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SELF-DETECTION OF THE ac JOSEPHSON CURRENT*

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In carrying out current-voltage (I-V) measurements on a large number of Pb-(Pb oxide)-Pb superconducting tunnel junctions[1] exhibiting dc Josephson currents,[2-4] we have observed a temperature-independent resonance-shaped peak at applied voltages V_0 less than the energy gap, 2Δ. This structure is only readily apparent at low temperatures (1.2°K to 1.6°K) and when the sample is in a magnetic field large enough to quench the dc Josephson current.[5] In Fig. 1 we show a typical series of I-V curves taken at 1.2°K for several different values of magnetic field,[6] the field being oriented in the plane of the junction. For our sample geometry, such characteristic curves are generally independent of the particular orientation of the field in the junction plane. However, when the field is perpendicular to the junction, the peak completely disappears.

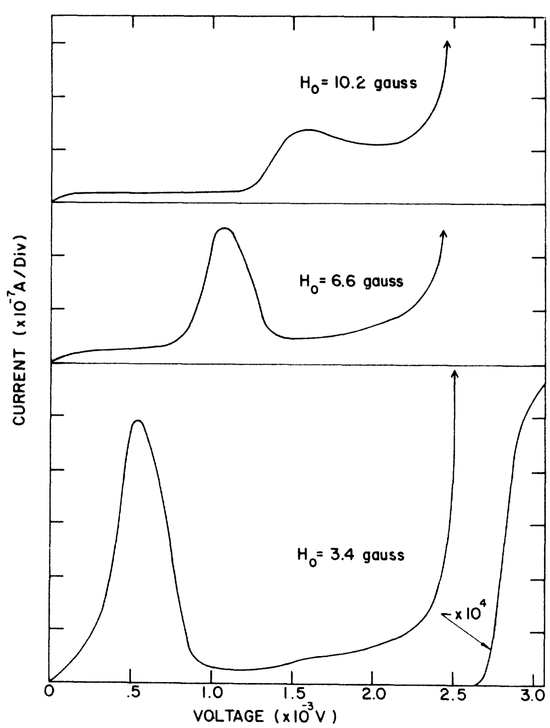

FIG. 1. Typical experimental I-V curves obtained for three different values of the magnetic field.

From a detailed analysis of I-V curves such as given in Fig. 1, we have been able to obtain the following information:

(a) The voltage V_p at which the peak occurs is approximately a linear function of the applied magnetic field H_0, the ratio V_p/H_0 being 0.19 ± 0.02 when V_p is expressed in millivolts and H_0 is expressed in gauss.

(b) The height of the peak is a strong function of magnetic field and consequently of voltage. For small voltages it varies as V_p^{-2}.

(c) The peak height for a given sample depends upon the magnitude of the zero-field dc Josephson current; the larger the Josephson current, the larger the peak. Indeed, for high-resistance samples which exhibit a dc Josephson current of only 10^{-2} to 10^{-3} of the maximum theoretical value, no peak is observed (at least down to 1.2°K).

The behavior described in (a) and (b) above is graphically presented in Fig. 2 where we plot, for the same sample shown in Fig. 1, the position of the peak as a function of magnetic field and the height of the peak as a function of voltage. Data are given only out to fields of order 10 gauss, corresponding to voltages of about $6\Delta/5$, because the peak becomes very small and badly smeared at larger fields (see Fig. 1).

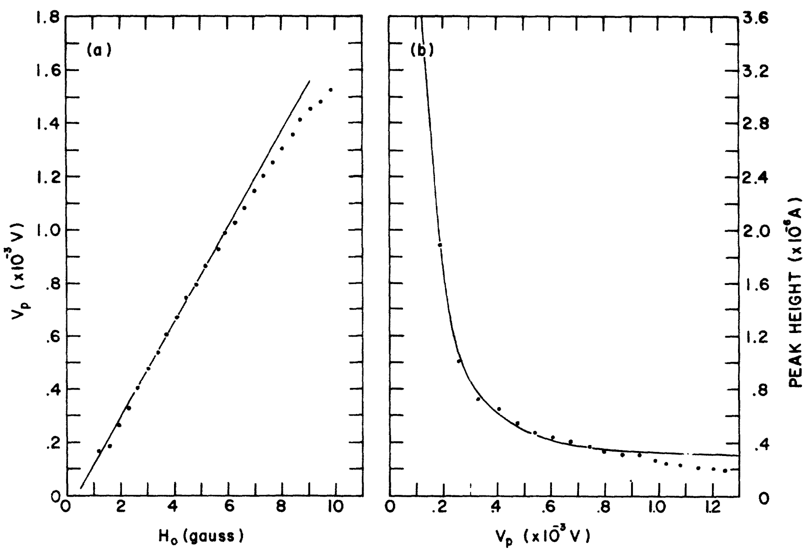

FIG. 2. Experimental points and theoretical curves showing (a) the dependence of peak position V_p on applied magnetic field, H_0, and (b) the dependence of peak height on V_p.

The essential features of our experimental data can be explained by considering the coupling of the Josephson ac current to the electromagnetic modes of the junction. The ac current has a spatial variation due to the external magnetic field as well as a time variation due to the dc bias voltage. This is apparent from the equations given by Josephson[7] for the current density between two superconductors separated by a thin oxide layer:

$$j(\vec{r},t)=j_1 \sin\varphi(\vec{r},t), \tag{1}$$

where j_1 is the normal-state current density at a junction voltage of $\frac{1}{2}\pi\Delta$ and φ is the phase difference between the two superconductors comprising the junction. This phase is determined by

$$\frac{\partial\varphi}{\partial t}=\frac{2eV}{\hbar}; \quad \nabla\varphi=\frac{2ed}{\hbar c}(\vec{H}\times\vec{n}). \tag{2}$$

Here, V is the total barrier voltage, both ac and dc, $\vec{H}$ the total magnetic field, $\vec{n}$ a unit vector normal to the junction, and $d=2\lambda+l$, where λ is the penetration depth and l the barrier thickness. For a dc bias voltage V_0, and an applied magnetic field H_0 in the plane of the junction and oriented along the y direction (see Fig. 3), Eq. (2) yields $\varphi=\omega t-kz$, where $\omega=2eV_0/\hbar$ and $k=2edH_0/\hbar c$.

A simple calculation of the coupling of the ac Josephson current to the junction fields can be carried out if the junction is treated as two semi-infinite pieces of superconductor separated by a thin oxide layer. The thickness of the layer is 10-20 Å so that variations of the field amplitudes with the x coordinate (see Fig. 3) in the oxide are completely negligible. The E_x component of the field is screened in the superconductor so that the integral of $\nabla\times\vec{E}=(1/c)(\partial\vec{H}/\partial t)$ over the surface $abcd$ (Fig. 3) which extends into the superconductors well beyond the penetration depth is[8]

$$l\frac{\partial E_x^{\,0}}{\partial z}=-\frac{(2\lambda+l)}{c}\frac{\partial H_y^{\,0}}{\partial t}. \tag{3}$$

Here, $E_x^{\,0}$ and $H_y^{\,0}$ are field amplitudes in the ox-

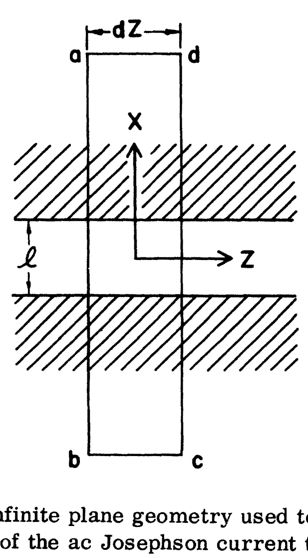

FIG. 3. Infinite plane geometry used to determine the coupling of the ac Josephson current to the electromagnetic field in the junction.

ide and the penetration depth λ is defined by

$$\lambda = \int_{\frac{1}{2}l}^{\infty} \frac{H_y(x)}{H_y^{\,0}}\,dx. \qquad (4)$$

In a similar way, taking the normal of the surface along the z axis,

$$l\frac{\partial E_x^{\,0}}{\partial y} = \frac{(2\lambda + l)}{c}\frac{\partial H_z^{\,0}}{\partial t}. \qquad (5)$$

The x component of $\nabla\times\vec{H} = (4\pi/c)\vec{j} + (\epsilon/c)(\partial\vec{E}/\partial t)$ in the oxide layer is

$$\frac{\partial H_z^{\,0}}{\partial y} - \frac{\partial H_y^{\,0}}{\partial z} = \frac{4\pi}{c} j_x + \frac{\epsilon}{c}\frac{\partial E_x^{\,0}}{\partial t}, \qquad (6)$$

where ϵ is the dielectric constant of the oxide layer and j_x is the driving current density given by Eq. (1). It follows from (3), (5), and (6) that the ac voltage across the barrier, $v = lE_x^{\,0}$, satisfies

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{\bar{c}^2}\frac{\partial^2}{\partial t^2}\right)v = \frac{4\pi l}{\epsilon\bar{c}^2}\frac{\partial j_x}{\partial t}, \qquad (7)$$

where $\bar{c} = c(l/d\epsilon)^{1/2}$.[9] Since the present experimental data correspond to a region in which $v/V_0 \ll 1$, the drive term in Eq. (7) can be linearized to $(4\pi l/\epsilon\bar{c}^2)j_1\omega\cos(\omega t - kz)$. Representing the losses (dissipative and radiative) by a Q factor, the solution of Eq. (7) is

$$v = v_0\cos(\omega t - kz + \theta),$$

$$v_0 = \frac{(4\pi l/\epsilon\omega)j_1}{\{[1-(k\bar{c}/\omega)^2]^2 + (1/Q)^2\}^{1/2}}$$

$$\theta = \tan^{-1}\frac{1/Q}{[1-(k\bar{c}/\omega)^2]}. \qquad (8)$$

The above ac voltage can in turn give rise to a dc current.[10] Since V in Eq. (2) is the total voltage $V_0 + v$, the phase φ is modified by the additional term[11] $(v_0/V_0)\sin(\omega t - kz + \theta)$ and the current density becomes

$$j = j_1\sin[\omega t - kz + (v_0/V_0)\sin(\omega t - kz + \theta)]. \qquad (9)$$

To first order in v_0/V_0, the dc part of Eq. (9) is

$$j_{\rm dc} = j_1\left(\frac{v_0}{V_0}\right)\sin\theta$$

$$= j_1\left(\frac{4\pi l j_1}{\epsilon\omega V_0}\right)\frac{1/Q}{[1-(k\bar{c}/\omega)^2]^2 + (1/Q)^2}. \qquad (10)$$

Equation (10) shows that the resulting dc current has a resonance shape. The maximum at $(\omega/k) = \bar{c}$ occurs when the phase velocity associated with the current-density distribution matches the phase velocity of the electromagnetic fields. Using the previously given values for k, $\bar{c}$, and ω, this phase-matching condition gives rise to the following linear relationship between the bias voltage at which the peak occurs and the applied magnetic field:

$$V_p = (ld/\epsilon)^{1/2}H_0. \qquad (11)$$

This equation is compared to our experimental data in Fig. 2(a). Taking λ to be[12] 400 Å and l to be 15 Å,[13] the best fit at low voltages is obtained for $\epsilon = 3.8$. It should be noted, however, that as the voltage increases beyond Δ, the ratio V_p/H_0 decreases and the current resonance broadens. In this region the photon energy is greater than 2Δ and absorption due to the breaking of pairs can take place. We believe this loss mechanism is responsible for these effects.

From Eq. (10), one can also obtain the dependence of the peak dc current on voltage. At resonance the current density is

$$(j_{\rm dc})_p = j_1 Q 4\pi l j_1/\epsilon\omega V_p, \qquad (12)$$

and since $\omega = 2eV_p/\hbar$, it varies as $1/V_p^{\,2}$. This dependence is compared to that experimentally observed in Fig. 2(b) where we fit Eq. (12) to our data. Using the previously derived value for ϵ, we find best agreement for $Q = 3.5$. The mechanism responsible for this low Q is being studied.

Our theory also explains the dependence of the peak height on the magnitude of the zero-field dc Josephson current as previously described; Eq. (12) shows that $(j_{\rm dc})_p \sim j_1^{\,2}$.[14] The disappearance of the peak when the magnetic field is perpendicular to the junction follows from Eq. (10) since $k = 0$ for this case.

That this simple model works so well may seem peculiar since it neglects the actual boundary conditions appropriate to the experimental junction. Furthermore, the wavelength of the mode in a field of 1 gauss is comparable to the junction dimensions in the direction of propagation. One might therefore expect that the detailed mode structure characteristic of the junction geometry should enter. However, for $Q \simeq 4$ and a junction dimension of 0.25 mm, the half-width of the peak becomes equal to the mode spacing at the resonant frequency corresponding to a magnetic field of about 2 gauss. Consequently, the geometrical mode structure is washed out in the region where present experimental data were taken.

It is worth emphasizing the importance of the role played by the constant magnetic field H_0. Without it, the power transferred to the electromagnetic field is reduced by a factor Q^{-2}. The application of H_0 alters the spatial distribution of the ac Josephson current density and allows it to couple to a mode having an inductive field extending into the penetration layer. This inductance can "resonate out" the capacitance of the junction.

It should be possible to construct junctions with Q's sufficiently large that $v_0/V_0 \approx 1$. Under these conditions the power transferred to the field will be of the order of microwatts. In this case, our linear approximation of the drive in Eq. (7) fails and the system of Eqs. (1), (2), and (7) must be reinvestigated.

Finally, we mention that structure in addition to the main peak has been observed in a few samples. This structure is presently being investigated as well as the general effect of junction size and geometry.

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[1]Our junctions usually consisted of crossed perpendicular Pb strips 20 mm long, 0.25 mm wide, and between 2000 and 3000 Å thick. The oxide barrier layers were thermally grown in pure dried oxygen, typical junction resistances being between 0.5 and 10 ohms. The Josephson dc currents were between 0.1 and 0.6 of their maximum theoretical values and the energy gaps for our Pb films were $(2.67 \pm 0.04) \times 10^{-3}$ eV.

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EFFECT OF ELECTRON CONCENTRATION ON MAGNETIC EXCHANGE INTERACTIONS IN RARE EARTH CHALCOGENIDES

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The magnetic moments of the lanthanides are localized in the $4f$ electron shells and the ferromagnetic or antiferromagnetic interactions observed in the lanthanide metals[1] and their metallic alloys[2] have been interpreted by the Ruderman-Kittel-Yosida-type indirect exchange[3] resulting from the polarization of the conduction electrons. We have examined in semiconducting lanthanide compounds the relationship between magnetic transition temperature and electrical properties which can be implied from the theory of indirect exchange via conduction electrons. We reported previously experimental evidence for such correlation in the bcc Th_3P_4 compounds Gd_2Se_3 and Gd_2S_3.[4] In these compounds doping with excess Gd or Y up to 0.1 atom per molecule increased the electrical conductivity by orders of magnitude and changed the magnetic properties of Gd_2Se_3 continuously from antiferromagnetic ($T_N = 5\,°K$) to ferromagnetic ($T_C = 80\,°K$) without variation in crystal structure.

The magnetic exchange interactions in NaCl-