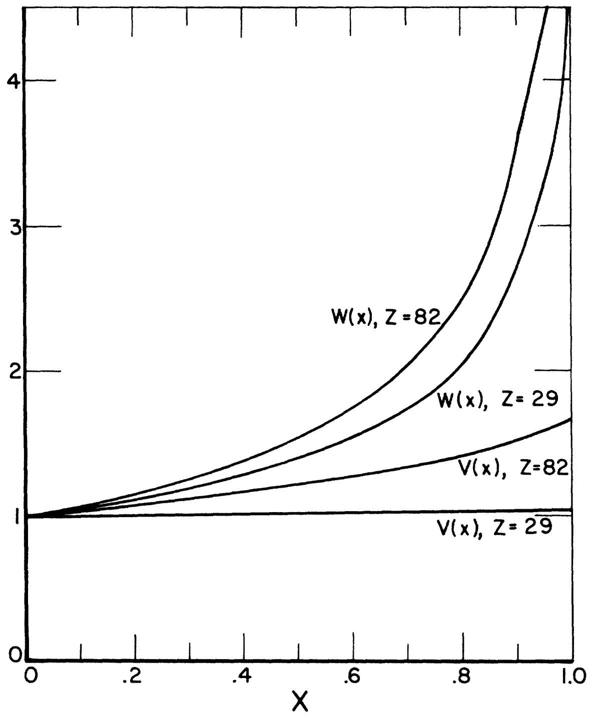

FIG. 3. The functions $V(x)$ and $W(x)$ for copper and lead.

A graph of the hypergeometric functions[3] $V = V(x) = F(ia, -ia; 1; x)$ and $W = W(x) = F(1 + ia, 1 - ia; 2; x)$ as a function of x is given for $Z = 29$ and $Z = 82$, $0 \leq x < 1$, in Fig. 3. The units of energy and momenta are mc^2 and mc throughout.

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GRAVITY: A UNIVERSAL REGULATOR?

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In view of the suggestive, although nonrigorous, results which have been obtained in several recent attempts[1-4] to obtain quantitative information from nonrenormalizable field theories, it is of interest to see how well some of the methods which have been introduced fare in a completely different setting, namely that of quantized general relativity. General relativity describes the arena in which the rest of physics happens, and it is of interest to determine at how many different levels it can make its influence felt. It has, for example, been repeatedly speculated[5-8] that quantum gravidynamics may remove the divergences in conventional field theories by providing a natural cutoff associated with the well known fundamental length 10^{-33} cm.[9]

In this note we show that very simple results can be obtained, which tend to confirm these speculations in at least one nonpathological case —two scalar particles exchanging gravitons in the ladder approximation. The "leading terms" of the Bethe-Salpeter amplitudes can be summed exactly, and, owing to certain remarkable cancellations, the sum of the ladder-type contributions to the gravitational self-energy can be expanded in a power series in the bare mass, with no approximations whatever. Furthermore, these results can be extended to the case of charged scalar particles, with one or more of the graviton rungs replaced by photons, and a simple expression can be obtained for the lowest order electromagnetic self-energy. The self-energies found in this way are finite, although not always small.

It is convenient to use "absolute units" in which $\hbar = c = 16\pi G = 1$, where G is the gravitation constant. If the DeDonder gauge[10] is employed for the graviton propagators, then the total Bethe-

Salpeter amplitude is given by

$$X(q,p_2,p_1)=\sum_{n=0}^{\infty}X_{n+1}(q,p_2,p_1), \tag{1}$$

where X_n is the n-rung amplitude:

$$X_1=[(q+p_2)\cdot(q-p_2)(q+p_1)\cdot(q-p_1)+(q+p_2)\cdot(q-p_1)(q-p_2)\cdot(q+p_1)$$
$$-(q+p_2)\cdot(q+p_1)(q-p_2)\cdot(q-p_1)-m_0^2(q+p_2)\cdot(q+p_1)-m_0^2(q-p_2)\cdot(q-p_1)-2m_0^4](p_2-p_1)^{-2}, \tag{2}$$

$$X_{n+1}=(-i)^n(2\pi)^{-4n}\int dk_1\cdots\int dk_n X_1(q,p_2,k_n)$$
$$\times X_1(q,k_n,k_{n-1})\cdots X_1(q,k_1,p_1)S_0(q+k_n)S_0(q-k_n)\cdots S_0(q+k_1)S_0(q-k_1), \tag{3}$$

$$S_0(k)\equiv(k^2+m_0^2)^{-1}. \tag{4}$$

Here $q+p_1, q-p_1$ are the initial momenta and $q+p_2, q-p_1$ the final momenta of the scalar particles, and m_0 is their bare mass (assumed the same for both). All propagators are understood to contain the usual infinitesimal negative imaginary term, and the flat space-time metric is taken in the form (-1, 1, 1, 1).

Expression (3) is $2(n-1)$-fold divergent at high momenta and hence high-energy damping of the total amplitude may immediately be suspected. (We ignore the low-momentum divergence arising from the long range of the gravitational interaction.) Using the symbol "~" to denote the usual "leading-term" approximation[2-4] of neglecting q and m_0 in comparison to the k's, and taking note of the cancellation of the second and third terms inside the brackets of Eq. (2) in the limit $q\to 0$,

$$X_1(0,p_2,p_1)=[S_0^{-1}(p_2)S_0^{-1}(p_1)-m_0^2(p_2+p_1)^2-3m_0^4](p_2-p_1)^{-2}, \tag{5}$$

we find

$$X_{n+1}\sim(-i)^n(2\pi)^{-4n}(q+p_2)\cdot(q-p_2)(q+p_1)\cdot(q-p_1)\int dk_1\cdots\int dk_n(p_2-k_n)^{-2}(k_n-k_{n-1})^{-2}\cdots(k_1-p_1)^{-2} \tag{6}$$

in which complete cancellation has occurred between vertices and scalar propagators.

Adding together the modified amplitudes (6), we now obtain

$$X(q,p_2,p_1)\sim[(q+p_2)\cdot(q-p_1)(q-p_2)\cdot(q+p_1)-(q+p_2)\cdot(q+p_1)(q-p_2)\cdot(q-p_1)-m_0^2(q+p_2)\cdot(q+p_1)$$
$$-m_0^2(q-p_2)\cdot(q-p_1)-2m_0^4](p_2-p_1)^{-2}+(q+p_2)\cdot(q-p_2)(q+p_1)\cdot(q-p_1)Y(p_2-p_1), \tag{7}$$

where the function Y satisfies the integral equation

$$Y(p)=p^{-2}-i(2\pi)^{-4}\int(p-k)^{-2}Y(k)dk. \tag{8}$$

In coordinate space this equation takes the trivial form

$$Z(x)=G(x)[1-iZ(x)], \tag{9}$$

where Z is the Fourier transform of Y and $G(x)$ is the Feynman Green's function for a massless field:

$$G(x)=(2\pi)^{-2}i(x^2+i0)^{-1}. \tag{10}$$

The solution of (8) is immediate:

$$Z(x)=\frac{G(x)}{1+iG(x)}=\frac{i}{(2\pi)^2}\frac{1}{x^2-(2\pi)^{-2}+i0} \tag{11}$$

or, in ordinary cgs units,

$$Z(x)=\frac{i}{(2\pi)^2}\frac{1}{x^2-\lambda^2+i0} \tag{11'}$$

where

$$\lambda=(4\hbar G/\pi c^3)^{1/2}=1.82\times10^{-33}\text{ cm}. \tag{12}$$

We thus see that in the limit of very high momentum transfer, in which the first term on the right of (7) becomes negligible compared to the second, the singularity of the effective gravitational interaction is displaced off the light cone in coordinate space and onto a hyperboloid lying at a distance λ in spacelike directions. This is roughly equivalent to endowing the scalar particles with the properties of hard spheres of diameter λ.

Corresponding to each ladder diagram there is a self-energy diagram which is obtained by closing off the top of the ladder and setting $q=0$. In view of the fact that the singularity of the scalar propagator which effects the closure does not coincide with that of the effective gravitational interaction $Z(x)$, one may anticipate that the sum of all such self-energy diagrams is finite. This sum is given by

$$\begin{aligned}\Sigma(p^2)&=i(2\pi)^{-4}\int S_0(k)X(0,k,p)dk\\&=i(2\pi)^{-4}\int[Y(k-p)S_0^{-1}(p)-m_0^2S_0(k)(k+p)^2(k-p)^2+im_0^2(2\pi)^{-4}\int Y(k-k')S_0(k')(k'+p)^2(k'-p)^{-2}dk'\\&\quad+im_0^2S_0(k)(2\pi)^{-4}\int(k+k')^2(k-k')^2Y(k'-p)dk'S_0^{-1}(p)+m_0^2(2\pi)^{-8}\int dk'\int dk''\,Y(k-k')S_0(k')\\&\quad\times(k'+k'')^2(k'-k'')^2Y(k''-p)S_0^{-1}(p)+\cdots]dk.\end{aligned} \tag{13}$$

The terms in m_0^2 are included to make up for the fact that the first term is obtained by neglecting the second and third terms inside the brackets of Eq. (5) when the X_1's are inserted into (3) to obtain X. Additional corrections, of order m_0^4, m_0^6, etc., can also be obtained in a straightforward manner if desired.

Care must be exercised in the evaluation of the above integrals, some of which are divergent standing alone. In particular, the k integration must be reserved to the last. Otherwise we should be tempted to use

$$(2\pi)^{-4}\int Y(k)dk=Z(0)=-i\text{ (in absolute units)}, \tag{14}$$

leading to a trivial and incorrect cancellation between the second and third and fourth and fifth terms inside the brackets of (13). Instead we use the relation

$$\delta(k)-i(2\pi)^{-4}Y(k)=i(2\pi)^{-2}\Box_kY(k), \tag{15}$$

which yields

$$\Sigma(p^2)=i(2\pi)^{-4}S_0^{-1}(p)\int Y(p-k)\{1-im_0^2\Box_k\int S_0(k)\Box_{k'}[(k+k')^2(k-k')^{-2}S_0(k')]dk'+\cdots\}dk. \tag{16}$$

The operator $\Box_{k'}$ transforms the divergent integrals into convergent ones, which makes it then safe to use (14). The integral above is easily evaluated, and we obtain

$$\begin{aligned}\Sigma(p^2)&=S_0^{-1}(p)[1-im_0^2(2\pi)^{-2}\int Y(p-k)\Box_kS_0(k)dk+\cdots],\\&=S_0^{-1}(p)-m_0^2+im_0^2(2\pi)^{-4}S_0^{-1}(p)\int Y(p-k)S_0(k)dk+\cdots.\end{aligned} \tag{17}$$

To present accuracy $S_0(k)$ in the final integrand may be replaced by $1/k^2$. In view of Eq. (8) this replacement yields

$$\Sigma(p^2)=p^2+m_0^2S_0^{-1}(p)[p^{-2}-Y(p)]+\cdots. \quad (18)$$

For spacelike p's the function $Y(p)$ has the form

$$Y(p)=-\frac{i}{8\pi}\frac{H_1^{(2)}(z)}{z}, \quad z\equiv\frac{(p^2)^{1/2}}{2\pi}>0, \quad (19)$$

its value for timelike p's being obtained by continuation in the lower half p^2 plane. Let us now introduce a wave-function-renormalization factor ξ multiplying the original scalar Lagrangean. It is not difficult to see that the radiatively corrected scalar propagator becomes

$$S(p^2)=\xi^{-1}[p^2+m_0^2+\Sigma(p^2)]^{-1}. \quad (20)$$

If the experimental mass is denoted by m then m_0 and ξ are determined by

$$m^2=m_0^2+\Sigma(-m^2), \quad \xi^{-1}=1+\Sigma'(-m^2). \quad (21)$$

Making use of the series expansion of the Hankel function (19) one finds, in ordinary units,

$$m_0^2=2m^2[1+\frac{m^2}{4\mu^2}(\ln\frac{4\mu^2}{m^2}+0.154)+\cdots], \quad (22)$$

$$\xi^{-1}=2[1+\frac{m^2}{4\mu^2}(\ln\frac{4\mu^2}{m^2}-0.846)+\cdots], \quad (23)$$

where

$$\mu\equiv\hbar/\lambda c=(\pi\hbar c/4G)^{1/2}=1.93\times10^{-5}\text{ g}\approx10^{19}\text{ BeV}. \quad (24)$$

For ordinary elementary particles the logarithmic terms are negligible, and we have $m^2=\frac{1}{2}m_0^2$, $\xi=\frac{1}{2}$.

In view of the restricted class of self-energy diagrams considered, these precise expressions are not, of course, to be taken seriously. However there is no reason to believe that the inclusion of other diagrams will qualitatively change the picture. In principle, the graviton ladder diagrams may be summed also for particles of spin $\frac{1}{2}$ and 1, although the complications of spin present serious computational obstacles in these cases. For spin 1 it is interesting to discover that the gradient terms in the particle propagator, which are responsible for the "bad" divergences which characterize vector theories, drop out when they are juxtaposed to graviton vertices. In this connection it is also worthy of note that functions like $Y(p)$, which have the exponentially damped or oscillatory high-energy behavior characteristic of Bessel functions, are very powerful regulators.

The ability of gravity to act as a regulator is not limited to gravitational self-energies alone. For example, a class of gravitational corrections to the lowest order electromagnetic self-energy of a charged particle is obtained by replacing a graviton line by a photon line in all possible ways in all of the ladder graphs. In the case of scalar particles it turns out that this gives a contribution to the self-energy function $\Sigma(p^2)$ which is of exactly the same form as the terms in m_0^2 in (13), but with m_0^2 replaced by $-4\pi\alpha$. The corresponding electromagnetic contribution to the self-mass is

$$\delta m^2=2\pi\alpha\mu^2+\text{logarithmic and other terms}. \quad (25)$$

The fact that this is by no means small is, of course, directly related to the well-known quadratic divergence of the self-energy in the absence of gravity. The question as to whether gravity can similarly serve as a regulator for vacuum polarization and vertex corrections, which lead to observable effects, is under current investigation.

We note finally that a self-energy function of the form (18) satisfies several important field-theoretical consistency criteria. First of all, it shows no tendency to lead to ghost states and negative probabilities; the solutions of Eqs. (21) are unique for most parameter values even when electromagnetic interactions are included and the bare mass is imaginary (negative m_0^2). In this respect the use of gravity as a regulator appears to differ from the use of any other physical field. Secondly, the form of the modified propagator Eq. (20) undergoes a smooth transition from $(p^2+m^2)^{-1}$ near the mass shell to approximately $S_0(p)$ far from the mass shell, showing that the singularity of the modified propagator in coordinate space is of the same strength as that of the bare propagator, in conformity with Lehmann's theorem.[11] Actually, this last property is a pure bonus, since one of the assumptions of the theorem, namely, the existence of an energy-momentum four-vector which generates infinitesimal displacements of local quantities, fails to hold in gravidynamics by reason of the general coordinate invariance of the theory and the resulting complete arbitrariness of the background coordinates.

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[10]The deDonder gauge of gravidynamics is the analog of the Lorentz gauge of electrodynamics. With any other gauge the graviton propagator is more singular, and gives rise to extra divergences in the theory. However, these are spurious divergences which cancel one another owing to gauge invariance. Just as in electrodynamics, the cancellation is an intricate one, involving all diagrams of a given order and not the ladder diagrams only. Moreover, as Feynman has pointed out (unpublished), in gravidynamics a previously unsuspected set of diagrams must be included, involving fictitious vector particles interacting with the gravitons only. [See B. S. DeWitt, Phys. Rev. Letters 12, 742 (1964).] Since the present investigation is restricted to the ladder graphs the results obtained are not gauge invariant. It is believed, however, that if the methods of references 2, 3, and 4 have any validity at all, their use in the present case in conjunction with the least divergent gauge leads to a modified scalar propagator [Eq. (20)] whose general analytical properties do not differ greatly from those of the correct propagator. As for justifying the restriction to ladder graphs, although it is true that when the rungs correspond to zero-mass quanta these graphs dominate all others at low energies, unfortunately we can say nothing about the high-energy regime which is important here. In this respect the present investigation suffers from the same defects as those of the cited references.

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SELF-CONSISTENCY OF HIGHER SYMMETRY UNIVERSES*

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According to the strongest version of the bootstrap conjecture of Chew and Frautschi,[1] it should be possible to rule out all strong interaction symmetries other than the observed one, $SU(2)\otimes Y$ (Y is hypercharge, baryon number plus strangeness) by requiring all particles to generate themselves self-consistently. Present knowledge of dynamics, however, still permits the construction of a wide range of higher symmetry universes which are self-consistent in first approximation.[2] About SU(2) dynamics it is known that pion-pion scattering and pion-nucleon scattering with production of ρ and $(\frac{3}{2},\frac{3}{2})$ N^* resonances, respectively, can be made self-consistent in first approximation.[3,4] Now suppose one replaces π, ρ, N, and N^* by multiplets Π, V, B, and B^*, still $J^P=0^-$, 1^-, $\frac{1}{2}^+$, and $\frac{3}{2}^+$, respectively, but now (except for B^*) transforming as the adjoint representation of a compact simple group G.[5] Simultaneously, in the scattering calculation, one replaces SU(2) crossing matrices by G crossing matrices. Then the $\Pi\Pi$ problem is still self-consistent, and, at least for $G=SU(n)$, the ΠB problem is also. Let $C_t(X,Y)$ [or $C_u(X,Y)$ denote the crossing matrix element giving the force exerted by crossed t (or u) channel Y on s channel X. Then for $\Pi\Pi$ scattering in which V bootstraps itself, the relevant elements are $C_t(V,V)$ and $C_u(V,V)$, which Cutkosky has shown are always attractive[6]:

$$C(V,V)=C(\rho,\rho)=\tfrac{1}{2}. \quad (1)$$

[If $C_t(X,Y)=C_u(X,Y)$, we drop the subscript.] Further,[6,7]

$$C(V',V)=0,\quad C_t(S,V)=+1=-C_u(S,V), \quad (2)$$

where V' is any other channel capable of supporting a vector-meson resonance and S is the scalar representation, i.e., the vacuum Regge trajectory. Hence the presence of crossed resonance