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SPIN SPLITTING OF THE GIANT QUANTUM OSCILLATIONS IN GALLIUM

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The purpose of the present note is to report the observation of a spin splitting of the giant quantum oscillations in gallium and to indicate how information concerning the effective g factor can be obtained from this splitting. The theory of the giant oscillations for a longitudinal sound wave propagated in a parallel magnetic field was developed by Gurevich et al.[1] A method for determining the effective mass from the line shape of the absorption peaks has already been outlined.[2] The spikelike giant oscillations were recently observed in bismuth[3] and in gallium,[2] but the spin splitting of the absorption peaks has not been observed before.

For a longitudinal sound wave propagated in a parallel magnetic field the absorption coefficient as a function of the magnetic field exhibits two series of spikes associated with the two spin directions. Each spike occurs when a quantized electronic level passes through the Fermi surface. If the g factor is such that the two series of spikes are only slightly displaced from each other and if the temperature is not sufficiently low, the two series may overlap. As a result the observed oscillation pattern will consist of a single series of giant absorption peaks.[2,3] However, as the temperature approaches absolute zero each giant absorption peak, which is a superposition of two adjacent spikes, will split into two subpeaks, one associated with Landau level n and spin up and the other with Landau level $n+l$ and spin down. If the first subpeak has a maximum at the field H_1 and the second at H_2 then

$$(n+\tfrac{1}{2})\beta H_1+\tfrac{1}{2}g\mu_B H_1=\zeta \tag{1}$$

and

$$(n+l+\tfrac{1}{2})\beta H_2-\tfrac{1}{2}g\mu_B H_2=\zeta, \tag{2}$$

where μ_B is the Bohr magneton, ζ is the Fermi energy, and g is the appropriate g factor which determines the spin splitting. The parameter β is related to the effective mass m^* by the relation

$$\beta=e\hbar/m^*c. \tag{3}$$

From Eqs. (1) and (2) we obtain

$$g=\frac{\zeta}{\mu_B}\left(\frac{1}{H_1}-\frac{1}{H_2}\right)+l\frac{\beta}{\mu_B} \tag{4}$$

or

$$g=2\frac{m_0}{m^*}\left[\frac{H_2-H_1}{H_1H_2(\Delta H^{-1})}+l\right], \tag{5}$$

where m_0 is the free electron mass and $\Delta H^{-1}=\beta/\zeta$ is the de Haas–van Alphen period.[4] If the two subpeaks are completely separated from each other, then one can measure the quantity $|(H_2-H_1)/H_1H_2|$. The effective mass m^* can be deduced from the line shape of either subpeak[2] or by some other method (e.g., temperature dependence of the amplitude of the de Haas–van Alphen oscillations). The integer l and the sign of H_2-H_1 cannot be obtained by this method. Consequently the g factor cannot be determined uniquely by the use of Eq. (5). However, if it is known that the spin splitting is small in comparison with the separation of the Landau levels, then $l=0$ and

$$|g|=\left|\frac{2m_0}{m^*(\Delta H^{-1})}\left(\frac{H_2-H_1}{H_1H_2}\right)\right|. \tag{6}$$

A relation which is equivalent to Eq. (6) has already been given by Rodriguez[5] who has considered the special case of $l=0$. In those instances when it is possible to observe the giant oscillations which correspond to $n=0, 1, 2, \cdots$ one can determine $|l|$ by the following rule. If the peak which occurs at the highest magnetic field splits into two subpeaks, at $T\to 0°K$, then $l=0$; otherwise, $l\neq 0$. If the first L peaks which are observed at the highest fields do not split into two subpeaks but the $(L+1)$st peak does split, then $|l|=L$. This rule will not hold if g

is an exact even multiple of m_0/m^* in which case the spin splitting cannot be observed. However, if g is an even multiple of m_0/m^* then $|l|$ may be determined from a plot of the amplitude of the peaks versus the field at which they occur. Such a plot should show an abrupt change (increase) in the amplitude after the first $|l|$ peaks.[6]

When the two subpeaks are not completely separated, H_1 and H_2 cannot be measured directly, and a more careful analysis of the line shape of the combined absorption peak is necessary. According to reference 2 this line shape is given by

$$\Gamma_i = A\left\{\cosh^{-2}\left[\beta\frac{(1+\alpha)H-\bar{H}}{2kT\bar{H}(\Delta H^{-1})}\right] + \cosh^{-2}\left[\beta\frac{(1-\alpha)H-\bar{H}}{2kT\bar{H}(\Delta H^{-1})}\right]\right\}, \qquad (7)$$

where, from the definition of α,

$$\alpha = \tfrac{1}{2}\bar{H}(\Delta H^{-1})(B_1 - B_2) = \tfrac{1}{2}\bar{H}(\Delta H^{-1})[g\mu_B/\beta - l] \qquad (8)$$

or

$$g = \frac{2m_0}{m^*}\left[\frac{2\alpha}{\bar{H}(\Delta H^{-1})} + l\right]. \qquad (9)$$

Experimentally, one can determine β and $|\alpha|$ by fitting the line shape of the absorption peak to Eq. (7). By using Eq. (9) one can then determine the g factor. Again, since l and the sign of α are unknown, this determination of the g factor is not unique. However, if it is known that $l=0$, then

$$|g| = |4\alpha m_0/\bar{H}(\Delta H^{-1})m^*|. \qquad (10)$$

Experiments were performed on three single crystals of gallium which were grown from high-purity (99.9999 %) bars obtained from Alcoa. The attenuation of 30- to 90-Mc/sec longitudinal waves propagated along the b axis of these crystals was measured in a parallel magnetic field up to 110 kG. These measurements were performed at liquid helium temperatures down to 1.1°K. The spikelike giant oscillations were observed in all cases. At temperatures below ~2.5°K a splitting of the first absorption peak (i.e., the one which occurs at the highest magnetic field) was noticed. At still lower temperatures ($T \lesssim 1.5$°K), the splitting of the next absorption peak (in direction of decreasing field) became apparent. Figure 1 shows a recorder tracing of the attenuation of a 30-Mc/sec longitudinal sound wave at 1.55°K. Note the splitting of the first peak in Fig. 1.[7] Figure 2 shows the first absorption peak for a 50-Mc/sec sound wave at 1.14°K. The line shape of the first and second absorption peaks, measured at various temperatures between 1.66°K and 1.14°K, were fitted to Eq. (7) by a computer which determined the best values of the parameters β and α. From these values we obtained $m^* = (0.066 \pm 0.007)m_0$ and $2|\alpha|/\bar{H}(\Delta H^{-1}) = 0.0313 \pm 0.0018$. The value for m^* lies between the values quoted by Shoenberg[8] and by Condon.[9] For the effective g value we obtain $g = 2(m_0/m^*)[l \pm (0.0313 \pm 0.0018)]$. The most likely possibilities for l are $l=0$ and $l=1$. For $l=0$ we obtain, using our value for m^*, $|g| = 0.95 \pm 0.11$, while for $l=1$ we obtain $|g| = 29 \pm 3$ or $|g| = 31 \pm 3$. The period of the giant oscillations was $(30.0 \pm 1.5)\times 10^{-7}$ G^{-1}, which agrees well with the value given by Shoenberg.[8] The amplitude of the first absorption peak for a 50-Mc/sec wave at 1.1°K was ~5 dB/cm.

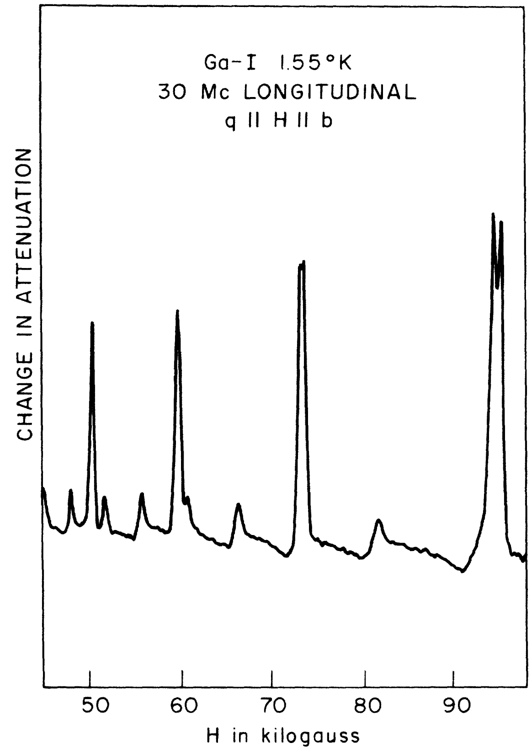

FIG. 1. Recorder tracing of the change in the ultrasonic attenuation of 30-Mc/sec longitudinal waves propagated along the b axis in gallium as a function of the magnetic field intensity at 1.55°K. The magnetic field was oriented along the b axis. Note the splitting of the first absorption peak.

A splitting of the giant absorption peaks may also occur if two close but nonidentical periods

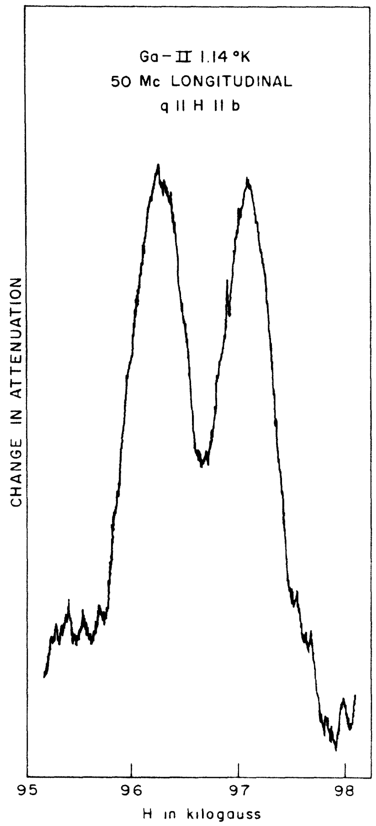

FIG. 2. Recorder tracing of the change in the ultrasonic attenuation of 50-Mc/sec longitudinal waves propagated along the b axis in gallium as a function of the magnetic field intensity at 1.14°K. The magnetic field was oriented along the b axis.

are present. Such a situation may arise if the sample contains two crystallites which are slightly misoriented relative to each other, or if two periods which are identical when the field is along a symmetry direction (b axis in our case) become nonidentical when the sample is misaligned and the field is not exactly along the symmetry direction. The fact that the same value for α was determined for all the three samples indicates that the observed splitting is not due to imperfections of the samples or to misalignment. Also the data of Shoenberg[8] suggest that the period which was investigated here is not degenerate near the b axis. Finally, one can show that for two close but nonidentical periods, $(1/H_1)-(1/H_2)$ is proportional to $(n+\frac{1}{2})$ and that $\alpha/\overline{H}$ should, in general, vary with n. As a consequence if g is evaluated from the splitting of two different absorption peaks, one should obtain different values. In our case n is sufficiently small ($n=3$ for the first peak) to observe a difference in the value of g, as determined from the first and second absorption peaks, if the splitting is due to two nonidentical periods. Since no such difference in the value of g was obtained from our data, we conclude that the splitting is due to spin.

The present experiment illustrates the usefulness of the giant oscillations in studying the Fermi surface of metals. From a single trace of the absorption coefficient as a function of the magnetic field one can determine not only the de Haas–van Alphen period[4] and the associated effective mass for a certain group of electrons, but one can also obtain information concerning the g factor of the same electrons. In the theory which was outlined in reference 2 and in the first paragraphs of the present paper, it was assumed that the energy E of the electron is a quadratic function of the crystal momentum $\hbar\vec{k}$. However, one can show that Eq. (6) of reference 2 and Eq. (5) of the present paper are approximately valid for an arbitrary dependence of E on $\vec{k}$ provided m^* is defined as

$$m^* = 2\pi^{-1}(dA/d\zeta). \qquad (11)$$

In Eq. (11) A is that cross-sectional area of the Fermi surface (in $\hbar\vec{k}$ space) which is perpendicular to the magnetic field and at which the group velocity of the electron in the direction of the sound propagation is equal to the sound velocity. Furthermore, it is possible to extend the theory to the case in which the magnetic field makes an arbitrary angle (but not a right angle) with the direction of the sound propagation.

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$I=0$ PION-PION AMPLITUDE AND σ DI-PION*

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Recently a great deal of attention has been given to the $I=0$ pion-pion amplitude whose form has been somewhat confusing both theoretically and experimentally. The problem is to combine consistently the so-called ABC enhancement[1] (or the s-wave virtual particle[2]) at threshold with the possible s-wave resonance at around 400 MeV.[3] The latter resonance, the σ di-pion, seems to explain a large number of phenomena in the right direction: nucleon-nucleon phase shifts,[4] the three-pion decay of the η and K mesons,[5] the K_1-K_2 mass difference,[6] the nucleon axial form factor.[7] It would, of course, play a role in many other processes where two pions enter in one channel or another, including pion-nucleon scattering[8] (where one crossed channel is $\pi\pi$).

Theoretically the σ di-pion would be an example of an unusual situation, from an intuitive potential point of view, of an s-wave resonance without an s-wave bound state in a purely attractive interaction and may throw some light on the nature of spectral functions.

We show in this note that the assumed existence of the ABC enhancement together with the σ di-pion and unitarity implies that the whole $I=0$ pion-pion amplitude is almost exactly known up to over 400 MeV. The whole amplitude represents an entirely s-wave continuous virtual particle, and therefore the entire trajectory can be used whenever the $I=0$, 2π system is exchanged such as in the phenomena of references 4-8.

The pion-pion trajectory in the complex angular momentum plane at threshold as derived from unitarity is given by[9]

$$\alpha(\nu)=\alpha(0)-\frac{Y_\nu}{Y_l}\nu+\frac{C\nu^2}{Y_l}+O(\nu^3)-\frac{1}{Y_l}(-\nu)^{\alpha(\nu)+\frac{1}{2}}, \quad (1)$$

where Y_ν, Y_l, C, and $\alpha(0)$ are real parameters. The scattering length a is given in terms of these parameters by

$$a=-2\frac{\cos\pi\alpha(0)}{\alpha(0)Y_l}<-\frac{2}{\alpha(0)Y_l}. \quad (2)$$

Suppose that the ν^2 term is discarded in Eq. (1), as is commonly done near the threshold; then in the case when Imα is small, or equivalently, Y_l is large,

$$\alpha(0)\cong(Y_\nu/Y_l)\nu_R, \quad (3)$$

where ν_R is the square of the center-of-mass momentum at the resonance. Hence

$$a<-2/Y_\nu\nu_R,$$

or, introducing also the full width Γ of the resonance given by

$$\tfrac{1}{2}Y_\nu\cong-\nu_R^{1/2}/s_R^{1/2}\Gamma,\quad s_R=4(\nu_R+1),$$

$$a<s_R^{1/2}\Gamma/\nu_R^{3/2}. \quad (4)$$

Assuming that the mass of σ is ~400 MeV with a width 100 MeV,[5] we get

$$a<0.47.$$

To obtain a large scattering length of the order of 1, the width of the resonance must be increased by a factor of more than two with the resonance position fixed at 400 MeV, or the position of the resonance must be lowered to ~350 MeV with the width fixed at ~125 MeV. Either alternative (or any suitable combination) gives rise to violations of unitarity[10] even at values of s near

$$s_R-\tfrac{1}{2}s_R^{1/2}\Gamma,$$

if a Breit-Wigner shape is assumed for the resonance. The imposition of unitarity causes the peak to be strongly distorted and seriously shifted compared with the Breit-Wigner form. Curve 1