is limited by the small lattice of spins considered, and a larger lattice would almost certainly give slightly different results. However, the method certainly provides useful results, and indicates the power of a Monte Carlo approach to this type of problem.

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INFLUENCE OF THE PHONON SPECTRUM ON THE DENSITY OF ELECTRON STATES IN SUPERCONDUCTING TANTALUM

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A tunnelling experiment has been performed on superconducting tantalum which suggests that electron-phonon interactions are responsible for the superconductivity in Ta, as they are in nontransition-group metals. This conclusion results from the observation of the influence of the phonon spectrum on the density of states of Ta; this is the first report of such an influence in a transition metal.

Superconducting metals can be divided into two groups, transition and nontransition metals. The critical temperatures of the superconductors in the transition group have a periodic dependence on their number of valence electrons. This is Matthias's rule.[1] Nontransition elements do not obey such a rule and this difference among others has led to the suggestion[2] that perhaps the mechanisms for the superconductivity in the two groups are different.

The success of the electron-phonon interaction in nontransition-group superconductors is epitomized by the detailed agreement of the tunnelling experiments on Pb by Rowell, Anderson, and Thomas[3] and the theoretical calculations of Schrieffer, Scalapino, and Wilkins,[4] and Scalapino and Anderson.[5] The current-voltage (I-V) characteristic of the Pb tunnel junctions showed structure which could be classified into two groups, the relatively large structure which is due to the peaks in the phonon spectrum $g(\nu)$ vs ν and the smaller structure which is associated with the Van Hove and other critical points in $g(\nu)$ vs ν. The density of states and hence the I-V characteristic reflect the sum of both these effects.

We have made similar tunnelling measurements on Ta to measure the effect of its phonon spectrum.

The tunnel junctions were made on bulk specimens of Ta because these can be made much more pure than films. Best commercial quality Ta was rolled into a foil 0.003 in. thick, and from this were cut pieces 1 in. long and 0.1 in. wide. After a chemical etch, the specimens were outgassed by heating to their melting points over many hours in a vacuum of $\lesssim 10^{-8}$ mm. The surface of the foil was then coated with GE varnish 7031 leaving clear a strip ~0.01 in. wide along the length of the specimen. This strip was then oxidized at 50°C in oxygen for a few hours and then cross strips of Ag were evaporated on. The junctions had a resistance of ~200 Ω and an area of ~10^{-3} cm^2.

At helium temperatures the junctions showed the usual I-V characteristic for a normal superconductor-insulator-metal tunnel junction. At the lowest temperature, 0.9°K, the first derivative (dV/dI) and the second derivative (d^2I/dV^2) of the I-V characteristic were measured as a function of dc bias. dV/dI was measured by applying a constant small ac current (0.5 μA) through the sample and measuring the ac voltage across it. d^2I/dV^2 was measured by applying a constant ac voltage (400 μV) across the junction and detecting the voltage at twice the frequency of the applied voltage.[6]

In Fig. 1 we show d^2I/dV^2 as a function of dc bias for the Ta-I-Ag (I represents "insulator") junction. The large fluctuations near the gap edge have been omitted. The voltages are measured from the gap Δ_0 which was found from a Ta-I-Pb junction to be $\Delta_0 = 0.7$ mV, in good agreement with Townsend and Sutton.[7]

It is immediately evident that there are two large dips in d^2I/dV^2 with minima at 11.4 mV and at 18.0 mV. The positions of these dips are in excellent agreement with what we might expect from the positions of the peaks in the phonon spectrum measured by inelastic neutron scattering.[8] There are two main peaks in the phonon spectrum. The lower one has a maximum at 11.5 mV and the higher one has a maximum at 17.9 mV.

Weaker structure in the density of states is caused by infinite discontinuities in $dg/d\nu$ or higher derivatives and these are associated with stationary points in the phonon dispersion curves.[5] These have been measured in symmetry directions,[8] and most are found to be within the width of the peaks and so we do not expect to see them resolved.[9] However, the large structure at 18 mV is not a simple dip but has a shape characteristic of a resonance in dI/dV. This is similar to the behavior of a Al-I-Pb junction[10] and it is probably caused similarly by the effect of both the spectrum peak and the critical points. The end of the phonon spectrum is at 21.1 mV, and a small, probably significant effect is observed here. There is a significant dip at 14.9 mV but the nearest measured critical point in the phonon spectrum is at 15.6 mV.

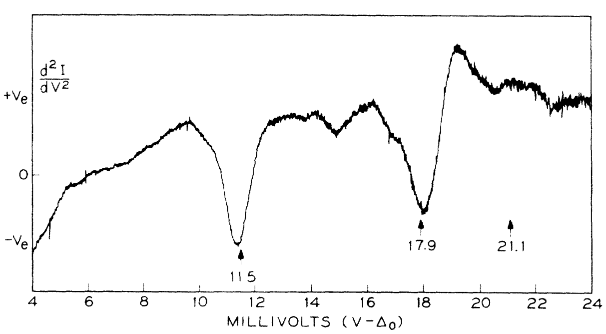

FIG. 1. Second derivative of the tunnel current of a Ta-I-Ag junction as a function of bias. The arrows mark the position of the two peaks in the phonon spectrum (11.5 mV and 17.9 mV) and the end of the spectrum at 21.1 mV.

The excellent agreement between the energy of the structure in the tunnel current and the energy of the peaks in the phonon spectrum leaves little doubt that electron-phonon interactions are present in Ta. We will next examine the size of the structure caused by the phonon peaks.

From Schrieffer, Scalapino, and Wilkins[4] we see that the change in the density of states is proportional to the real part of Δ^2/ϵ_p^2, where Δ is the complex gap and ϵ_p is the energy of the phonons. At a peak in the phonon spectrum the real part of Δ^2 decreases, which causes the density of states to decrease. The change in the real part of Δ^2 is proportional to Δ_0^2, where Δ_0 is the value of Δ at the gap edge; therefore Δ_0^2/ϵ_p^2 should be proportional to the size S of the step in conductance at the phonon peak energy. Adler[11] has pointed out that for Pb,[10] In,[12] and Sn,[10] $S^{-1}\Delta_0^2/\epsilon_p^2$ is approximately constant. If Δ_0 and ϵ_p are measured in millivolts and S expressed as a percent of dI/dV at ϵ_p, then $S^{-1}\Delta_0^2/\epsilon_p^2 \gtrsim 10^{-2}$. We find from the data of Bermon and Ginsberg[13] that Hg also falls into this scheme. We find from the measurement of dI/dV for the Ta-I-Ag junction that the step at 11.4 mV is ~0.45% which gives $S^{-1}\Delta_0^2/\epsilon_p^2 \sim 8\times10^{-3}$. This leads us to conclude that an electron-phonon interaction can equally well account for the superconductivity in Ta as in nontransition element superconductors.

Since the writing of this Letter, similar results have been obtained for Nb. The energies of the peaks in d^2I/dV^2 vs $(V-\Delta_0)$ again agree very well with the energies of the peaks in the phonon spectrum.[14]

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SPIN SPLITTING OF THE GIANT QUANTUM OSCILLATIONS IN GALLIUM

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The purpose of the present note is to report the observation of a spin splitting of the giant quantum oscillations in gallium and to indicate how information concerning the effective g factor can be obtained from this splitting. The theory of the giant oscillations for a longitudinal sound wave propagated in a parallel magnetic field was developed by Gurevich et al.[1] A method for determining the effective mass from the line shape of the absorption peaks has already been outlined.[2] The spikelike giant oscillations were recently observed in bismuth[3] and in gallium,[2] but the spin splitting of the absorption peaks has not been observed before.

For a longitudinal sound wave propagated in a parallel magnetic field the absorption coefficient as a function of the magnetic field exhibits two series of spikes associated with the two spin directions. Each spike occurs when a quantized electronic level passes through the Fermi surface. If the g factor is such that the two series of spikes are only slightly displaced from each other and if the temperature is not sufficiently low, the two series may overlap. As a result the observed oscillation pattern will consist of a single series of giant absorption peaks.[2,3] However, as the temperature approaches absolute zero each giant absorption peak, which is a superposition of two adjacent spikes, will split into two subpeaks, one associated with Landau level n and spin up and the other with Landau level $n+l$ and spin down. If the first subpeak has a maximum at the field H_1 and the second at H_2 then

$$(n+\tfrac{1}{2})\beta H_1+\tfrac{1}{2}g\mu_B H_1=\zeta \tag{1}$$

and

$$(n+l+\tfrac{1}{2})\beta H_2-\tfrac{1}{2}g\mu_B H_2=\zeta, \tag{2}$$

where μ_B is the Bohr magneton, ζ is the Fermi energy, and g is the appropriate g factor which determines the spin splitting. The parameter β is related to the effective mass m^* by the relation

$$\beta=e\hbar/m^*c. \tag{3}$$

From Eqs. (1) and (2) we obtain

$$g=\frac{\zeta}{\mu_B}\left(\frac{1}{H_1}-\frac{1}{H_2}\right)+l\frac{\beta}{\mu_B} \tag{4}$$

or

$$g=2\frac{m_0}{m^*}\left[\frac{H_2-H_1}{H_1H_2(\Delta H^{-1})}+l\right], \tag{5}$$

where m_0 is the free electron mass and $\Delta H^{-1}=\beta/\zeta$ is the de Haas–van Alphen period.[4] If the two subpeaks are completely separated from each other, then one can measure the quantity $|(H_2-H_1)/H_1H_2|$. The effective mass m^* can be deduced from the line shape of either subpeak[2] or by some other method (e.g., temperature dependence of the amplitude of the de Haas–van Alphen oscillations). The integer l and the sign of H_2-H_1 cannot be obtained by this method. Consequently the g factor cannot be determined uniquely by the use of Eq. (5). However, if it is known that the spin splitting is small in comparison with the separation of the Landau levels, then $l=0$ and

$$|g|=\left|\frac{2m_0}{m^*(\Delta H^{-1})}\left(\frac{H_2-H_1}{H_1H_2}\right)\right|. \tag{6}$$

A relation which is equivalent to Eq. (6) has already been given by Rodriguez[5] who has considered the special case of $l=0$. In those instances when it is possible to observe the giant oscillations which correspond to $n=0, 1, 2, \cdots$ one can determine $|l|$ by the following rule. If the peak which occurs at the highest magnetic field splits into two subpeaks, at $T\to 0°K$, then $l=0$; otherwise, $l\neq 0$. If the first L peaks which are observed at the highest fields do not split into two subpeaks but the $(L+1)$st peak does split, then $|l|=L$. This rule will not hold if g