-\lambda(-t^2)}}{(\Delta^2-m^2)^2}\right\}_{q,q_2}. \tag{8}$$

Here

$$q/u = 2u^{-2}[(u^2+m_2^2-m^2)^2 = 4m_2^2u^2]^{1/2},$$

$$(-t^2)_{\substack{\max\\ \min}} = 2W^{-2}\{(W^2-M_1^2)^2-m^2(W^2+M_1^2)-(W^2+M_1^2-m^2)u^2$$
$$\pm[W^2-2(M_1^2+m^2)W^2+(M_1^2-m^2)^2]^{1/2}[u^4-2(W^2+M_1^2)u^2+(W^2-M_1^2)^2]^{1/2}\}, \tag{9}$$

and

$$\Delta^2-m^2 = -a+b\cos\theta_2, \quad \omega^2 = A-B\cos\theta_2+C\sin\theta_2\cos\varphi_2, \tag{10}$$

where the coefficients a, b, A, B, and C are positive functions of W^2, u^2, and t^2.

As a result of the factor $\exp[-\lambda(-t^2)]$, the integrand in (8) tends rapidly to zero as $(-t^2)$ increases from its lowest value, and the main contribution to the t^2 integral comes from a region of values of the variable in the vicinity of the lower limit, $(-t^2)_{\min}$. Evaluation of the coefficients A, B, and C demonstrates that for values of $(-t^2)$ near this limit ω^2 is given to an excellent approximation by the simple form

$$\omega^2 \cong A-B\cos\theta_2. \tag{11}$$

The use of this approximation considerably simplifies the integration in (8) without appreciably altering the results of an exact evaluation. With the use of Eq. (11) the integrand is independent of φ_2 and $d(\cos\theta_2)$ is linearly related to $d\omega^2$. It follows that $d\sigma/du^2$ can be rewritten,

$$\frac{d\sigma}{du^2} = \left(\frac{g^2}{4\pi}\right)\frac{(m_2^2-4m^2)}{4F_I^2}\left(\frac{d\sigma}{d\Omega}\right)_0 \int_{(-t^2)_{\min}}^{(-t^2)_{\max}} d(-t^2)\int_{\omega_0^2}^{\omega_{\max}^2} d\omega^2 e^{-\lambda(-t^2)}\frac{1}{B}\frac{q}{u}\frac{(-\omega^2)}{[a-b/B(A-\omega^2)]^2}, \tag{12}$$

$$\omega_{\max}^2 = A+B.$$

The integration over ω^2 is restricted to values of the final pion-nucleon energy for which the diffraction-peak parametrization of the pion-nucleon cross section [Eq. (4)] is expected to be valid.

The solid curve of Fig. 2 represents a plot of $d\sigma/du^2$ as a function of the center-of-mass energy of the pi-rho system for a value of λ of 6 $(\text{BeV}/c)^{-2}$ and an incident pion lab momentum of 3.65 BeV/c [$W^2 = 7.76$ (BeV)^2]. The value of λ was taken in agreement with π-p scattering data[4] corresponding to values of ω^2 between the maximum obtainable value of 4.16 (BeV)^2 and a chosen value of ω_0^2 of 2.70 (BeV)^2. The latter value corresponds to a pion lab momentum of approximately 1 BeV/c. The shape of the function $d\sigma/du^2$ is rather insensitive to the value chosen for ω_0. The predominance of small values for the π-ρ center-of-mass energy is interpreted kinematically by saying that both the pi and the rho tend to go forward after production.

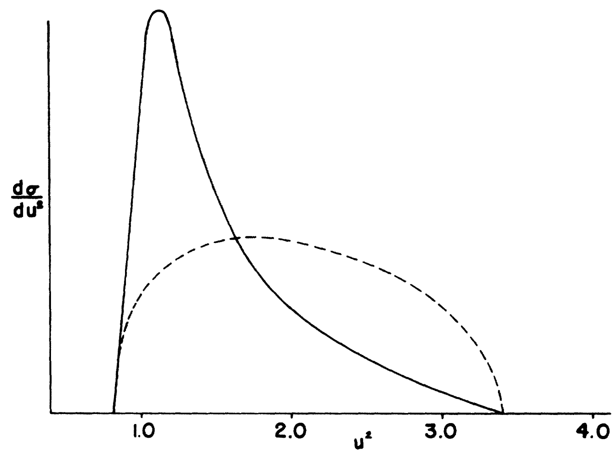

FIG. 2. Plot of the differential cross section obtained from the diagram of Fig. 1(a) as a function of the squared mass of the pi-rho system. The peak in the mass spectrum results from the assumption that the virtual exchanged pion is diffraction scattered from the nucleon. The dashed curve corresponds to phase space.

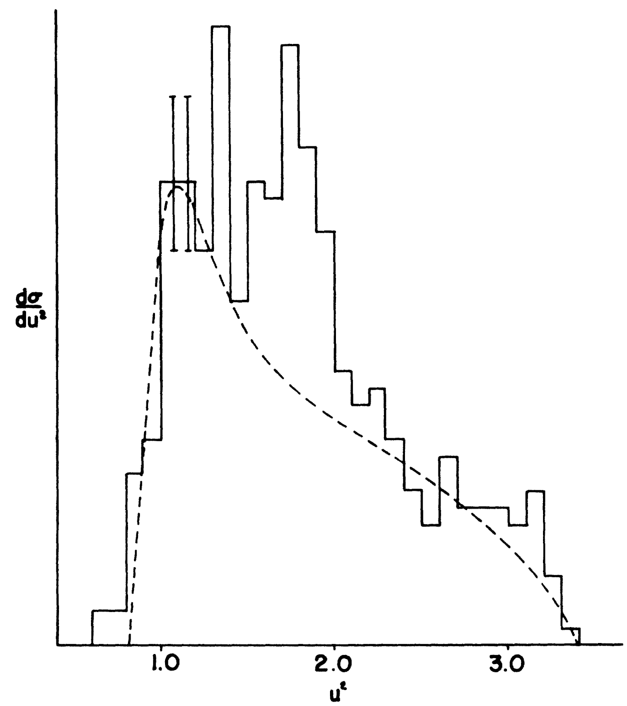

FIG. 3. Sketch of the data of reference 1 as a function of the squared mass of the pi-rho system. The dashed curve represents a fit to the data obtained by combining the present calculation with phase space so as to obtain reasonable agreement at the upper and lower extremes of the spectrum.

The histogram of Fig. 3 is a sketch of the data of reference 1 at the pion lab momentum cited above. Subsequent experiments[3] indicate that the broad peak in the data exhibited at low values of u^2 consists of two adjacent peaks at approximately $(1.08\text{ BeV})^2$ and $(1.32\text{ BeV})^2$. The magnitude of the observed total cross section reported in reference 1 is 0.86 mb.

The nature of the present calculation is such that a reliable value for the total cross section is not readily obtained.[7] The method followed here for handling the form factor and the spin terms appears in a similar calculation to underestimate the cross section by as much as a factor of two (while closely reproducing the observed spectrum shape).[5] Further, it is expected that rho-nucleon diffraction scattering associated with the diagram of Fig. 1(b) should contribute appreciably to the peak in the low mass region of the π-ρ spectrum. Finally the present calculation excludes all events with $\omega^2 < \omega_0^2$ for which the diffraction peak parametrization is not valid.[8] These events as well as the contributions of other possible diagrams are expected to add to the spectrum roughly in accord with the dashed phase-space plot of Fig. 2. Taking account of these considerations we are led to conclude on the basis of the present calculation that it is possible to obtain a total cross section as large as 0.4 or 0.5 mb, approximately one fourth of which is represented by a peak in the low mass region of the π-ρ spectrum. Since the measured total cross section considerably exceeds this estimate it is likely that a π-ρ resonance does enhance the reaction. This conclusion is further supported by the fact that the calculated peak in the spectrum appears to be rather narrower than that observed. It is suggested that the peak observed in the data of reference 1 is a combination of two adjacent peaks,[3] the first of which arises from the kinematical effect discussed here and the second of which represents a true resonance. The pi-rho system associated with the kinematical peak does not occur in a definite angular momentum state, but the sharp rise of the calculated cross section at threshold suggests that the s-wave term is predominant. The pi-rho angular distribution in this peak should be independent of the azimuthal angle φ_2 between the planes $(\vec{p}_1, \vec{p}_2)$ and $(\vec{p}_2, \vec{q}_2)$, and the rho should exhibit in its own rest frame a $\cos^2\theta$ decay angular distribution with respect to the incident pion. Further, the kinematical peak given by the present calculation should become larger and broader as the incident pion momentum is increased.

It is a pleasure to thank Professor Marc Ross for suggesting this investigation and for much advice and encouragement during the course of the analysis. The author wishes in addition to acknowledge useful discussions with Professor L. W. Jones.

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[6]We follow the notation of E. Ferrari and F. Selleri, Nuovo Cimento, Suppl. 24, 453 (1962).

[7]For a value of $(d\sigma/d\Omega)_0$ equal to 11 mb/sr, the value obtained by integration of Eq. (12) over u^2 is 0.03 mb.

[8]The analysis of the experimental data excluded only those events with values of ω in the N^* mass band.

SPIN AND UNITARY SPIN INDEPENDENCE OF STRONG INTERACTIONS*

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The purpose of this Letter is twofold. We want first to point out that the group SU(4) introduced by Wigner[1] to classify nuclear states can be extended to the relativistic domain and it is, therefore, relevant for particle physics. We will next show that when strangeness is taken into account the group SU(4) becomes enlarged to[2] SU(6) which contains, as a subgroup, $SU(3)\otimes[SU(2)]_q$. $[SU(2)]_q$ is the unitary subgroup (little group) of the Lorentz group that leaves invariant the momentum four-vector q.

The group we consider here embodies SU(3) and the ordinary spin in the same way as Wigner's SU(4) embodies isotopic spin and ordinary spin. Preliminary results on the classification of particles based on SU(6) seem encouraging enough to motivate a study of this group.[3]

We begin by discussing the first point. Let us assume that the ρ, ω, and π mesons are coupled to the nuclear field through a symmetrical Lagrangian of the form

$$L_{NM} = g\{\bar{\psi}\gamma_\mu\psi\omega_\mu + \bar{\psi}\gamma_\mu\tau^a\psi\rho_\mu{}^a + i\bar{\psi}\gamma_5\gamma_\mu\tau^a\psi\varphi_\mu{}^a\}, \quad (1)$$

where a denotes the isotopic spin index. Let us further impose the subsidiary conditions

$$\partial_\mu\omega_\mu = 0, \quad \partial_\mu\rho_\mu{}^a = 0,$$
$$\partial_\lambda\varphi_\mu{}^a - \partial_\mu\varphi_\lambda{}^a = 0, \quad (2)$$

which insure that $\omega_\mu, \rho_\mu{}^a, \varphi_\mu{}^a$ describe, respectively, particles with $(J=1^-, T=0)$, $(J=1^-, T=1)$, and $(J=0^-, T=1)$. The pion field π^a is related to the axial vector field $\varphi_\mu{}^a$ through $\varphi_\lambda{}^a = (1/\mu)\partial_\lambda\pi^a$, μ being the mass common to all mesons.

The conditions (2) are compatible with the equations of motion only if L includes, besides L_{NM} [Eq. (2)], additional terms such that the mesons (ρ, ω, π) are coupled to conserved currents. Thus ω and ρ are coupled to the conserved baryon and isotopic-spin currents, respectively, while the pion is coupled to a conserved axial-vector current.

It can now be shown[4] that L is invariant under a group[5] $\mathcal{G}_4$ which induces for each momentum q of the mesons a unitary unimodular transformation among the 15 degenerate states ω, ρ, and π. In counting the multiplicity we include, for a given momentum, the spin states just as for Wigner's supermultiplets. Under this transformation the nucleon $(S=\frac{1}{2}, T=\frac{1}{2})$ transforms like the four-dimensional representation of the group.

In the nonrelativistic limit, L_{NM} gives rise to a potential which describes spin- and isospin-independent exchange forces (Majorana forces) between nucleons. This potential is, therefore, invariant under Wigner's group SU(4). If now a purely spin-dependent perturbation is introduced, ω and ρ remain degenerate whereas the pion splits from them within the supermultiplet. We note that ω, ρ, and π are associated with the adjoint representation of SU(4). When this representation is reduced under the subgroup $SU(2)\otimes[SU(2)]_q$ it splits into states with $(J=1^-, T=0)$, $(J=1^-, T=1)$, and $(J=0^-, T=1)$.

These considerations are readily extended to include strange particles. In this case the SU(2)