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$I=0$ PION-PION AMPLITUDE AND σ DI-PION*

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Recently a great deal of attention has been given to the $I=0$ pion-pion amplitude whose form has been somewhat confusing both theoretically and experimentally. The problem is to combine consistently the so-called ABC enhancement[1] (or the s-wave virtual particle[2]) at threshold with the possible s-wave resonance at around 400 MeV.[3] The latter resonance, the σ di-pion, seems to explain a large number of phenomena in the right direction: nucleon-nucleon phase shifts,[4] the three-pion decay of the η and K mesons,[5] the K_1-K_2 mass difference,[6] the nucleon axial form factor.[7] It would, of course, play a role in many other processes where two pions enter in one channel or another, including pion-nucleon scattering[8] (where one crossed channel is $\pi\pi$).

Theoretically the σ di-pion would be an example of an unusual situation, from an intuitive potential point of view, of an s-wave resonance without an s-wave bound state in a purely attractive interaction and may throw some light on the nature of spectral functions.

We show in this note that the assumed existence of the ABC enhancement together with the σ di-pion and unitarity implies that the whole $I=0$ pion-pion amplitude is almost exactly known up to over 400 MeV. The whole amplitude represents an entirely s-wave continuous virtual particle, and therefore the entire trajectory can be used whenever the $I=0$, 2π system is exchanged such as in the phenomena of references 4-8.

The pion-pion trajectory in the complex angular momentum plane at threshold as derived from unitarity is given by[9]

$$\alpha(\nu)=\alpha(0)-\frac{Y_\nu}{Y_l}\nu+\frac{C\nu^2}{Y_l}+O(\nu^3)-\frac{1}{Y_l}(-\nu)^{\alpha(\nu)+\frac{1}{2}}, \tag{1}$$

where Y_ν, Y_l, C, and $\alpha(0)$ are real parameters. The scattering length a is given in terms of these parameters by

$$a=-2\frac{\cos\pi\alpha(0)}{\alpha(0)Y_l}<-\frac{2}{\alpha(0)Y_l}. \tag{2}$$

Suppose that the ν^2 term is discarded in Eq. (1), as is commonly done near the threshold; then in the case when $\mathrm{Im}\,\alpha$ is small, or equivalently, Y_l is large,

$$\alpha(0)\cong(Y_\nu/Y_l)\nu_R, \tag{3}$$

where ν_R is the square of the center-of-mass momentum at the resonance. Hence

$$a<-2/Y_\nu\nu_R,$$

or, introducing also the full width Γ of the resonance given by

$$\tfrac{1}{2}Y_\nu\cong-\nu_R^{1/2}/s_R^{1/2}\Gamma,\quad s_R=4(\nu_R+1),$$

$$a<s_R^{1/2}\Gamma/\nu_R^{3/2}. \tag{4}$$

Assuming that the mass of σ is ~400 MeV with a width 100 MeV,[5] we get

$$a<0.47.$$

To obtain a large scattering length of the order of 1, the width of the resonance must be increased by a factor of more than two with the resonance position fixed at 400 MeV, or the position of the resonance must be lowered to ~350 MeV with the width fixed at ~125 MeV. Either alternative (or any suitable combination) gives rise to violations of unitarity[10] even at values of s near

$$s_R-\tfrac{1}{2}s_R^{1/2}\Gamma,$$

if a Breit-Wigner shape is assumed for the resonance. The imposition of unitarity causes the peak to be strongly distorted and seriously shifted compared with the Breit-Wigner form. Curve 1

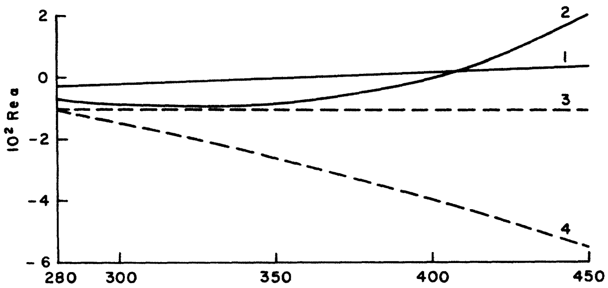

FIG. 1. Real part of the various trajectories as a function of the two-pion mass. (See text for the explanation of curves 1-4.)

in Fig. 1 shows a trajectory for such a case with the choice of the parameters as shown in Table I. The corresponding cross section is shown in curve 1 of Fig. 2.

Consistency between the large scattering length[8] $a \cong 1.32$ and a resonance whose width is <~100 MeV can be achieved, however, only if the ν^2 in Eq. (1) is kept. There are now four parameters to adjust and, at present, only three pieces of data. A unique solution is not obtainable without more information. With the scattering length fixed at 1.32 and the resonance at 400 MeV with width of 100 MeV, the parameters of the trajectory are given in Table I for various values of $\alpha(0)$. A typical trajectory and the corresponding cross section are given by the curves labeled "2" in Figs. 1 and 2. For comparison we have included two other possible trajectories adjusted to give the right scattering length. Whether the trajectory actually cuts the line $l=0$ (curve 2) or not (curve 3) depends on a more solid confirmation of the existence of the σ di-pion.

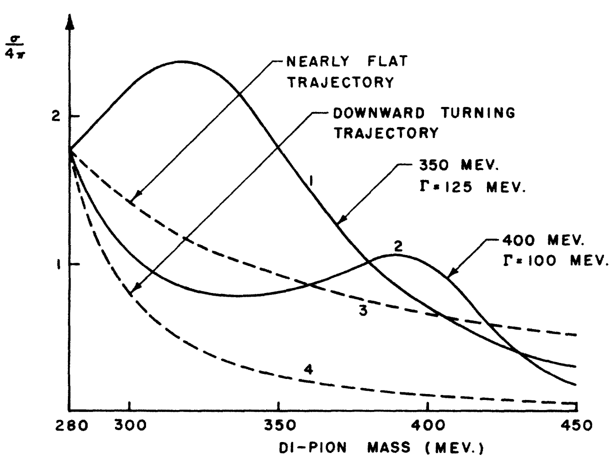

FIG. 2. Total $\pi\pi$, $I=0$, cross section corresponding to trajectories of Fig. 1.

Table I. Possible sets of parameters for the pion-pion trajectory.

Curve No.	Mass (MeV)	Width (MeV)	$\alpha(0)$	Y_l	Y_ν	C
1	350	125	-0.002	755	-2.70	0.0
2	400	100	-0.007	211	1.08	2.46
...	400	100	-0.014	109	1.08	2.48
...	400	100	-0.020	75	1.08	2.52
...	400	100	-0.032	48	1.08	2.59
3	...	...	-0.01	150	0.0	0.0
4	...	...	-0.01	150	4.0	0.0

In any case, a large $\pi\pi$ scattering length and a large cross section around 400 MeV implies a trajectory very close to $l=0$. The $\pi\pi$ system represents then a virtual particle over the whole range of energy from threshold up to a mass of 400 MeV. It is this fact which is most likely causing the difficulties of clearly establishing the σ di-pion as a particle and establishing its mass.[11] We should raise the question whether a continuous virtual particle should be included in any scheme of particle classification. As far as calculations are concerned, the $I=0$, $\pi\pi$ amplitude would then consist entirely of an s-wave contribution of the form

$$A(s,t)=\beta(\nu)/\alpha(\nu), \tag{5}$$

with $\alpha(\nu)$ given by (1) and the residue $\beta(\nu)$ given by[9]

$$\beta(\nu)\approx b(\nu)\cong(\nu+\mu^2)^{1/2}/Y_l. \tag{6}$$

One can then consider the exchange of the whole trajectory or the whole amplitude (5), rather than the exchange of a single particle in bootstrap calculations, for example.

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[10]From unitarity we have $\mathrm{Im}A^{-1}=(s-4)/(4s)^{1/2}$, but from the Breit-Wigner formula $\mathrm{Im}A^{-1}=(\sqrt{s_R})-4/s_R$; s_R is the fixed position of the resonance.

[11]It should be remarked that if the $\pi\pi$ cross section is nearly flat, such as curves 2 or 3 in Fig. 2, it would be hard to see it experimentally in the usual comparison of an effective $\pi\pi$ mass with the phase-space curve unless the absolute value of the phase-space normalization is known.

MAGNETIC MOMENT OF THE Λ*

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We have measured the magnetic moment of the Λ, using 8553 decays $\Lambda\to p+\pi^-$ from Λ produced via the reaction $\pi^-+p\to\Lambda+K^0$ in the Alvarez 72-inch hydrogen bubble chamber. The Λ's are almost completely polarized, for all production angles (see Fig. 1). Of the 8553 Λ's, 6263 are produced by 1030-MeV/c π^- in an average field of 15.5 kG, and 2290 at 1170 MeV/c in 17.9 kG. At 17.9 kG the Λ-spin precession is 2.7 deg per Λ mean life for $|\mu_\Lambda|=1$. (The unit is $e\hbar/2m_pc$, one Bohr nucleon magneton. In these units the neutron has $\mu_n=-1.91$.) We find

$$\mu_\Lambda=-1.39\pm0.72.$$

Our result can be compared with the two previous measurements. Cool et al.[1] find $\mu_\Lambda=-1.5\pm0.5$. Kernan et al.[2] find $\mu_\Lambda=0.0\pm0.6$. Our result is in good agreement with that of Cool et al. It is also consistent with the theoretical prediction of Coleman and Glashow,[3] $\mu_\Lambda=\frac{1}{2}\mu_n=-0.95$, based on SU(3) symmetry.[4]

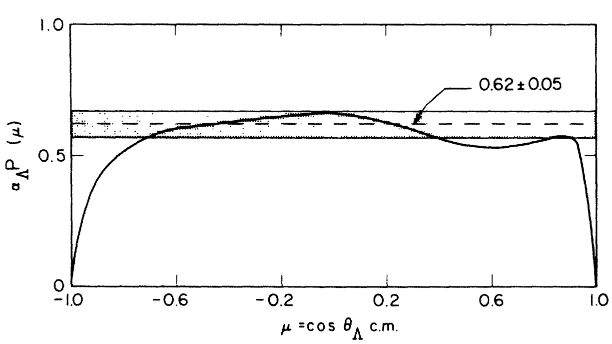

FIG. 1. Decay asymmetry of Λ. The curve was calculated from the least-squares fits to $\sigma(\mu)$ and $\sigma(\mu)\alpha_\Lambda P(\mu)$. The value $\alpha_\Lambda P(\mu)=0.62$ corresponds to $|P|=1$, according to J. W. Cronin and O. E. Overseth [Phys. Rev. 129, 1795 (1963)].

The equation of motion of $\vec{\mathrm{P}}$, the Λ polarization in the Λ rest frame, is

$$d\vec{\mathrm{P}}/dt=(e/m_pc)\mu_\Lambda\vec{\mathrm{P}}\times\vec{\mathrm{H}}'\equiv M\vec{\mathrm{P}}\times\vec{\mathrm{H}}', \tag{1}$$

where $\vec{\mathrm{H}}'$ is the magnetic field in the Λ frame, t is the Λ proper time, and $M\equiv e\mu_\Lambda/m_pc$ includes the sign (positive if the magnetic moment is along $+\vec{\mathrm{P}}$). We detect the precession of $\vec{\mathrm{P}}(t)$ by means of the precession of the decay asymmetry $\alpha_\Lambda\vec{\mathrm{P}}(t)=\alpha_\Lambda P(\mu)\hat{n}(t)$, where $t=0$ at production and

$$\hat{n}(0)=(\hat{\pi}_{\mathrm{inc}}\times\hat{\Lambda})/|\hat{\pi}_{\mathrm{inc}}\times\hat{\Lambda}|;$$

$\hat{\pi}_{\mathrm{inc}}$ and $\hat{\Lambda}$ are the directions of the incident π and of the Λ in the laboratory frame, and μ is the cosine of the Λ production angle in the c.m. frame. Because Eq. (1) is homogeneous in P, the signs of α_Λ and of $P(\mu)$ are irrelevant.

In our experiment the precession angles are small. We therefore write the solution of Eq. (1), including only the terms linear in t:[5]

$$\alpha_\Lambda\vec{\mathrm{P}}(t)=\alpha_\Lambda P(\mu)\{\hat{n}(0)+M[\hat{n}(0)\times\vec{\mathrm{H}}']t\}. \tag{2}$$

Figure 1 shows $\alpha_\Lambda P(\mu)$, which is obtained from the decay asymmetry by omission of the correlation with the magnetic-field direction. By our sign convention, $\alpha_\Lambda P(\mu)$ is positive for all values of μ. The decay pions prefer to go along $+\hat{n}(0)$ at $t=0$.

We now choose a particular coordinate system. The incident π^- are horizontal. The laboratory field H is approximately uniform and points vertically upwards. We choose Cartesian axes $\hat{z}=\hat{\pi}_{\mathrm{inc}}$, $\hat{x}=$ vertical (upwards), and $\hat{y}=\hat{z}\times\hat{x}$. Then $\vec{\mathrm{H}}=H\hat{x}$, with H positive.[6] In ordinary spherical coordinates, the x, y, and z components of $\hat{\Lambda}$ are $\sin\theta\cos\varphi$, $\sin\theta\sin\varphi$, and $\cos\theta$. The compo-