

# Floquet Superradiance Lattices in Thermal Atoms

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Floquet modulation has been widely used in optical lattices for coherent control of quantum gases, in particular for synthesizing artificial gauge fields and simulating topological matters. However, such modulation induces heating which can overwhelm the signal of quantum dynamics in ultracold atoms. Here we report that the thermal motion, instead of being a noise source, provides a new control knob in Floquet-modulated superradiance lattices, which are momentum-space tight-binding lattices of collectively excited states of atoms. The Doppler shifts combined with Floquet modulation provide effective forces along arbitrary directions in a lattice in frequency and momentum dimensions. Dynamic localization, dynamic delocalization, and chiral edge currents can be simultaneously observed from a single transport spectrum of superradiance lattices in thermal atoms. Our Letter paves a way for simulating Floquet topological matters in room-temperature atoms and facilitates their applications in photonic devices.

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Collective interaction between atoms and light plays an important role in quantum optics [1,2]. A most famous example is the Dicke superradiance featured by enhanced decay rates of collectively excited atoms [3], in particular for atoms confined in a small volume compared with the light wavelength. In the opposite limit, i.e., when the size of the atomic cloud is much larger than the light wavelength, Scully *et al.* [4,5] found that the light momentum can be stored in atoms prepared in the so-called timed Dicke states (TDSs) [6–8]. The stored momentum can be released in the directional superradiant emission [4]. More interestingly, such TDSs with different momenta can be coupled by multiple lasers to form momentum-space superradiance lattices (SLs) [9,10], which provide a new platform for quantum simulation.

In contrast to real-space optical lattices that require ultracold temperature, SLs can be implemented in both cold atoms [11–13] and room-temperature atoms [14–16], since TDSs are robust to the center-of-mass motion of atoms [17–20]. Such a robustness can be understood from a distinct feature of the momentum-space SL, i.e., its Brillouin zone (BZ) is in real space. Here the positions of atoms play the same role as the crystal momenta of electrons in solids. Atoms moving through the real-space BZs follow the same dynamics of electrons in a DC electric field. Therefore, the atomic motion, instead of being a source of noise, can simulate an effective electric field in momentum space, which has been used to measure the geometric Zak phase [16,21,22].

Here we show that SLs can also be used to study the rich physics in Floquet (temporally periodic) systems [23–27], where the thermal motion induced DC field significantly enriches the tunability. Being a widely used technique in quantum engineering [28–39], Floquet driving [40–43] can extend the 1D SLs into (1 + 1)D (momentum-frequency dimensional) lattices, allowing us to simulate higher-dimensional lattice dynamics [44–47]. In the (1 + 1)D SLs, there is a constant effective electric field in the frequency dimension, determined by the Floquet modulation frequency [48]. On the other hand, the atomic velocity provides a DC field along the momentum dimension with a strength proportional to the Doppler shift. By resolving the direction of the total field with a spectroscopic method, we observe dynamic localization, dynamic delocalization, and chiral edge currents [49–65] in SLs of thermal atoms.

*Experimental setup and theoretical model.*—The experiment is implemented in a vapor cell of natural abundance rubidium atoms in a standing-wave-coupled electromagnetically induced transparency configuration [see Fig. 1(a)]. A weak probe field couples the ground state  $|g\rangle \equiv |5^2S_{1/2}, F=1\rangle$  to an excited state  $|b\rangle \equiv |5^2P_{1/2}, F=2\rangle$  in the  $^{87}\text{Rb}$  D1 line. The excited state is resonantly coupled to a metastable state  $|a\rangle \equiv |5^2S_{1/2}, F=2\rangle$  by two strong laser fields. The probe field pumps the atoms to the TDS,

$$|b_{k_p}\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{ik_p \cdot r_j} |g_1 \dots b_j \dots g_N\rangle, \quad (1)$$

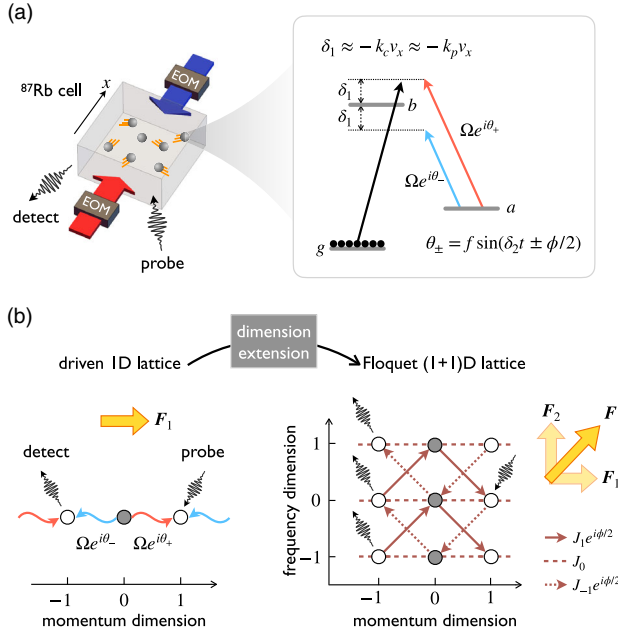


FIG. 1. (a) Schematic configuration of the experimental setup. The coupling fields are phase modulated by two EOMs at a RF frequency  $\delta_2 = 80$  MHz. Inset: the atomic levels in the reference frame of the atoms with velocity  $v_x$ . The coupling fields have opposite Doppler shifts  $\pm\delta_1$  and periodically modulated phases  $\theta_{\pm}(t)$ . (b) Mapping between the dynamically modulated 1D SL and the  $(1+1)$ D Floquet SL, with white and gray circles labeling the  $b$ - and  $a$ -sublattice sites, respectively. Left: the driven 1D momentum-space lattices with phase-modulated hopping strengths. The yellow arrow  $F_1$  indicates the direction of atomic motion induced DC electric force. Right: the equivalent  $(1+1)$ D frequency-momentum lattices. The yellow arrow  $F$  indicates the total DC electric force with components  $F_1$  and  $F_2$  along the momentum and frequency dimensions. The probed and detected sites are marked schematically.

where  $k_p$  is the wave vector of the probe field, and  $r_j$  is the position of the  $j$ th atom. The standing-wave-coupling fields couple  $|b_{k_p}\rangle$  to other TDSs with discrete momenta to form SLs [9], in which the directional superradiant emission from the TDS  $|b_{-k_p}\rangle$  is collected along  $-k_p$  direction [4–8], such that the lattice transport from  $|b_{k_p}\rangle$  to  $|b_{-k_p}\rangle$  can be investigated. We can tune the probe frequency to measure the transport at different energies, similar to tuning the Fermi energy of electrons in solids.

In our experiment on 1D SLs, the probe field propagates in the  $+x$  direction with a wave vector  $k_p$ , parallel to the two coupling laser fields with wave vectors  $\pm k_c$  and  $k_p \approx k_c$ . The phases of the coupling fields are modulated by two electro-optic modulators (EOMs) independently,  $\theta_{\pm}(t) = f \sin(\delta_2 t \pm \phi/2)$ , where  $f$  and  $\delta_2$  are the modulation depth and frequency, and  $\phi$  controls the phase delay between the two radio-frequency (RF) driving fields. The interaction between the atoms and the coupling fields is described by a Floquet driven SL [see Fig. 1(b)]. The

EOMs introduce periodic modulation of the coupling strengths in the SL Hamiltonian (we set  $\hbar = 1$ ; see details in the Supplemental Material [66]),

$$H_d = \sum_j \Omega a_{2j}^\dagger [e^{-i\theta_+(t)} b_{2j+1} + e^{-i\theta_-(t)} b_{2j-1}] + \text{H.c.}, \quad (2)$$

where  $\Omega$  is the Rabi frequency of the coupling fields,  $a_{2j}^\dagger$  and  $b_{2j+1}^\dagger$  are creation operators in the  $j$ th unit cell of the SL,  $a_{2j}^\dagger \equiv 1/\sqrt{N} \sum_l e^{2ijk_c x_l} |a_l\rangle \langle g_l|$ , and  $b_{2j+1}^\dagger \equiv 1/\sqrt{N} \sum_l e^{i(2j+1)k_c x_l} |b_l\rangle \langle g_l|$  with  $x_l$  being the position of the  $l$ th atom and  $N$  the total number of atoms. They create TDSs from the ground state, e.g.,  $a_{2j}^\dagger |g_1, g_2, \dots, g_N\rangle = 1/\sqrt{N} \sum_l \exp(2ijk_c x_l) |g_1 \dots a_l \dots g_N\rangle$ , which carries a momentum of  $2jk_c$ .

Using the Floquet theorem for  $H_d$ , we obtain an effective  $(1+1)$ D Hamiltonian  $V_2 + H_s$  [66], including a potential energy in the frequency dimension [74],

$$V_2 = -\delta_2 \sum_{j,m} m (a_{2j,m}^\dagger a_{2j,m} + b_{2j+1,m}^\dagger b_{2j+1,m}), \quad (3)$$

and the static lattice Hamiltonian  $H_s = \sum_n H_s^{[n]}$  with

$$H_s^{[n]} = \sum_{j,m} \Omega e^{in\phi/2} a_{2j,m}^\dagger [J_{-n}(f) b_{2j+1,m+n} + J_n(f) b_{2j-1,m-n}] + \text{H.c.}, \quad (4)$$

which couples lattice sites along the crystal direction  $\hat{e}_1 + n\hat{e}_2$  (denoted by Miller index  $[1, n]$ ) with  $\hat{e}_1$  and  $\hat{e}_2$  being the unit vectors along the momentum and frequency dimensions. Here  $J_n$  is the  $n$ th order Bessel function and  $d_{j,m}^\dagger$  ( $d = a, b$ ) is the creation operator for  $|d_{j,m}\rangle \equiv e^{-im\delta_2 t} |d_j\rangle$  [43,66]. We can understand  $|d_{j,m}\rangle$  as a replica of  $|d_j\rangle$  that carries  $-m\delta_2$  energy from the periodic driving [75]. The  $n$ th order hopping coefficient carries a phase  $\pm n\phi/2$ , which can synthesize a gauge field in the lattice [see Fig. 1(b)]. The linear potential along the frequency dimension  $V_2$  can be characterized by a force  $F_2 = \delta_2 \hat{e}_2$ . Throughout the experiment we set  $\delta_2 = 80$  MHz and  $\Omega = 25$  MHz to guarantee that the driving frequency is much larger than the hopping strengths.

For moving atoms, the photons in the two coupling fields along  $\pm x$  directions have extra energies due to the Doppler shifts  $\pm\delta_1$  with  $\delta_1 = -k_c v_x$  and  $v_x$  being the atomic velocity in  $x$  direction. Since the TDSs in the SL are created by absorbing and emitting photons in the coupling fields, the SL acquires a linear potential along the momentum dimension (in the lab reference frame),

$$V_1 = -\delta_1 \sum_{j,m} [2ja_{2j,m}^\dagger a_{2j,m} + (2j+1)b_{2j+1,m}^\dagger b_{2j+1,m}], \quad (5)$$

which has the effect of a force  $F_1 = \delta_1 \hat{e}_1$ . Consequently, the force associated to the total linear potential  $V_1 + V_2$

is  $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \delta_1 \hat{e}_1 + \delta_2 \hat{e}_2$ . In our experiment, the hopping of excitations along the force direction is inhibited because the potential difference is much larger than the coupling strengths.

**Lattice transport measurement.**—We can tune the probe frequency to measure the transport in SLs with different values of  $\mathbf{F}$ . The interaction Hamiltonian between the probe field and the atoms is  $H_p = \sqrt{N}\Omega_p e^{-i\Delta_p t} b_{1,0}^\dagger + \text{H.c.}$ , where  $\Delta_p = \nu_p - \omega_{bg}$  with  $\nu_p$  being the probe field frequency and  $\omega_{bg}$  being the atomic transition frequency in the lab reference frame. The probe field couples the ground state to a TDS  $|b_{1,0}\rangle$ , which has a potential energy  $-\delta_1$  proportional to the atomic velocity, such that we need to detune the probe field  $\Delta_p \approx -\delta_1$  to excite the TDSs of atoms with the corresponding velocities. Since  $\mathbf{F}_1$  is also proportional to the atomic velocity, we can measure lattice dynamics with different  $\mathbf{F}_1$  at the corresponding probe detunings. In other words, with the probe detuning  $\Delta_p$ , we measure the transport from  $|b_{1,0}\rangle$  to  $|b_{-1,m}\rangle$  in an effective force  $\mathbf{F} = -\Delta_p \hat{e}_1 + \delta_2 \hat{e}_2$ . Such transport is experimentally measured from the superradiant directional emission of  $|b_{-1,m}\rangle$  along the  $-x$  direction, which satisfies the phase-matching condition  $-2k_c + k_p \approx -k_p$  [66]. Therefore, we can simultaneously measure the dynamics with different effective forces in a single transport spectrum by detuning the probe field frequency. The coherent probe field may also create more than one excitation which, however, have the same dynamic response as the TDSs due to the bosonic nature of atomic excitations in a weak probe field [9,14].

We show a typical spectrum for the transport from  $|b_{1,0}\rangle$  to  $|b_{-1,m}\rangle$  in Fig. 2. Because of the Stark localization, the transport is only significant when the two states have the same potential energy, i.e., when the force points in a direction perpendicular to a crystal direction of the  $(1+1)$ D SL. As a result, the reflection is featured by discrete peaks at  $\Delta_p = n\delta_2$ . The reflection peak at  $\Delta_p = n\delta_2$  is contributed by the transport from  $|b_{1,0}\rangle$  to  $|b_{-1,-2n}\rangle$  along the crystal direction  $[1, n]$  (dotted lines in the insets), perpendicular to  $\mathbf{F}$ . From the reflection spectra we are able to investigate the dynamic localization and delocalization by simultaneously tuning the modulation depths of the two EOMs while keeping their phase delay  $\phi = \pi$  fixed. We can also tune  $\phi$  to introduce effective magnetic fluxes in the  $(1+1)$ D SLs to investigate the Floquet chiral edge currents.

**Dynamic localization and delocalization.**—We measure the reflection spectra as a function of the probe detuning  $\Delta_p$  and the modulation depth  $f$  [see Fig. 3(a)]. The transport along the crystal direction  $[1, n]$  is demonstrated near the detuning  $\Delta_p = n\delta_2$  and governed by the interaction Hamiltonian  $H_s^{[n]}$  in Eq. (4). Remarkably, the transport is totally inhibited when the modulation depth  $f$  reaches to the zeros of the corresponding Bessel functions  $J_n(f)$  [marked by black arrows in Fig. 3(b)], where the effective

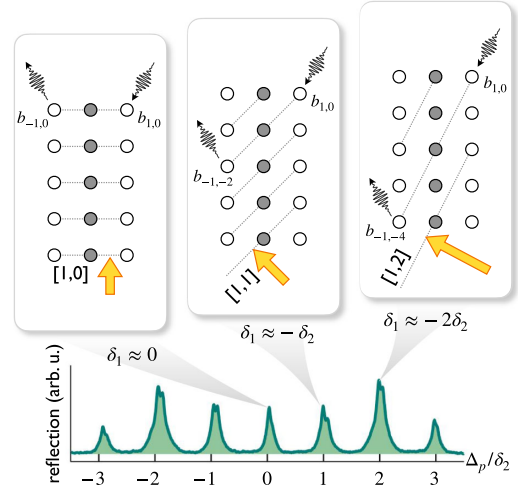


FIG. 2. Correspondence between peaks at  $\Delta_p = n\delta_2$  in the reflection spectrum and the dynamics along the crystal direction  $[1, n]$  (dashed lines). The  $n$ th peak is contributed by the atoms in the velocity group with a Doppler shift  $\delta_1 = -n\delta_2$ . The effective force  $\mathbf{F} = -\Delta_p \hat{e}_1 + \delta_2 \hat{e}_2$  makes the dynamics of  $(1+1)$ D lattices dominated by transitions along the crystal direction  $[1, n]$ . The reflection signal shows the transport from the probed state  $|b_{1,0}\rangle$  to the detected site  $|b_{-1,-2n}\rangle$ . The phase delay  $\phi = \pi$  and modulation depth  $f = 3$ .

hopping strength is zero in  $H_s^{[n]}$ . In particular, at the detuning  $\Delta_p = 0$ , the reflectivity vanishes at  $f = 2.4, 5.5$ , and  $8.7$ , which are the first three zeros of  $J_0(f)$ , characterizing the dynamic localization [49,50]. For parameters in our experiment, the localization at zeros of  $J_n(f)$  can be clearly identified up to  $|n| = 4$ , a significant improvement compared with other platforms [51,52].

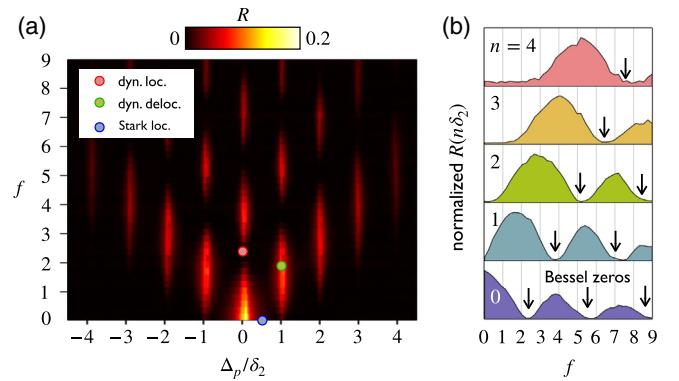


FIG. 3. Dynamic localization and delocalization. (a) Experimental reflection spectra as a function of the probe detuning  $\Delta_p$  and the modulation depth  $f$ . We denote the points for Stark localization, dynamic localization, and dynamic delocalization with dots in blue, red, and green colors. (b) The reflections at  $\Delta_p = n\delta_2$  characterizing transport along crystal directions  $[1, n]$  ( $n = 0, 1, 2, 3, 4$ ) are measured as functions of  $f$ , where the transport is completely inhibited at the zero points (marked by black arrow) of  $J_n(f)$ . We set the phase delay  $\phi = \pi$ .

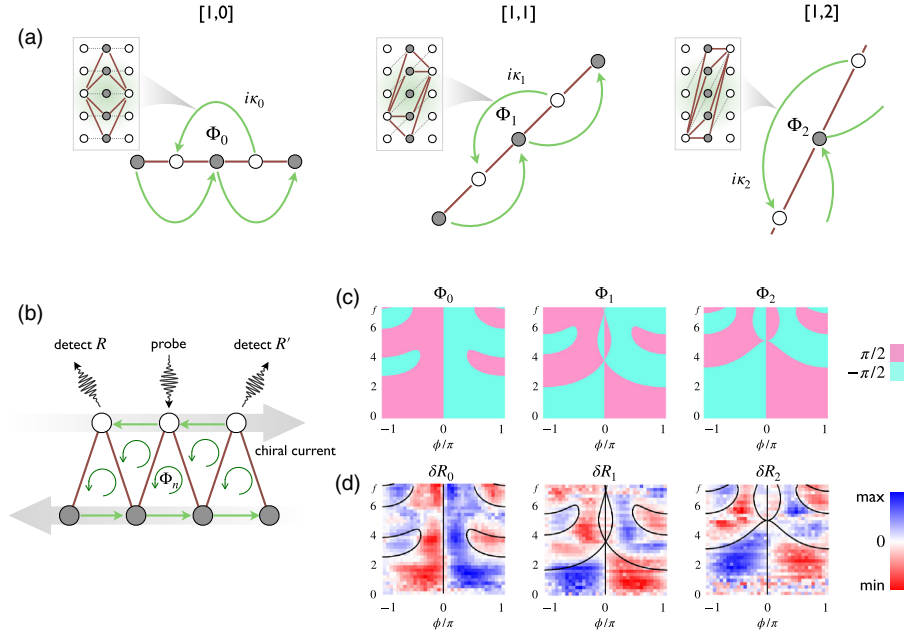


FIG. 4. Floquet chiral currents. (a) The schematics of the effective Hamiltonian along a composite dimension of momentum and frequency. The NNN hoppings (green lines with arrows indicating the transition directions attached with a phase  $\pi/2$ ) along the crystal direction  $[1, n]$  ( $n = 0, 1, 2$ ) are induced by all possible second-order transition paths (insets). (b) The zigzag chiral ladder along  $[1, n]$  [replotted from the reduced 1D lattices in (a)] with an effective magnetic flux  $\Phi_n$  in a unit loop. The two sublattices are the two edges which host chiral edge currents with directions (along thick arrows) determined by  $\Phi_n$ . The green arrows denote the transition directions attached with a phase  $\pi/2$ . (c) Phase diagram of  $\Phi_n$  as functions of  $\phi$  and  $f$ . (d) The phase diagram of chiral currents measured by  $\delta R_n$  which are consistent with the corresponding phase diagram of  $\Phi_n$  with phase boundaries indicated by solid lines.

The order  $n$  is limited by the width of the Doppler broadening, which has a FWHM of 500 MHz at the experiment's temperature of 60 °C.

The measured transport can be understood as the interplay between the lattice dynamics in the AC and DC field [49–51, 53–60] in a 1D SL. The oscillating phases  $\theta_{\pm}(t)$  introduce a uniform AC field  $(\partial\theta_-/\partial t - \partial\theta_+/\partial t)/2 = f\delta_2 \sin(\delta_2 t)$  [43]. We observe three important effects in the presence of both the AC and DC field as being marked in Fig. 3(a). Without AC and DC fields, i.e., for  $\delta_1 = f = 0$  near  $\Delta_p = 0$ , the 1D SL transport signal is strong from  $|b_1\rangle$  to  $|b_{-1}\rangle$ . When we keep  $f = 0$  and detuning  $\Delta_p$  away from zero, the signal disappears due to Stark localization in a DC field (see the blue point at  $\Delta_p = \delta_2/2$  and  $f = 0$ ). When we keep  $\Delta_p = 0$  and increase  $f$ , an effective AC field leads to the dynamic localization at  $f = 2.4$  (see the red dot) [49, 50, 53–56]. With both the AC and DC fields, the transport is recovered when the modulation frequency of the AC field is on resonance with the Bloch frequency of the DC field (see the green dot for  $\Delta_p = \delta_2$ ), which is called dynamic delocalization or photon-assisted tunneling [51, 57–60].

**Phase diagram of chiral currents.**—Chiral edge currents have been observed in zigzag SLs [14]. Floquet engineering can bring new control knobs to induce such unidirectional transport [12, 61–65]. Near  $\Delta_p = n\delta_2$ , the

Hamiltonian  $H_s^{[n]}$  introduces equipotential transitions while  $H_s^{[m]}$ 's with  $m \neq n$  provide second-order transitions via intermediate states [see green lines in Fig. 4(a) and derivation in the Supplemental Material [66]],

$$K^{[n]} = \sum_{j,m} i\kappa_n (b_{2j-1,m-n}^\dagger b_{2j+1,m+n} - a_{2j,m}^\dagger a_{2j+2,m-2n}) + \text{H.c.}, \quad (6)$$

with  $\kappa_n = -\sum_{p=1}^{\infty} 2 \sin(p\phi) \Omega^2 J_{p+n}(f) J_{p-n}(f) / p\delta_2$  [66]. When  $\phi = \pi$ , we obtain  $\kappa_n = 0$  for dynamic localization and delocalization. In general, for  $\phi \neq 0$  or  $\pi$ ,  $H_s^{[n]}$  and  $K^{[n]}$  induce the nearest-neighbor and next-nearest-neighbor (NNN) hoppings [see red lines and green arrows in Figs. 4(a) and 4(b)]. The phase factors in  $K^{[n]}$  bring a magnetic flux in a unit closed loop [circular arrows in Fig. 4(b)] [14, 76],

$$\Phi_n = \frac{\pi}{2} (-1)^n \text{sgn}[\kappa_n], \quad (7)$$

which depends on both  $f$  and  $\phi$  and exhibits rich structures along different crystal directions [see Fig. 4(c)].

The direction of the synthesized magnetic field, i.e., the sign of  $\Phi_n$ , determines the unidirectional transport along



the  $b$  sublattice, regardless of the strength  $\kappa_n$  [14]. The chiral current flows to the right and left for  $\Phi_n = \pi/2$  and  $-\pi/2$ , which results in an asymmetric distribution of steady-state probabilities in the SL with respect to the probed site. Such an asymmetry can be measured by comparing the reflection signals of two probe fields [14,77] in opposite directions and with opposite detunings. We define  $R$  and  $R'$  as the reflectivities of the probe field incident along  $+x$  and  $-x$  directions.  $R(n\delta_2)$  characterizes the transport from the site  $|b_{1,0}\rangle$  to  $|b_{-1,-2n}\rangle$  while  $R'(-n\delta_2)$  describes the transport from the site  $|b_{-1,0}\rangle$  to  $|b_{1,2n}\rangle$  [66]. The chiral current along the crystal direction  $[1, n]$  is measured by  $\delta R_n \equiv R(n\delta_2) - R'(-n\delta_2)$  [see Fig. 4(d)], which agrees with the phase diagrams in Fig. 4(c).

In conclusion, we demonstrate that the SL provides a highly tunable platform to study the Floquet dynamics in thermal atoms. We simultaneously observe dynamic localization and delocalization from a single transport spectrum. We also experimentally measure the phase diagrams of chiral edge currents in a  $(1+1)$ D lattice with both artificial magnetic fluxes and electric fields. We can increase hopping strengths between TDSs to overwhelm the linear potentials, such that the motion of excitations is governed by the underlying 2D Bloch bands with the effective electric force being a perturbation. In such a regime topological nonequilibrium phenomena such as quantized energy pumping [37,38,44–47] and space-time crystals [75,78–81] can be investigated. Furthermore, we can introduce interaction in SLs by using additional lasers to couple atoms to the Rydberg states [82] to realize many-body phenomena beyond the short-range interaction [66], such as the interplay between long-range nonlocal interaction and dynamic localization-delocalization transition [83,84].

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- [1] M. Gross and S. Haroche, Superradiance: An essay on the theory of collective spontaneous emission, *Phys. Rep.* **93**, 301 (1982).

- [2] S. Inouye, A. P. Chikkatur, D. M. Stamper-Kurn, J. Stenger, D. E. Pritchard, and W. Ketterle, Superradiant rayleigh scattering from a Bose-Einstein condensate, *Science* **285**, 571 (1999).
- [3] R. H. Dicke, Coherence in spontaneous radiation processes, *Phys. Rev.* **93**, 99 (1954).
- [4] M. O. Scully, E. S. Fry, C. H. Ooi, and K. Wódkiewicz, Directed Spontaneous Emission from an Extended Ensemble of  $N$  Atoms: Timing is Everything, *Phys. Rev. Lett.* **96**, 010501 (2006).
- [5] M. O. Scully and A. A. Svidzinsky, The super of superradiance, *Science* **325**, 1510 (2009).
- [6] M. O. Araújo, I. Krešić, R. Kaiser, and W. Guerin, Superradiance in a Large Cloud of Cold Atoms in the Linear-Optics Regime, *Phys. Rev. Lett.* **117**, 073002 (2016).
- [7] S. J. Roof, K. J. Kemp, M. D. Havey, and I. M. Sokolov, Observation of Single-Photon Superradiance and the Cooperative Lamb Shift in an Extended Sample of Cold Atoms, *Phys. Rev. Lett.* **117**, 073003 (2016).
- [8] Y. He, L. Ji, Y. Wang, L. Qiu, J. Zhao, Y. Ma, X. Huang, S. Wu, and D. E. Chang, Geometric Control of Collective Spontaneous Emission, *Phys. Rev. Lett.* **125**, 213602 (2020).
- [9] D. W. Wang, R. B. Liu, S. Y. Zhu, and M. O. Scully, Superradiance Lattice, *Phys. Rev. Lett.* **114**, 043602 (2015).
- [10] D. W. Wang, H. Cai, L. Yuan, S. Y. Zhu, and R. B. Liu, Topological phase transitions in superradiance lattices, *Optica* **2**, 712 (2015).
- [11] L. Chen, P. Wang, Z. Meng, L. Huang, H. Cai, D. W. Wang, S. Y. Zhu, and J. Zhang, Experimental Observation of One-Dimensional Superradiance Lattices in Ultracold Atoms, *Phys. Rev. Lett.* **120**, 193601 (2018).
- [12] P. Wang, L. Chen, C. Mi, Z. Meng, L. Huang, S. N. Khan, H. Cai, D. W. Wang, S. Y. Zhu, and J. Zhang, Synthesized magnetic field of a sawtooth superradiance lattice in Bose-Einstein condensates, *npj Quantum Inf.* **6**, 18 (2020).
- [13] C. Mi, K. S. Nawaz, L. Chen, P. Wang, H. Cai, D. W. Wang, S. Y. Zhu, and J. Zhang, Time-resolved interplay between superradiant and subradiant states in superradiance lattices of Bose-Einstein condensates, *Phys. Rev. A* **104**, 043326 (2021).
- [14] H. Cai, J. Liu, J. Wu, Y. He, S. Y. Zhu, J. X. Zhang, and D. W. Wang, Experimental Observation of Momentum-Space Chiral Edge Currents in Room-Temperature Atoms, *Phys. Rev. Lett.* **122**, 023601 (2019).
- [15] Y. He, R. Mao, H. Cai, J. X. Zhang, Y. Li, L. Yuan, S. Y. Zhu, and D. W. Wang, Flat-Band Localization in Creutz Superradiance Lattices, *Phys. Rev. Lett.* **126**, 103601 (2021).
- [16] R. Mao, X. Xu, J. Wang, C. Xu, G. Qian, H. Cai, S. Y. Zhu, and D. W. Wang, Measuring Zak phase in room-temperature atoms, *Light Sci. Appl.* **11**, 291 (2022).
- [17] G. L. Celardo, G. G. Giusteri, and F. Borgonovi, Cooperative robustness to static disorder: Superradiance and localization in a nanoscale ring to model light-harvesting systems found in nature, *Phys. Rev. B* **90**, 075113 (2014).
- [18] F. Damanet, D. Braun, and J. Martin, Cooperative spontaneous emission from indistinguishable atoms in arbitrary motional quantum states, *Phys. Rev. A* **94**, 033838 (2016).

- [19] S. L. Bromley, B. Zhu, M. Bishof, X. Zhang, T. Bothwell, J. Schachenmayer, T. L. Nicholson, R. Kaiser, S. F. Yelin, M. D. Lukin, A. M. Rey, and J. Ye, Collective atomic scattering and motional effects in a dense coherent medium, *Nat. Commun.* **7**, 11039 (2016).
- [20] P. Weiss, A. Cipris, M. O. Araújo, R. Kaiser, and W. Guerin, Robustness of Dicke subradiance against thermal decoherence, *Phys. Rev. A* **100**, 033833 (2019).
- [21] J. Zak, Berry's Phase for Energy Bands in Solids, *Phys. Rev. Lett.* **62**, 2747 (1989).
- [22] D. Xiao, M. C. Chang, and Q. Niu, Berry phase effects on electronic properties, *Rev. Mod. Phys.* **82**, 1959 (2010).
- [23] A. Eckardt, Colloquium: Atomic quantum gases in periodically driven optical lattices, *Rev. Mod. Phys.* **89**, 011004 (2017).
- [24] T. Oka and S. Kitamura, Floquet engineering of quantum materials, *Annu. Rev. Condens. Matter Phys.* **10**, 387 (2019).
- [25] M. S. Rudner and N. H. Lindner, Band structure engineering and non-equilibrium dynamics in Floquet topological insulators, *Nat. Rev. Phys.* **2**, 229 (2020).
- [26] C. Weitenberg and J. Simonet, Tailoring quantum gases by Floquet engineering, *Nat. Phys.* **17**, 1342 (2021).
- [27] A. de la Torre, D. M. Kennes, M. Claassen, S. Gerber, J. W. McIver, and M. A. Sentef, Colloquium: Nonthermal pathways to ultrafast control in quantum materials, *Rev. Mod. Phys.* **93**, 041002 (2021).
- [28] T. Oka and H. Aoki, Photovoltaic hall effect in graphene, *Phys. Rev. B* **79**, 081406(R) (2009).
- [29] N. H. Lindner, G. Refael, and V. Galitski, Floquet topological insulator in semiconductor quantum wells, *Nat. Phys.* **7**, 490 (2011).
- [30] G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat, T. Uehlinger, D. Greif, and T. Esslinger, Experimental realization of the topological Haldane model with ultracold fermions, *Nature (London)* **515**, 237 (2014).
- [31] M. C. Rechtsman, J. M. Zeuner, Y. Plotnik, Y. Lumer, D. Podolsky, F. Dreisow, S. Nolte, M. Segev, and A. Szameit, Photonic floquet topological insulators, *Nature (London)* **496**, 196 (2013).
- [32] M. S. Rudner, N. H. Lindner, E. Berg, and M. Levin, Anomalous Edge States and the Bulk-Edge Correspondence for Periodically Driven Two-Dimensional Systems, *Phys. Rev. X* **3**, 031005 (2013).
- [33] S. Mukherjee, A. Spracklen, M. Valiente, E. Andersson, P. Öhberg, N. Goldman, and R. R. Thomson, Experimental observation of anomalous topological edge modes in a slowly driven photonic lattice, *Nat. Commun.* **8**, 13918 (2017).
- [34] Q. Cheng, Y. Pan, H. Wang, C. Zhang, D. Yu, A. Gover, H. Zhang, T. Li, L. Zhou, and S. Zhu, Observation of Anomalous  $\pi$  Modes in Photonic Floquet Engineering, *Phys. Rev. Lett.* **122**, 173901 (2019).
- [35] L. Jiang, T. Kitagawa, J. Alicea, A. R. Akhmerov, D. Pekker, G. Refael, J. I. Cirac, E. Demler, M. D. Lukin, and P. Zoller, Majorana Fermions in Equilibrium and in Driven Cold-Atom Quantum Wires, *Phys. Rev. Lett.* **106**, 220402 (2011).
- [36] P. Titum, E. Berg, M. S. Rudner, G. Refael, and N. H. Lindner, Anomalous Floquet-Anderson Insulator as a Nonadiabatic Quantized Charge Pump, *Phys. Rev. X* **6**, 021013 (2016).
- [37] M. H. Kolodrubetz, F. Nathan, S. Gazit, T. Morimoto, and J. E. Moore, Topological Floquet-Thouless Energy Pump, *Phys. Rev. Lett.* **120**, 150601 (2018).
- [38] F. Nathan, R. Ge, S. Gazit, M. Rudner, and M. Kolodrubetz, Quasiperiodic Floquet-Thouless Energy Pump, *Phys. Rev. Lett.* **127**, 166804 (2021).
- [39] G. Li, L. Wang, R. Ye, S. Liu, Y. Zheng, L. Yuan, and X. Chen, Observation of flat-band and band transition in the synthetic space, *Adv. Opt. Photonics* **4**, 036002 (2022).
- [40] G. Floquet, On linear differential equations with periodic coefficients, *Ann. Sci. Ec. Norm. Super. Ser.* **12**, 47 (1883).
- [41] J. H. Shirley, Solution of the Schrödinger Equation with a Hamiltonian Periodic in Time, *Phys. Rev.* **138**, B979 (1965).
- [42] N. Goldman and J. Dalibard, Periodically Driven Quantum Systems: Effective Hamiltonians and Engineered Gauge Fields, *Phys. Rev. X* **4**, 031027 (2014).
- [43] A. Gómez-León and G. Platero, Floquet-Bloch Theory and Topology in Periodically Driven Lattices, *Phys. Rev. Lett.* **110**, 200403 (2013).
- [44] I. Martin, G. Refael, and B. Halperin, Topological Frequency Conversion in Strongly Driven Quantum Systems, *Phys. Rev. X* **7**, 041008 (2017).
- [45] E. Boyers, P. J. D. Crowley, A. Chandran, and A. Sushkov, Exploring 2D Synthetic Quantum Hall Physics with a Quasiperiodically Driven Qubit, *Phys. Rev. Lett.* **125**, 160505 (2020).
- [46] D. M. Long, P. J. D. Crowley, and A. Chandran, Nonadiabatic Topological Energy Pumps with Quasiperiodic Driving, *Phys. Rev. Lett.* **126**, 106805 (2021).
- [47] D. Malz and A. Smith, Topological Two-Dimensional Floquet Lattice on a Single Superconducting Qubit, *Phys. Rev. Lett.* **126**, 163602 (2021).
- [48] A. Dutt, Q. Lin, L. Yuan, M. Minkov, M. Xiao, and S. Fan, A single photonic cavity with two independent physical synthetic dimensions, *Science* **367**, 59 (2020).
- [49] D. Dunlap and V. Kenkre, Dynamic localization of a charged particle moving under the influence of an electric field, *Phys. Rev. B* **34**, 3625 (1986).
- [50] M. Holthaus, Collapse of Minibands in Far-Infrared Irradiated Superlattices, *Phys. Rev. Lett.* **69**, 351 (1992).
- [51] C. Sias, H. Lignier, Y. P. Singh, A. Zenesini, D. Ciampini, O. Morsch, and E. Arimondo, Observation of Photon-Assisted Tunneling in Optical Lattices, *Phys. Rev. Lett.* **100**, 040404 (2008).
- [52] A. Szameit, I. L. Garanovich, M. Heinrich, A. A. Sukhorukkov, F. Dreisow, T. Pertsch, S. Nolte, A. Tünnermann, and Y. S. Kivshar, Polychromatic dynamic localization in curved photonic lattices, *Nat. Phys.* **5**, 271 (2009).
- [53] K. Madison, M. Fischer, R. Diener, Q. Niu, and M. G. Raizen, Dynamical Bloch Band Suppression in An Optical Lattice, *Phys. Rev. Lett.* **81**, 5093 (1998).
- [54] S. Longhi, M. Marangoni, M. Lobino, R. Ramponi, P. Laporta, E. Cianci, and V. Foglietti, Observation of Dynamic Localization in Periodically Curved Waveguide Arrays, *Phys. Rev. Lett.* **96**, 243901 (2006).

- [55] H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo, Dynamical Control of Matter-Wave Tunneling in Periodic Potentials, *Phys. Rev. Lett.* **99**, 220403 (2007).
- [56] A. Szameit, I. L. Garanovich, M. Heinrich, A. A. Sukhorukov, F. Dreisow, T. Pertsch, S. Nolte, A. Tünnermann, S. Longhi, and Y. S. Kivshar, Observation of Two-Dimensional Dynamic Localization of Light, *Phys. Rev. Lett.* **104**, 223903 (2010).
- [57] E. Haller, R. Hart, M. J. Mark, J. G. Danzl, L. R. Illner, and H. Nägerl, Inducing Transport in a Dissipation-Free Lattice with Super Bloch Oscillations, *Phys. Rev. Lett.* **104**, 200403 (2010).
- [58] R. Ma, M. E. Tai, P. M. Preiss, W. S. Bkar, J. Simon, and M. Greiner, Photon-Assisted Tunneling in a Biased Strongly Correlated Bose Gas, *Phys. Rev. Lett.* **107**, 095301 (2011).
- [59] Y. A. Chen, S. Nascimbène, M. Aidelsburger, M. Atala, S. Trotzky, and I. Bloch, Controlling Correlated Tunneling and Superexchange Interactions with ac-Driven Optical Lattices, *Phys. Rev. Lett.* **107**, 210405 (2011).
- [60] L. Yuan and S. Fan, Three-Dimensional Dynamic Localization of Light from a Time-Dependent Effective Gauge Field for Photons, *Phys. Rev. Lett.* **114**, 243901 (2015).
- [61] M. Mancini, G. Pagano, G. Cappellini, L. Livi, M. Rider, J. Catani, C. Sias, P. Zoller, M. Inguscio, M. Dalmonte, and L. Fallani, Observation of chiral edge states with neutral fermions in synthetic Hall ribbons, *Science* **349**, 1510 (2015).
- [62] B. K. Stuhl, H. I. Lu, L. M. Ayccock, D. Genkina, and I. B. Spielman, Visualizing edge states with an atomic Bose gas in the quantum Hall regime, *Science* **349**, 1514 (2015).
- [63] L. F. Livi, G. Cappellini, M. Diem, L. Franchi, C. Clivati, M. Frittelli, F. Levi, D. Calonico, J. Catani, M. Inguscio, and L. Fallani, Synthetic Dimensions and Spin-Orbit Coupling with an Optical Clock Transition, *Phys. Rev. Lett.* **117**, 220401 (2016).
- [64] M. Atala, M. Aidelsburger, M. Lohse, J. T. Barreiro, B. Paredes, and I. Bloch, Observation of chiral currents with ultracold atoms in bosonic ladders, *Nat. Phys.* **10**, 588 (2014).
- [65] F. A. An, E. J. Meier, and B. Gadway, Direct observation of chiral currents and magnetic reflection in atomic flux lattices, *Sci. Adv.* **3**, e1602685 (2017).
- [66] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.129.273603> for the experimental setup, the derivation of the Hamiltonian and the proposal on introducing interactions. It includes Refs. [67–73].
- [67] H. Sambe, Steady states and quasienergies of a quantum-mechanical system in an oscillating field, *Phys. Rev. A* **7**, 2203 (1973).
- [68] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley-VCH Verlag GmbH, Weinheim, Germany, 1998).
- [69] F. Ripka, H. Kübler, R. Löw, and T. Pfau, A room-temperature single-photon source based on strongly interacting Rydberg atoms, *Science* **362**, 446 (2018).
- [70] D. S. Ding, Z. K. Liu, B. S. Shi, G. C. Guo, K. Mølmer, and C. S. Adams, Enhanced metrology at the critical point of a many-body Rydberg atomic system, *Nat. Phys.* **18**, 1447 (2022).
- [71] N. Henkel, R. Nath, and T. Pohl, Three-Dimensional Roton Excitations and Supersolid Formation in Rydberg-Excited Bose-Einstein Condensates, *Phys. Rev. Lett.* **104**, 195302 (2010).
- [72] G. Pupillo, A. Micheli, M. Boninsegni, I. Lesanovsky, and P. Zoller, Strongly Correlated Gases of Rydberg-Dressed Atoms: Quantum and Classical Dynamics, *Phys. Rev. Lett.* **104**, 223002 (2010).
- [73] J. Honer, H. Weimer, T. Pfau, and H. P. Büchler, Collective Many-Body Interaction in Rydberg Dressed Atoms, *Phys. Rev. Lett.* **105**, 160404 (2010).
- [74] D. Yu, B. Peng, X. Chen, X.-J. Liu, and L. Yuan, Topological holographic quench dynamics in a synthetic frequency dimension, *Light Sci. Appl.* **10**, 209 (2021).
- [75] Q. Gao and Q. Niu, Floquet-Bloch Oscillations and Intra-band Zener Tunneling in an Oblique Spacetime Crystal, *Phys. Rev. Lett.* **127**, 036401 (2021).
- [76] D. Hügél and B. Paredes, Chiral ladders and the edges of quantum Hall insulators, *Phys. Rev. A* **89**, 023619 (2014).
- [77] Y. He, J. Wu, Y. Hu, J. X. Zhang, and S. Y. Zhu, Unidirectional reflectionless anti-parity-symmetric photonic lattices of thermal atoms, *Phys. Rev. A* **105**, 043712 (2022).
- [78] T. Morimoto, H. C. Po, and A. Vishwanath, Floquet topological phases protected by time glide symmetry, *Phys. Rev. B* **95**, 195155 (2017).
- [79] S. Xu and C. Wu, Space-Time Crystal and Space-Time Group, *Phys. Rev. Lett.* **120**, 096401 (2018).
- [80] Y. Peng and G. Refael, Floquet Second-Order Topological Insulators from Nonsymmorphic Space-Time Symmetries, *Phys. Rev. Lett.* **123**, 016806 (2019).
- [81] Y. Peng, Topological Space-Time Crystal, *Phys. Rev. Lett.* **128**, 186802 (2022).
- [82] Y. Li, H. Cai, D. W. Wang, L. Li, J. Yuan, and W. Li, Many-Body Chiral Edge Currents and Sliding Phases of Atomic Spin Waves in Momentum-Space Lattice, *Phys. Rev. Lett.* **124**, 140401 (2020).
- [83] A. Cao, R. Sajjad, H. Mas, E. Q. Simmons, J. L. Tanlimco, E. Nolasco-Martinez, T. Shimasaki, H. Esat Kondakci, V. Galitski, and D. M. Weld, Interaction-driven breakdown of dynamical localization in a kicked quantum gas, *Nat. Phys.* **18**, 1302 (2022).
- [84] J. H. S. Toh, K. C. McCormick, X. Tang, Y. Su, X. W. Luo, C. Zhang, and S. Gupta, Many-body dynamical delocalization in a kicked one-dimensional ultracold gas, *Nat. Phys.* **18**, 1297 (2022).