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*Permanent address: Department of Physics, University of North Carolina, Chapel Hill, North Carolina. This work was performed during tenure of a fellowship of the National Science Foundation.

[1]T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962); T. D. Lee, Phys. Rev. 128, 898 (1962).

[2]G. Feinberg and A. Pais, Phys. Rev. 131, 2724 (1963); 133, B477 (1964).

[3]Y. Pwu and T. T. Wu, Phys. Rev. 133, B778 (1964).

[4]M. A. Baqi Bég, Ann. Phys. (N.Y.) 27, 183 (1964).

[5]W. Pauli, Helv. Phys. Acta Suppl. 4, 69 (1956).

[6]O. Klein, Niels Bohr and the Development of Physics, edited by W. Pauli, with the assistance of L. Rosenfeld and V. Weisskopf (Pergamon Press, London, 1955), p. 96; Nuovo Cimento Suppl. 6, 344 (1957).

[7]L. D. Landau, in Niels Bohr and the Development of Physics, edited by W. Pauli, with the assistance of L. Rosenfeld and V. Weisskopf (Pergamon Press, London, 1955), p. 52.

[8]S. Deser, Rev. Mod. Phys. 29, 417 (1957).

[9]For the role played by this length in the theory of measurement, see T. Regge, Nuovo Cimento 7, 215 (1958); and B. S. DeWitt, in Gravitation: An Introduction to Current Research, edited by L. Witten (John Wiley & Sons, Inc., New York, 1962).

[10]The deDonder gauge of gravidynamics is the analog of the Lorentz gauge of electrodynamics. With any other gauge the graviton propagator is more singular, and gives rise to extra divergences in the theory. However, these are spurious divergences which cancel one another owing to gauge invariance. Just as in electrodynamics, the cancellation is an intricate one, involving all diagrams of a given order and not the ladder diagrams only. Moreover, as Feynman has pointed out (unpublished), in gravidynamics a previously unsuspected set of diagrams must be included, involving fictitious vector particles interacting with the gravitons only. [See B. S. DeWitt, Phys. Rev. Letters 12, 742 (1964).] Since the present investigation is restricted to the ladder graphs the results obtained are not gauge invariant. It is believed, however, that if the methods of references 2, 3, and 4 have any validity at all, their use in the present case in conjunction with the least divergent gauge leads to a modified scalar propagator [Eq. (20)] whose general analytical properties do not differ greatly from those of the correct propagator. As for justifying the restriction to ladder graphs, although it is true that when the rungs correspond to zero-mass quanta these graphs dominate all others at low energies, unfortunately we can say nothing about the high-energy regime which is important here. In this respect the present investigation suffers from the same defects as those of the cited references.

[11]H. Lehmann, Nuovo Cimento 11, 342 (1954).

SELF-CONSISTENCY OF HIGHER SYMMETRY UNIVERSES*

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According to the strongest version of the bootstrap conjecture of Chew and Frautschi,[1] it should be possible to rule out all strong interaction symmetries other than the observed one, $SU(2)\otimes Y$ (Y is hypercharge, baryon number plus strangeness) by requiring all particles to generate themselves self-consistently. Present knowledge of dynamics, however, still permits the construction of a wide range of higher symmetry universes which are self-consistent in first approximation.[2] About SU(2) dynamics it is known that pion-pion scattering and pion-nucleon scattering with production of ρ and $(\frac{3}{2},\frac{3}{2})$ N^* resonances, respectively, can be made self-consistent in first approximation.[3,4] Now suppose one replaces π, ρ, N, and N^* by multiplets Π, V, B, and B^*, still $J^P=0^-$, 1^-, $\frac{1}{2}^+$, and $\frac{3}{2}^+$, respectively, but now (except for B^*) transforming as the adjoint representation of a compact simple group G.[5] Simultaneously, in the scattering calculation, one replaces SU(2) crossing matrices by G crossing matrices. Then the $\Pi\Pi$ problem is still self-consistent, and, at least for $G=SU(n)$, the ΠB problem is also. Let $C_t(X,Y)$ [or $C_u(X,Y)$ denote the crossing matrix element giving the force exerted by crossed t (or u) channel Y on s channel X. Then for $\Pi\Pi$ scattering in which V bootstraps itself, the relevant elements are $C_t(V,V)$ and $C_u(V,V)$, which Cutkosky has shown are always attractive[6]:

$$C(V,V)=C(\rho,\rho)=\tfrac{1}{2}. \qquad (1)$$

[If $C_t(X,Y)=C_u(X,Y)$, we drop the subscript.] Further,[6,7]

$$C(V',V)=0, \quad C_t(S,V)=+1=-C_u(S,V), \qquad (2)$$

where V' is any other channel capable of supporting a vector-meson resonance and S is the scalar representation, i.e., the vacuum Regge trajectory. Hence the presence of crossed resonance

V need not imply the presence of any additional multiplets V', and the force exerted by V on the vacuum trajectory is not so great that unitarity will surely be violated in a crossed channel, as happens in a case discussed by Chew.[8]

Now let $G = SU(n)$. In ΠB scattering, crossed channel B supports B^*, and reciprocally B^* supports B. Each of the states $\underline{1}_3, \underline{8}_3, \underline{8}'_3, \underline{10}_3, \underline{10}_3^*$, $\underline{27}_3$ occurring in the decomposition of the SU(3) direct product $\Pi \otimes B = \underline{8}_3 \otimes \underline{8}_3$ has its generalization to an SU(n) state $\underline{1}_n$, $\underline{8}_n$, $\underline{8}'_n$, etc.[9] B couples to some linear combination of the ΠB states $\underline{8}_n$ and $\underline{8}_n'$ (D and F states in the notation of Gell-Mann):

$$|B\rangle = \sin\theta\, |\Pi B; \underline{8}_n'\rangle + \cos\theta\, |\Pi B; \underline{8}_n\rangle. \qquad (3)$$

θ is Cutkosky's mixing parameter.[10] The choice of θ determines which channel B^* will resonate. θ, in turn, is not determined uniquely by the self-consistency requirement, at least in first order. It must be fixed by appeal to experiment. We want $B^* = \underline{10}_3$, which necessitates[10] $\theta \approx 33°$ (for $\theta \approx 0$, $B^* = \underline{10}_3 + \underline{10}_3^*$; for $\theta \approx \frac{1}{2}\pi$, $B^* = \underline{27}_3 + \underline{8}_3'$). The relevant crossing matrix elements are, for all n and θ,[11]

$$C_u(B^* = \underline{10}_n, B) = 2\cos^2\theta (n^2-4)^{-1} + 2\sin\theta\cos\theta (n^2-4)^{-1/2}, \qquad (4)$$

$$C_u(B, B^* = \underline{10}_n) = \tfrac{1}{2}\cos^2\theta + \tfrac{1}{2}(n^2-4)^{1/2}\sin\theta\cos\theta. \qquad (5)$$

Since

$$\lim_{n\to\infty} C_u(B^* = \underline{10}_n, B) = 0,$$

the choice $B^* = \underline{10}_n$ will not work for large n (similarly for the choices $B^* = \underline{10}_n + \underline{10}_n^*$, $B^* = \underline{27}_n$). What does work is either B and B^* both $\underline{8}_n$ (D) or B and B^* both $\underline{8}_n'$ (F). Again the mechanism is a reciprocal bootstrap, B^* supporting B, and B supporting B^*:

$$C_u(B, B^*) = \tfrac{1}{2}(\sin^2\theta - \cos^2\theta)(\sin^2\theta^* - \cos^2\theta^*) + \tfrac{1}{2}[(n^2-12)/(n^2-4) - 1]\cos^2\theta\cos^2\theta^*. \qquad (6)$$

B couples to the linear combination of F and D states given by Eq. (3), while B^* couples to a similar combination with θ^* replacing θ. $C_u(B^*, B)$ follows from Eq. (6) by interchanging θ and θ^*:

$$C_u(B^*, B) = C_u(B, B^*). \qquad (7)$$

For large n the square bracket in Eq. (6) is negligible; maxima are attained for $\sin\theta = \sin\theta^* = 1$ (both F) or $\cos\theta = \cos\theta^* = 1$ (both D). Thus a first-approximation SU(n) universe of the familiar type (0^- mesons and $\frac{1}{2}^+$ baryons) exists for every n.[12,13]

On the other hand, we may expect that, in the normal course of progress in the understanding of higher symmetry crossing matrices, the bootstrap requirement will succeed in ruling out, even in first approximation, a very large class of universes: those in which the baryons and mesons are assigned to multiplets of dimension much greater than that of the adjoint representation of the group. From an approximate calculation of the crossing matrices for scattering of large-isospin particles in SU(2), it follows that the formation of large-isospin states in an SU(2) $\otimes Y$ universe is much less favored because the relevant crossing matrix elements are going to zero as I increases.[7,14] The expressions in reference 7 follow from quite general group-theoretical principles applied to SU(2), and it would be amazing if similar results did not hold for the higher groups. That is, for all groups only lowest dimensional representations should be of dynamical significance.[15] Indeed, the greater simplicity of adjoint-representation scattering in SU(n) for n large suggests that this principle may apply with even greater force to the higher rank groups than to SU(2) (for n large one can get the adjoint representation of baryons B^* but no B^* multiplets of higher dimension).

So far we have been emphasizing how similar the crossing matrices of low- and high-rank groups G are. To the end of demonstrating a difference, we consider the $I = 0$ vacuum Regge trajectory. The mechanism proposed by Balázs[16] for generating the vacuum trajectory, ρ exchange in $\pi\pi$ scattering, generalizes unchanged to $\Pi\Pi$ scattering in G in virtue of Eq. (2). Therefore every G-symmetric adjoint-representation universe will possess a vacuum trajectory. Also, it will lie high in the ReJ vs momentum transfer plane: The V-exchange mechanism is sufficient to cause the S trajectory at zero momentum transfer to pass through Re$J = 1$, the highest value permetted by unitarity.[16]

First let us consider the force exerted by other particles on the vacuum trajectory. This force probably varies but little with rank of the group; for particle-antiparticle scattering of a d-dimensional representation of G,

$$C_t(S, X) = d_x/d, \qquad (8)$$

where d_x is the dimension of X.[7] As pointed out above both d_x and d are likely to be small: $d_x \cong d \cong O(d_R)$, d_R the dimension of the adjoint representation. Hence $C_t(S,X)$ is never likely to be much different from 1. On the other hand, the inverse force, that exerted on all other particles by the vacuum trajectory, varies in more interesting fashion. For elastic scattering of any two representations of G having dimension d_1 and d_2,[17,18]

$$C_t(X,S) = (d_1 d_2)^{-1/2}. \quad (9)$$

For general G we anticipate $d_1 \sim d_2 \sim O(d_R)$. For adjoint representation scattering in SU(n), $d_1 = d_2 = d_R = n^2-1$; the force exerted by the vacuum trajectory is falling off as 1/3, 1/8, 1/15, ⋯ as one goes up in the series SU(2), SU(3), SU(4), ⋯. There is a marked difference between SU(2) and even the lowest rank higher symmetries. Therefore it would be highly desirable to design projected dynamical calculations with the vacuum trajectory in such manner that the crossing matrix elements, and with them the rank of the group, become part of the input information.

The same remarks, of course, would apply not only to the vacuum trajectory but also to any other singlet. For example, the presence of two $J^P = 1^-$ singlets ω and φ(1040 MeV)[19] in the SU(2) limit suggests the presence of a singlet V_0 in addition to the octet V in the SU(3) limit; and since the crossing matrix elements (8) are relatively insensitive to the rank of the group, V_0 may be present in G as well.[20]

Should the strong-interaction symmetry observed in nature be described as a broken SU(3) symmetry or as a broken SU(4) symmetry? Present theoretical calculations cannot distinguish between SU(∞) and SU(3), let alone SU(4) and SU(3), so that some experimental information on this question would be desirable.[21] If the universe is a broken SU(4), then there should exist an additional hypercharge quantum number, call it Y_β to distinguish it from the familiar hypercharge $Y_\alpha = S_\alpha$ (strangeness) + baryon number; and particles carrying this quantum number should exist at higher energies than have up to now been studied. Experimentally, this quantum number would manifest itself as a new associated production rule, similar to the familiar one for S_α. The more probable circumstance a priori is that the particles $Y_\beta \neq 0$ should lie higher in mass than the particles $Y_\beta = 0$.[22] Consider first the way SU(3) symmetry is broken. Most probably, the necessary condition for a given SU(2) multiplet I to lie lowest in the broken SU(3) multiplet is that there exist a suitable SU(2) mechanism to bootstrap the multiplet I. By an SU(2) mechanism is meant one which would work even in the absence of the other members of the broken SU(3) multiplet. Certainly mechanisms for bootstrapping ρ and N^*, the two lowest-lying members of the V and B^* multiplets, were known before SU(3) symmetry appeared on the scene; and it is at least in line with the conjectured necessary condition that the two lowest lying members of the B and Π multiplets are just those required for the ρ and N^* bootstraps; in addition, since both N and π are $S_\alpha = 0$ particles, the universe observed from the low-energy end looks like a purely SU(2) universe. If we extend the idea from a broken SU(3) symmetry to a broken SU(4) symmetry, it follows that SU(4) would break so that the universe observed from the low-energy end would look like a self-consistent SU(3) universe, while the particles $Y_\beta \neq 0$ would lie higher in mass.[23,24] [Whether SU(4) symmetry exists or not, the requirement that the lowest-lying states form an approximately self-consistent subset is useful in understanding the way SU(3) symmetry breaks.]

The universes SU(5), SU(6), ⋯ become successively less attractive theoretically because, were particles $Y_\gamma \neq 0, Y_\delta \neq 0$, ⋯ present, they would act to produce a regular representation of baryons B^* in the SU(5), SU(6), ⋯ limit; this representation would break to give an octet of $\frac{3}{2}^+$ resonances lying lowest, rather than a decuplet. Of course if singlet trajectories have anything to do with the stability of the low-lying states, one should not expect an exact SU(4) symmetry [or an exact SU(3) symmetry for that matter], since $C_t(X,S) = O(1/15)$ [or $O(1/8)$] is practically zero; but rather a badly broken symmetry so as to give $C_t(X,S) = O(1/3)$.

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[1]G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961).

[2]By a first-approximation calculation is meant one in which only the nearest crossed singularities are taken into account and inelastic effects are replaced by cutoffs. The terms "higher symmetry" and "higher rank group"

will be used interchangeably. The rank of a group is the number of commuting linearly conserved observables. $SU(2)\otimes Y$ is rank two (I_3 and Y).

[3]F. Zachariasen and C. Zemach, Phys. Rev. 128, 849 (1962); and earlier references quoted therein.

[4]E. Abers and C. Zemach, Phys. Rev. 131, 2305 (1963); and earlier references quoted therein.

[5]Baryon number conservation is assumed also.

[6]R. E. Cutkosky, Phys. Rev. 131, 1888 (1963). Cutkosky's λ is $C_u(V,V)+C_t(V,V)$, and his λ_i are the $C_u(V',V)+C_t(V',V)$, in the notation of the present paper.

[7]D. E. Neville, to be published.

[8]G. F. Chew, University of California Radiation Laboratory Report No. 11151 (unpublished).

[9]D. E. Neville, Phys. Rev. 132, 844 (1963).

[10]R. E. Cutkosky, Ann. Phys. (N.Y.) 23, 415 (1963). See also R. E. Cutkosky, J. Kalckar, and P. Tarjanne, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 653; A. W. Martin and K. C. Wali, Nuovo Cimento 31, 1324 (1964).

[11]The proof is an application of the projection operator techniques described in references 6, 9, and a paper by R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev. Mod. Phys. 34, 1 (1962). Reference 9 [especially Table III and Eq. (2.22)] calculates $C(X,Y)$ for the limits of pure F or pure D coupling. The extra elements needed for a discussion of the general case follow readily from the two projection operators of Eq. (2.9) [provided one changes their signs! For instance, the projection operator which annihilates an initial two-particle F state is $(BA-AB)_j{}^i$].

[12]We have used the Gell-Mann–Ne'eman version of $SU(n)$ symmetry ($B=8_n$) rather than the Sakata version ($B=3_n$) as our example because ΠB scattering for the latter is not self-consistent for large enough n. Cutkosky, Kalckar, and Tarjanne (reference 10) have shown that for all B^*,

$$\lim_{n\to\infty} |C_u(B=3_n, B^*)| \leq n^{-1}.$$

[13]In addition to the linear combination $|B\rangle$, Eq. (3), there is an orthogonal linear combination $|B'\rangle$. Eigenstate B' does not resonate when B does: $C_u(B',B^*)$ is given by Eq. (6) after the interchange $\cos\theta \leftrightarrow -\sin\theta$, and for n large $C_u(B',B^*)=-C_u(B',B^*)$.

[14]The approximate equations for $C(X,Y)$ resemble WKBJ solutions to problems in quantum mechanics and are valid for "classical" isospins, those such that $I(I+1)/I^2 \cong 1$.

[15]In nuclear physics, systems with very large isospin are common. A high atomic-weight nucleus is very close to threshold for decay into a many-body system, and the effects in this physical channel are more important than exchanges in crossed channels. For states of high total energy, therefore, the statement in the text needs qualification. In the low-energy region, on the other hand, it seems reasonable to expect low-dimensional multiplets even in the presence of collective effects. Firstly, a low-dimensional state has the extra advantage: Crossed as well as physical channels can contribute to its stability. Secondly, many-body interactions seem to presuppose strong few-body interactions, hence to presuppose the existence of a sector of the total scattering problem in which crossed channels are important. We thank Dr. Roland Omnes for pointing out the significance of physical channels effects.

[16]Louis A. P. Balázs, Phys. Rev. 132, 867 (1963).

[17]D. Amati, L. L. Foldy, A. Stanghellini, and L. Van Hove, to be published.

[18]Equations (8) and (9) hold not only for simple groups G, but also for compact Lie groups generally.

[19]P. Schlein, W. Slater, L. Smith, D. Stork, and H. Ticho, Phys. Rev. Letters 10, 368 (1963); P. Connolly, E. Hart, K. Lai, G. London, G. Moneti, R. Rau, N. Samios, I. Skillicorn, S. Yamamoto, M. Goldberg, M. Gundzik, J. Leitner, and S. Lichtman, Phys. Rev. Letters 10, 371 (1963).

[20]The short-range repulsion and attraction observed in NN and $N\bar{N}$ scattering, respectively, are evidence that one ω (or φ) coupling, that to $N\bar{N}$, is quite strong. See J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960); G. Breit, Proc. Natl. Acad. Sci. U. S. 46, 746 (1960).

[21]If singlets are dynamically important, then self-consistent symmetries would tend to be low-rank symmetries or badly broken symmetries $G_1\otimes G_2\otimes G_3\otimes\cdots$ with each factor G_1 of low rank; then the irreducible representations would be of small dimension and the elements (9) would be large.

[22]Previous theoretical estimates, however, based on the extension of the SU(3) mass-splitting formula to SU(4), have placed the particles $Y_\beta \neq 0$ at roughly the same mass as the particles $Y_\beta=0$. P. Tarjanne and V. Teplitz, Phys. Rev. Letters 11, 447 (1963).

[23]SU(4) predicts additional mesons $\bar{K}', \bar{K}^{*\prime}, \omega', \eta'$ with $Y_\beta=+1$ plus their antiparticles with $Y_\beta=-1$ (the notation implies the conventional strangeness S_α, spin parity, and isospin; e.g., η' has $S_\alpha=0$, $J^P=0^-$, $I=0$); as well as additional baryons Ξ', Λ' with $Y_\beta=+1$; and N', Λ'' with $Y_\beta=-1$. SU(4) also predicts the quantum numbers of some $Y_\beta=0$ resonances: φ(1040 MeV) (reference 19) should have $IJ^P=0(1)^-$; Y_0^*(1404 MeV) [see the report by B. P. Gregory, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 779 ff, for a summary and bibliography of the data on the Y_0^*] should have $IJ^P=0(\frac{1}{2})^+$; and the recently discovered 959-MeV $\pi\pi\eta$ resonance [G. R. Kalbfleisch, L. W. Alvarez, A. Barbaro-Galtieri, O. I. Dahl, P. Eberhard, W. E. Humphrey, J. S. Lindsey, D. W. Merrill, J. J. Murray, A. Rittenberg, R. R. Ross, J. B. Slater, F. T. Shively, D. M. Siegal, G. A. Smith, and R. D. Tripp, Phys. Rev. Letters 12, 527 (1964)] should have $IJ^P=00^-$.

[24]Some particles $Y_\beta \neq 0$ might be so long lived as to leave easily visible tracks in emulsion. If the nonleptonic weak-interaction selection rule were $(|\Delta S_\beta|, |\Delta S_\alpha|, \Delta I)=(1,0,0$ only or 1 only or both 0 and 1), then all charged $Y_\beta\neq 0$ mesons and baryons (of mass

≥ 1 BeV and ≥ 1.5 BeV, say) in Π and B could decay to $K\pi$, $N\pi$, $\Lambda\pi$, $\Sigma\pi$, or $\Xi\pi$ final states. For comparison, known lifetimes, for $K_1{}^0 \rightarrow 2\pi$, $\Lambda \rightarrow N\pi$, $\Xi \rightarrow \Lambda\pi$, all $(0,1,\frac{1}{2})$ processes, are $O(10^{-10}$ sec) (see report by Gregory, reference 23, pp. 783, 839). If the selection rule were $(1,1,\frac{1}{2})$, some mesons would decay predominantly by leptonic modes; known lifetimes, $\pi \rightarrow l\nu$ and $K^{\pm} \rightarrow l\nu$, are $O(10^{-8}$ sec). As is well known, of course, one must be prepared for factor-of-ten errors in extrapolations of weak decay lifetimes because universal Fermi interaction is so badly violated.

ERRATA

HIGH-DENSITY BEHAVIOR AND DYNAMICAL STABILITY OF NEUTRON STAR MODELS.

C. W. Misner and H. S. Zapolsky [Phys. Rev. Letters 12, 635 (1964)].

Equation (7a) should read

$$\epsilon = a/16\pi r^2 \tag{7a}$$

instead of the incorrect expression which appears in the Letter,

$$\epsilon = a/16\pi^2.$$

Also, the last word in reference 9 should read "Eq. (9)" instead of "Eq. (7)."

EXCITATION OF LONGITUDINAL PLASMA OSCILLATIONS NEAR CYCLOTRON HARMONICS.

S. J. Buchsbaum and A. Hasegawa [Phys. Rev. Letters 12, 685 (1964)].

Equation (2) contains a misprint. The second factor in the first term should be $(\omega_c{}^2 + \omega_p{}^2 - \omega^2)$.