Minimally Dissipative Information Erasure in a Quantum Dot via Thermodynamic Length

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In this Letter, we explore the use of thermodynamic length to improve the performance of experimental protocols. In particular, we implement Landauer erasure on a driven electron level in a semiconductor quantum dot, and compare the standard protocol in which the energy is increased linearly in time with the one coming from geometric optimization. The latter is obtained by choosing a suitable metric structure, whose geodesics correspond to optimal finite-time thermodynamic protocols in the slow driving regime. We show experimentally that geodesic drivings minimize dissipation for slow protocols, with a bigger improvement as one approaches perfect erasure. Moreover, the geometric approach also leads to smaller dissipation even when the time of the protocol becomes comparable with the equilibration timescale of the system, i.e., away from the slow driving regime. Our results also illustrate, in a single-electron device, a fundamental principle of thermodynamic geometry: optimal finite-time thermodynamic protocols are those with constant dissipation rate along the process.

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Landauer erasure represents one of the most paradigmatic protocols in stochastic and quantum thermodynamics. Its relevance is not only historical, as it was the first case in which a strong argument for the physicality of information was made, but also conceptual, as it shows how logical irreversibility inevitably leads to dissipation, and practical, as it imposes a fundamental bound on the minimal heat released by an operating computer with finite memory. In particular, Landauer's limit establishes that the minimal amount of heat dissipated in order to erase a bit is bounded by [1]

$$\Delta Q \geq -k_B T \Delta S, \tag{1}$$

where T is the temperature of the bath and ΔS is the difference in entropy between the final and the initial state, which turns out to be negative for erasing protocols.

Equality in Eq. (1) corresponds to an ideal isothermal process. This can only be realized in infinite time, which makes Landauer's limit *de facto* unattainable in practice. Nevertheless, it is a crucial task to minimize dissipation (i.e., ΔQ) in information-processing devices, and much experimental effort has been devoted to approach the Landauer's limit [2]. Experimental demonstrations of (almost-perfect) Landauer erasure have been reported in different platforms, including colloidal particles [3–6], nanomagnets [7–9], superconducting flux logic cells [10], underdamped micromechanical oscillators [11,12], and optomechanical systems [13] (see also related works in quantum systems such as nuclear magnetic resonance setups [14] and ion traps [15]).

Despite how well studied this problem is, in all the experimental works above the driving chosen in order to induce the erasure is linear in time. We show here that this is suboptimal, which is in agreement with previous theoretical works [16–26]. In particular, we study how to exploit the concept of thermodynamic length [19,27–36] to devise better erasing protocols in finite time. This quantity arises from the expansion of the entropy production for protocols that are performed in a long, but finite time. In this regime, optimal protocols are governed by the principle of constant dissipation rate, meaning that the optimal protocol is the one that allocates the dissipation in the most uniform way [35,37–39]. The corresponding thermodynamic metric also gives a prescription to find minimally dissipating drivings: in fact, the geodesics associated with this metric realize optimal protocols in the slow driving regime [19,27–29,33].

We experimentally demonstrate how this geometric approach can be exploited to minimize dissipation in a Landauer erasure protocol. Our device is based on a semiconductor quantum dot which allows for the manipulation of discrete energy levels, see Fig. 1. We study both the regime of slow driving, for which we demonstrate the

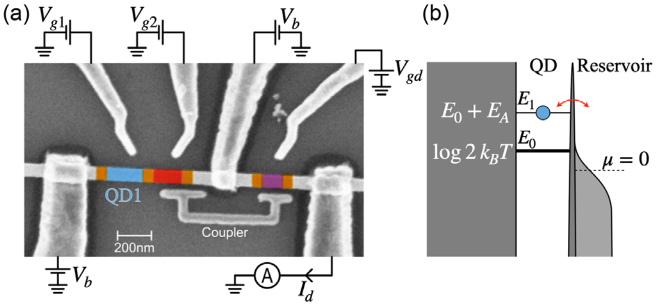

FIG. 1. (a) Scanning electron microscope image of the nanowire device. Embedded in the nanowire are three QDs, each aligned to one of the plunger gates V_{g1},V_{g2}, or V_{gd}. Contacts separate the device into one part with two QDs and one part with a single QD. The coupler couples the two systems together, allowing the current I_d through the lone QD to provide a measure of the charge state of the other system. Here, the QD involved in the experiment is marked in blue (close to the plunger gate with V_{g1}) while the quantum dot marked in red is tuned into Coulomb blockade. The sensor quantum dot is marked in purple and the tunnel barriers are colored orange. (b) The energy diagram for the protocol.

expected improvement, and the fast driving regime. For the latter, which is in principle outside of the realms of application of thermodynamic length, we still observe substantial reductions in dissipation compared to the linear drive. Finally, we show that the improvements become more and more relevant the closer one gets to complete erasure.

These results can be regarded as the experimental proof of principle for the relevance of thermodynamic length in devising optimal finite-time protocols. As it was argued theoretically in [19,33,35–39], thermodynamic length offers a flexible and powerful tool for minimizing dissipation. It is particularly interesting to see that even for a problem as well explored as the one of Landauer erasure it is possible to find an improvement over present experimental protocols.

Experimental setup.—The experiment is performed using the same device as in [40], shown in Fig. 1(a). Three quantum dots (QDs) are formed by polytype engineering in an InAs nanowire [41–44]. The occupancy $n \in \{0,1\}$ of a spin-degenerate energy level E in the leftmost QD (QD1) encodes the bit of information to be erased in the experiment. We drive the energy level with the plunger gate voltage V_{g1}, which has a lever arm $\alpha = 1.6\times 10^4\ k_BT/\mathrm{V}$. The rightmost QD is voltage biased with $V_b = 0.5$ mV and tuned so that the current I_d is sensitive to changes in the QD1 occupancy, giving a real-time probe of n [45,46]. The middle QD is kept in Coulomb blockade, reducing the system to the one shown in Fig. 1(b): a discrete QD1 energy level coupled to a fermionic reservoir at temperature $T =$ 100 mV (set by the cryostat temperature). Electrons tunnel between them with the rates $\Gamma_{\rm in} = 2\Gamma_0(1+bE)f(E)$ and $\Gamma_{\rm out} = \Gamma_0(1+bE)[1-f(E)]$, where $\Gamma_0 = 39$ Hz and $b = 0.0036/k_BT$ were determined using a feedback protocol [47], and $f(E) = 1/(1+e^{E/k_BT})$ is the Fermi-Dirac distribution. The average occupation at equilibrium for each energy is given by $n_{\rm eq}(E) \coloneqq 1/(1+\frac{1}{2}e^{E/k_BT})$, corresponding to the thermal state for a system with a degeneracy 2 in the $n=1$ state.

Erasure protocol.—The erasure protocol is realized as follows: first, the system is allowed to thermalize in contact with the reservoir bath while keeping its energy at $E_0 = \log 2\,k_BT$ corresponding to a 50%–50% occupation condition, see Fig. 1(b). Then, while still keeping it in contact with the bath, we ramp up the energy of the dot until we reach $E_1 \coloneqq E_0 + E_A$, where E_A defines the driving amplitude. When $E_A \gg k_BT$, we have $n_{\rm eq}(E_1) \approx 0$, i.e., the dot is unoccupied with probability close to 1. As the last step, the energy is quenched back to E_0, so that the system is effectively erased.

We measure the heat ΔQ by monitoring the electron transitions: whenever the dot is occupied and an electron tunnels out, the energy at that time gets transferred to the reservoir where it dissipates and adds to ΔQ; similarly, if an electron tunnels into the dot that energy is subtracted. Moreover, since the quench is instantaneous, it does not contribute to the heat production, as the state of the system is unaffected by it. Repeating the protocol many times one can then compute the average heat $\langle\Delta Q\rangle$ simply by adding up the resulting heat for each round and dividing by the number of rounds. Alternatively, the same result can also be obtained from the average occupation $\langle n(t)\rangle$ thanks to the equality:

$$\langle\Delta Q\rangle = -\int_0^\tau dt\ \langle\dot{n}(t)\rangle\ E(t), \tag{2}$$

where τ is the total time of the protocol, while $E(t)$ is the drive used to interpolate between E_0 and E_1. The most usual choice is to take it to be a linear drive $E(t) \coloneqq E_0 + E_A\,t/\tau$, but in principle $E(t)$ could be any function satisfying $E(0) = E_0$ and $E(\tau) = E_1$. In fact, it turns out that the linear protocol is suboptimal.

An alternative protocol can be designed as follows. First, it should be noticed that in the limit of $(\Gamma_0\tau) \gg 1$, Eq. (2) can be brought to the form [48]

$$\langle\Delta Q\rangle = -k_BT\,\Delta S + k_BT\int_0^\tau dt\ g(t)\dot{E}(t)^2, \tag{3}$$

up to corrections of order $(\Gamma_0\tau)^{-2}$ and regardless of the particular choice of the protocol. The quantity $g(t)$ is called thermodynamic metric: it is always positive and depends smoothly on the drive $E(t)$. For reasons of space, we refer for the particular expression of the metric to the Supplemental Material [48]. The integral in Eq. (3) is a standard quantity in differential geometry, usually called energy functional. The name comes from the analogy with

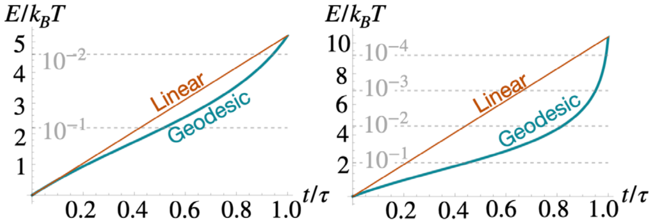

FIG. 2. Comparison between the linear drive and the geodesic drive for amplitudes $E_A = 5.2k_BT$ and $E_A = 10.4k_BT$. The dotted lines in gray indicate the equilibrium population of the excited state for each energy. As we can see, the geodesic drive allocates more time for the ramping up when the excited state is more occupied, while it becomes steeper at the end of the protocol.

the action of a particle moving with velocity $\dot{E}(t)$ and variable mass $g(t)$. Interestingly, thanks to the form of the dissipation in Eq. (3), we can automatically construct minimally dissipating drives simply by solving the geodesic equation for $E(t)$. Further details are provided in the Supplemental Material [48] and in [19,35].

The corresponding trajectory is shown in Fig. 2 for two driving amplitudes. Compared with the linear drive, the geodesic one allocates more time to ramp up the energy when the QD is occupied with larger chance (at low E) and becomes steeper towards the end of the protocol. This can be intuitively understood as follows: since the dissipation is linear in $\langle \dot{n}(t)\rangle$ while $\langle n(t)\rangle$ decreases exponentially with $E(t)$, it is better to allocate more time at the beginning, when the variation $\langle \dot{n}(t)\rangle$ is big, and to reserve little time to the final jump in the energy, because an exponentially small amount of the tunneling events take place at large E. Notice that this reasoning is justified by the fact that for slow driving $\langle n(t)\rangle \simeq n_{\mathrm{eq}}[E(t)]$. Still, we show that this intuition is also relevant for drives where $\Gamma_0\tau \simeq 3$ (which we dub fast driving regime).

The reasoning above intuitively captures a characteristic of geodesic drives: it can be proven that the entropy production rate, i.e., the integrand in Eq. (3), is constant along optimal protocols [35,37–39]. This effect is exemplified in Fig. 3, where we plot the heat production rate both in the

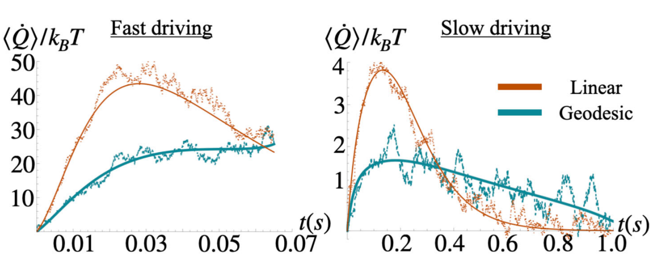

FIG. 3. Entropy production during Landauer erasure for amplitude $E_A = 10.4k_BT$ and erasure times of $\tau = 0.07$ and $\tau = 1.0$ s. The continuous lines are the theoretical predictions, while the dotted lines are the experimental data. The derivative has been performed by regularising the experimental data through a mean filter.

fast and in the slow driving regime ($\Gamma_0\tau \simeq 40$ for the latter), comparing the behavior of a linear drive with the one of the geodesic. We see that for the nonoptimized drive, the heat production peaks at the beginning, while decreasing towards zero at the end of the protocol. For geodesic drives instead, the heat is produced more uniformly along the protocol. (The fact that the entropy production rate is not perfectly constant along the trajectory arises from finite time effects. We numerically verified that increasing $\Gamma_0\tau$ makes the heat production closer to a constant value).

Comparison between linear and geodesic drive.—In this section we compare the performance of the geodesic protocol with the usual choice of a linear drive. The data are presented in Fig. 4.

The two plots on the left represent the quality of erasure as a function of the driving amplitude. This quantity is measured by the percentage of residual population in the dot at the end of the drive or, equivalently, with the population probability $p(n=0)$. On the right of Fig. 4, we also plot the dissipated heat as a function of the driving amplitude E_A. The above plots refer to the slow driving [$\tau = 1.0$ s, $(\Gamma_0\tau) = 39$], while the two on the bottom refer to the fast driving regime [$\tau = 0.07$ s, $(\Gamma_0\tau) = 2.73$]. These experimental results are complemented by numerical simulations in the Supplemental Material [48], where we confirm that the point $\tau = 1.0$ s is deep in the slow driving regime, whereas for $\tau = 0.07$ s the slow driving approximation (3) breaks down.

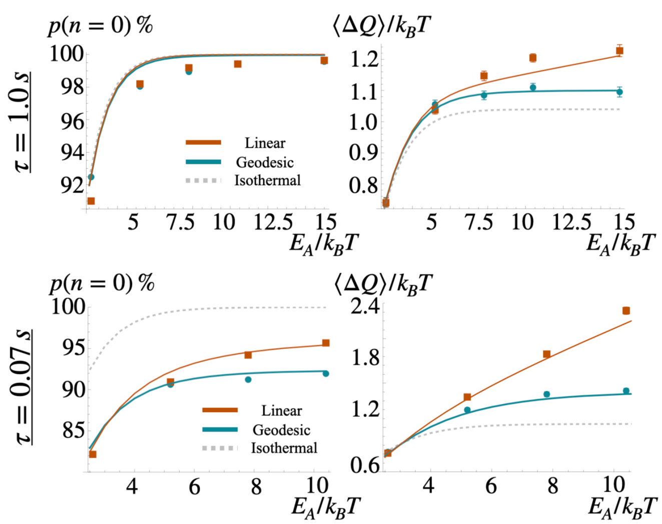

FIG. 4. In the two left panels we plot the heat as a function of the erasure quality of the protocol [given by $1 - \langle n(\tau)\rangle = p(n=0)$, the average population in the empty state]. The panel above refers to $\tau = 1.0$ s, while the bottom one corresponds to $\tau = 0.07$ s (same convention used for the right panels). On the right, we plot the heat produced as a function of the driving amplitude. The continuous lines are the theoretical predictions, while the dotted lines corresponds to the experimental data. The gray dotted line corresponds to the ideal case, that is the maximal erasure for each E_A on the left, and the Landauer's limit $k_BT\Delta S$ on the right.

When the amplitude is small, we see that the results given by the geodesic drive are similar to the ones for the linear drive (we even find a marginally better erasure in the 1 s drive). This can be explained by the fact that for small amplitudes the geodesic does not depart much from the linear drive, see Fig. 2. For larger amplitudes, we can appreciate the strength of geodesic protocols. In the slow driving regime (top figures), we observe that the geodesic drive dissipates less (right top Fig. 4) for a similar quality of erasure (left top Fig. 4). In fact, the dissipation grows linearly as the amplitude E_A increases for the linear protocol (i.e., as the quality of the erasure increases), whereas it tends to a constant for the geodesic drive. This makes geodesic drives more and more relevant when one wants to reach higher erasing quality. Indeed, the geodesic drive stays much closer to the Landauer's limit of $k_B T \Delta S$. These results demonstrate the reduction in dissipation when erasing a qubit in the slow driving regime theoretically predicted in previous works [19]. In the Supplemental Material [48], we complement these experimental results with numerical simulations that show the reduction of dissipation achieved through the geodesic drive as a function of τ.

Interestingly, as we depart from the slow driving regime (bottom Fig. 4), we observe a trade-off: on the one hand, the fast linear drive achieves a higher quality of erasure than the geodesic protocol. This happens because for such a short protocol duration, the system does not have enough time to respond to the steep ramp at the end of the geodesic drive making that energy range effectively lost in the erasure attempt. On the other hand, the dissipation produced by the geodesic protocol saturates, as expected from the theory [48], so one can achieve the same erasing precision as the one given by the linear drive at the same dissipation just by increasing the amplitude.

It should be noticed that if one allows for a small extra time at the end of the protocol in which the system thermalizes at a fixed energy, the difference in the quality of erasure between the linear and the geodesic drive would disappear. On the other hand, since the biggest contribution to the dissipation comes from the initial part of the protocol, if one can allow for this additional time, this would make the geodesic drive preferable because it would give the same erasure quality at lower dissipation. This intuition is made precise in the Supplemental Material [48] via numerical simulations of the process. In this way, there is a trade-off between the precision of erasure and time at optimal dissipation, or between dissipation and quality of erasure for a fixed time.

Conclusions.—The present Letter shows the relevance of thermodynamic length in the design of minimally dissipating experimental protocols. In particular, by considering the erasure of information in a quantum dot we showed that even in such a well-studied protocol a simple application of our method decreases the amount of dissipation released during the driving. This comes at the cost of a small decrease in the quality of the erasure, which can arguably be recovered by allowing a small transient at the end of the transformation, or by moving to bigger driving amplitudes (thanks to the saturation of the dissipated heat shown on the right of Fig. 4). Moreover, we showed that even if the thermodynamic length in principle should only apply to the slow driving limit, it improves also on relatively fast protocols, proving the wide applicability of this approach.

This universality comes from an underlying physical principle: the dissipation rate in optimal protocols should be constant along the trajectory [35,37–39]. In this sense, what the geodesic drive does is allocate the heat production in a more uniform way compared with the one arising from the naive choice of a linear drive. This fact is of key importance in the interpretation of the shape of the geodesic protocols and provides an intuitive method to develop optimal drives.

Beyond the minimization of average dissipation, the geodesic drives considered here can also become useful for the minimization of work and heat fluctuations [31,54], for probabilistic work extraction [55,56], and for increasing the efficiency of thermal machines [57–60]. Future works include the implementation of optimal protocols in the fast driving regime [20–22,24,61] and observing effects arising from quantum coherence in erasure processes [23,62,63].

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[48] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.129.270601 which includes Refs. [49–53], for a discussion of the experimental apparatus, an analytical example explaining how to use of the thermodynamic metric formalism, together with further numerical simulations corroborating some of the claims in the text.

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