

Glueball Dark Matter Revisited

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We revisit the possibility that dark matter is composed of stable scalar glueballs of a confining dark SU(3) gauge theory coupled only to gravity. The relic abundance of dark glueballs is studied for the first time in a thermal effective theory accounting for strong-coupling dynamics. An important ingredient of our analysis is the use of an effective potential for glueballs that is fitted by lattice simulations. We predict the relic abundance to be in the range $0.12\zeta_T^{-3}\Lambda/(137.9 \text{ eV}) \lesssim \Omega h^2 \lesssim 0.12\zeta_T^{-3}\Lambda/(82.7 \text{ eV})$, with Λ being the confinement scale, ζ_T the visible-to-dark sector temperature ratio, and the uncertainty is coming from the fit to lattice data. This prediction is an order of magnitude smaller than the existing glueball abundance results in the literature. Our framework can be easily generalized to different gauge groups and modified cosmological histories paving the way toward consistent exploration of strongly coupled dark sectors and their cosmological implications.

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Introduction.—Confining dark Yang-Mills sectors are often considered as a possible source of cold dark matter (CDM) in the Universe. In the simplest case, such dark gauge sectors are decoupled from the standard model (SM), except for the gravitational interaction. However, the strong self-interactions confine the gauge sector into composite objects such as glueballs, in the case of a dark sector only composed of dark gluons. In a minimal approach, the lightest composite state predicted by a pure strongly coupled gauge theory, the scalar dark glueball, is extensively discussed in the literature as a possible natural CDM candidate [1–14] (see also Ref. [15] for a more general discussion including “dark hadrons” and Ref. [16] for phenomenology of generic late-time forming DM). As an important case of self-interacting DM, this type of DM enables a consistent description of the structure of the Universe at small scales, in particular, helping resolve the so-called missing satellite problem [17] and the cusp-core problem in the CDM distribution at galactic scales [18,19]. Furthermore, strongly coupled dark Yang-Mills theories resembling quantum chromodynamics (QCD) in the standard model are physically motivated (e.g., these sectors show up frequently in string compactifications [20–30]) and a wealth of knowledge in nonperturbative QCD can be directly applied there. Note, since only a pure Yang-Mills

theory has robust and clean results available from lattice simulations [31], it has traditionally been the best starting point to study strongly coupled dark sectors rigorously.

With the presence of a first-order confinement-deconfinement phase transition at a critical temperature T_c [31–35], an analysis of relic abundance of this type of DM is nontrivial and requires a detailed knowledge of thermal field theory in a nonperturbative domain. The existing calculations predict that the relic abundance of dark glueballs overcloses the Universe for a confining sector with critical temperature above the electron volt scale, if that sector is not significantly cooler than the SM thermal bath. When multiple dark gauge sectors are present, a situation ubiquitous in string theory, this becomes a serious problem for phenomenology [36]. Therefore, a precise understanding of the cosmological generation of glueball DM, with the inclusion of strong-coupling effects, is necessary. In this Letter, we develop a novel approach to study the relic abundance of dark glueballs by using the well established low-energy effective model of glueball and gluon dynamics at finite temperatures [37]. We further constrain the effective model parameters by means of lattice results such as thermodynamic quantities and observables of the gluon condensate at finite temperature.

Our approach provides for the first time a rigorous theoretical treatment of the dark glueball dynamics yielding a prediction for the range of relic abundance $0.12\zeta_T^{-3}\Lambda/(137.9 \text{ eV}) \lesssim \Omega h^2 \lesssim 0.12\zeta_T^{-3}\Lambda/(82.7 \text{ eV})$, about an order of magnitude below the previous estimates in Refs. [1,36], depending on the visible-to-dark sector temperature ratio ζ_T . We confirm the linear dependence of the relic abundance

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with the confinement scale which is the essence of the dark glueball overproduction problem in the early Universe while the relic abundance itself is significantly reduced.

Glueball effective Lagrangian.—A first-principle’s treatment of the $SU(N)$ confinement-deconfinement phase transition is a tough theoretical challenge which requires a consistent description of a deeply nonperturbative dynamics. Lattice simulations represent a valuable tool to study phase transitions in Yang-Mills theories with and without matter fields (e.g., see Refs. [31,38,39]). At the same time, other complementary approaches have been used to understand different aspects of the strong-coupling effects, such as effective models and the functional renormalization group [40–59]. Here, we describe the dynamics of dark glueballs by means of an effective field theory [37].

At nonvanishing temperatures T , the \mathbb{Z}_N center of $SU(N)$ is a relevant global symmetry [60] and it is possible to construct a number of gauge invariant operators charged under \mathbb{Z}_N . The Polyakov loop is a remarkable example, defined as

$$\ell(x) = \frac{1}{N} \text{Tr}[\mathbf{L}] \equiv \frac{1}{N} \text{Tr} \left[\mathcal{P} \exp \left[ig \int_0^{1/T} A_0(\tau, \mathbf{x}) d\tau \right] \right], \quad (1)$$

where \mathcal{P} denotes path ordering, A_0 is the time component of the vector potential associated with this gauge group, g is the $SU(N)$ coupling constant, and (τ, \mathbf{x}) are Euclidean spacetime coordinates. The Polyakov loop is charged with respect to the center \mathbb{Z}_N of the $SU(N)$ gauge group [60] under which it transforms as $\ell \rightarrow z\ell$ with $z \in \mathbb{Z}_N$. Since the expectation value of the Polyakov loop vanishes at temperatures below the critical one and it is nonzero at higher temperatures, it is typically used as an order parameter for the Yang-Mills confinement phase transition at temperature $T_c \sim \Lambda$ [60]. This observation was exploited to model the phase transition in a mean field approach in terms of Polyakov loops known as the Polyakov loop model (PLM) [51]. This model captures the essential features of confinement phase transition in $SU(N)$ theories with $N \geq 2$ while PLM-inspired models were also proposed to understand the physics of heavy-ion collisions at the RHIC collider [57,58]. In [33], it has been shown that PLM can very well capture thermodynamic observables predicted by lattice simulations [31].

At temperatures around T_c , one can treat the glueball field \mathcal{H} and the Polyakov loop ℓ in a unified description, with an effective temperature-dependent potential given by [37]

$$V[\mathcal{H}, \ell] = \frac{\mathcal{H}}{2} \ln \left[\frac{\mathcal{H}}{\Lambda^4} \right] + T^4 \mathcal{V}[\ell] + \mathcal{H} \mathcal{P}[\ell] + V_T[\mathcal{H}], \quad (2)$$

where the first term is the zero-temperature glueball potential which can be obtained via the constraint of trace anomaly [61,62], Λ is the confinement scale of the theory,

and $\mathcal{V}[\ell]$ and $\mathcal{P}[\ell]$ are assumed to be real polynomials in ℓ and invariant under \mathbb{Z}_N , with coefficients that depend on fits to lattice data. Thermal corrections are included in $V_T[\mathcal{H}]$, which might involve terms that are nonanalytic in \mathcal{H} [48].

Note that (i) the potential in Eq. (2) reduces to the glueball dynamics at low temperatures and follows the PLM in the hot phase, (ii) the glueball field \mathcal{H} is a dimension four scalar field, and (iii) the term that couples \mathcal{H} and ℓ is the most general interaction term which can be constructed without spoiling the zero temperature trace anomaly [Eq. (21) of Ref. [62]].

In this simplified model we neglect the entire tower of heavier glueballs and pseudoscalar glueballs and the infinite series of dimensionless gauge invariant operators with different charges under \mathbb{Z}_N . Nevertheless, this model describes the essential features of the Yang-Mills phase transition. Below the critical temperature T_c the last term in Eq. (2) is negligible. Since the glueballs are relatively heavy compared to the Λ scale their temperature contribution $V_T[\mathcal{H}]$ can also be disregarded in the first approximation [37]. We leave a refined analysis accounting for thermal effects in the glueball potential for a future investigation.

In the opposite limit, $T \gg T_c$, in the deconfined phase, the term $T^4 \mathcal{V}[\ell]$ dominates, i.e., dark gluons are the dominant component. The precise relation between the confinement scale Λ and the critical temperature of the phase transition T_c depends mildly on the gauge group and matter structure of the theory and is determined by lattice simulations. In this Letter, we consider $T_c \sim 1.61\Lambda$ for $SU(3)$ (see, e.g., Ref. [63] for an arbitrary number of colors).

We consider the following Lagrangian for the glueball and Polyakov loop degrees of freedom [37,64,65]:

$$\mathcal{L} = \frac{c}{2} \frac{\partial_\mu \mathcal{H} \partial^\mu \mathcal{H}}{\mathcal{H}^{3/2}} - V[\mathcal{H}, \ell], \quad (3)$$

where

$$c = \frac{1}{2\sqrt{e}} \left(\frac{\Lambda}{m_{\text{gb}}} \right)^2 \quad (4)$$

is a constant determined by the glueball mass m_{gb} , that in the following is assumed to be $m_{\text{gb}} = 6\Lambda$ [66]. The Polyakov loop is a nondynamical order parameter and since it is assumed to be homogeneous in space, we ignore terms involving spatial derivatives of ℓ . This corresponds to neglecting the nontrivial dynamics of a first order phase transition, which proceeds via the formation of bubbles and their subsequent collisions. This could have a significant impact on the formation of glueballs, as observed in the presence of matter (see, e.g., Refs. [67,68]). The kinetic term for the glueball field \mathcal{H} is nonstandard, as it can be inferred from its dimensionality. For this reason, we write the glueball field \mathcal{H} in terms of a canonically normalized

TABLE I. Parameters of the effective potential in Eq. (5).

a_0	a_1	a_2	a_3	a_4	b_3	b_4
3.72	-5.73	8.49	-9.29	0.27	2.40	4.53

scalar field ϕ as $\mathcal{H} = 2^{-8}c^{-2}\phi^4$, and from this point on we refer to ϕ as the glueball field. It evolves according to the effective Lagrangian

$$\begin{aligned}
 \mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V[\phi, \ell], \\
 V[\phi, \ell] &= \frac{\phi^4}{2^8c^2} \left[2\ln\left(\frac{\phi}{\Lambda}\right) - 4\ln 2 - \ln c \right] \\
 &\quad + \frac{\phi^4}{2^8c^2}\mathcal{P}[\ell] + T^4\mathcal{V}[\ell], \\
 \mathcal{P}[\ell] &= c_1|\ell|^2, \\
 \mathcal{V}[\ell] &= -\frac{b_2(T)}{2}|\ell|^2 + b_4|\ell|^4 - b_3(\ell^3 + (\ell^*)^3), \\
 b_2(T) &= \sum_{i=0}^4 a_i \left(\frac{T_c}{T}\right)^i, \tag{5}
 \end{aligned}$$

where we have kept only the lowest order in $\mathcal{P}[\ell]$ satisfying the symmetries. The Polyakov loop potential $\mathcal{V}[\ell]$ is determined from symmetry arguments combined with fits to lattice thermodynamic quantities. Our choice here is taken from Ref. [33] and the numerical values of the constants are reported in Table I, for clarity.

Temperature dependence of the Polyakov loop.—The Lagrangian in Eq. (5) describes the evolution of the glueball-dark gluon system across the phase transition. This effective description is expected to be valid in a broad temperature range, except when the temperature is large $T \gg T_c$, where $V_T[\mathcal{H}]$ needs to be included. Since the Polyakov loop is a nondynamical degree of freedom, its temperature evolution is determined by the location of the minimum in the effective potential. Being the order parameter of the phase transition, ℓ approaches 1 at high temperatures and vanishes for temperatures below the critical one. The stationary points of ℓ are $\ell = 0$ and

$$|\ell_\pm| = \frac{3b_3}{4b_4} \left(1 \pm \sqrt{1 + \frac{512b_2(T)b_4 - 4c_1b_4}{c^2} \left(\frac{\phi}{T}\right)^4} \right), \tag{6}$$

representing two minima, $\ell = 0$ and $\ell = \ell_+$, separated by a maximum in $\ell = \ell_-$. The solution $\ell = 0$ denotes the confined phase and it is a global minimum only for temperatures below the critical temperature. In the deconfined phase, the solution $\ell = 0$ becomes metastable and $\ell = \ell_+$ becomes the global minimum. The Polyakov loop is then “integrated out” using its equation of motion $\ell = \ell(\phi, T)$, giving rise to a potential for the glueball

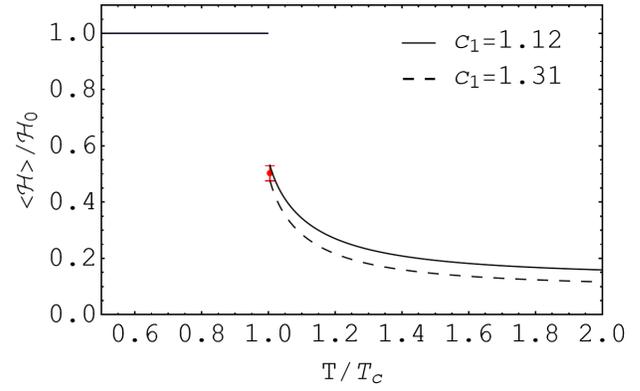


FIG. 1. Vacuum expectation value of the glueball field \mathcal{H} as a function of temperature. The field is normalized to its value in the confined phase. The discontinuity at $T = T_c$ is characteristic of a first-order phase transition and the value of the jump depends on the parameter c_1 , whose limiting values shown in this plot are obtained by a comparison with the lattice data [69]. The values shown correspond to 1σ uncertainty range. We do not use the lattice data for higher temperatures in the comparison, as our model neglects thermal corrections, which are increasingly relevant above T_c .

field in the form $V[\phi, T] = V[\phi, \ell(\phi, T)]$. Moreover, we set the zero-point energy of the glueball field to zero in order to properly describe glueballs as matter. The evolution of the glueball minimum in this new potential is shown in Fig. 1 in terms of the field \mathcal{H} and compared to lattice simulations. Below T_c , $\langle \mathcal{H} \rangle$ is constant with temperature and it discontinuously jumps to a lower value right above the critical temperature. We match the size of the discontinuity predicted in our potential to a result from lattice, given in Ref. [69] (the red point in Fig. 1). This constraint is enough to impose limitations on the value of c_1 in Eq. (5), the glueball-Polyakov loop coupling. We found this value to be $c_1 = 1.225 \pm 0.19$ at 95% C.L. The associated uncertainty of $\sim 20\%$ dominates the uncertainty in the glueball relic abundance in our analysis, such that $\sim \mathcal{O}(3\%)$ uncertainties on the fitting parameters in Table I have been ignored.

Cosmological evolution of the glueball field.—Thanks to the previous discussion, we are left with a relatively simple recipe to describe the glueball field dynamics across the phase transition. Note that the evolution can be treated as completely classical, since the effective Lagrangian in Eq. (5) fully accounts for quantum effects at tree level.

In a first approximation, the glueball field is homogeneous and evolves in an expanding Friedmann-Lemaître-Robertson-Walker (FLRW) Universe. The Klein-Gordon equation for a field in a FLRW metric reads

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V[\phi, T] = 0, \tag{7}$$

where the Hubble parameter H when glueballs form is approximately determined by the SM content of the

Universe, as it is assumed to have more degrees of freedom than the confining dark sector and, if there are no interactions with the SM, this sector is colder than the SM thermal bath. We denote the visible-to-dark sector temperature ratio by ξ_T . The photon temperature T_γ determines the Hubble parameter H and can be taken as a time variable in Eq. (7) by using

$$t = \frac{1}{2} \sqrt{\frac{45}{4\pi^3 g_{*,\rho}(T_\gamma)}} \frac{m_P}{T_\gamma^2}, \quad (8)$$

where m_P is the Planck mass and $g_{*,\rho}(T_\gamma)$ is the number of degrees of freedom of the SM bath at temperature $T_\gamma = \xi_T T$. Note that the dark sector temperature T is the one that governs the phase transition, i.e., entering in Eq. (2). In terms of this variable Eq. (7) reads

$$\frac{4\pi^3 g_{*,\rho} \xi_T^4 T^6}{45 m_P^2} \frac{d^2 \phi}{dT^2} + \frac{2\pi^3}{45 m_P^2} \frac{dg_{*,\rho}}{dT} \xi_T^4 T^6 \frac{d\phi}{dT} + \partial_\phi V[\phi, T] = 0, \quad (9)$$

where the second term can be neglected for a large range of temperatures as $g_{*,\rho}$ is constant except at a few isolated events (the QCD phase transition, for example). We consider it as a free parameter and take $g_{*,\rho} = 100$, which has very little impact on our final result. The visible-to-dark sector temperature ratio can be absorbed in an effective Planck mass, $M \equiv m_P / \xi_T^2$.

The nonperturbative dynamics of the system is encoded in Eq. (9) and, after the phase transition, we assume that the energy density stored in the glueball field gives precisely the DM relic density. From the particle physics point of view the evolution can be described as follows. In the deconfined phase the Universe is populated by dark gluons that form glueballs at the phase transition, thanks to the interaction term in Eq. (5). When the phase transition is completed, DM glueballs populate the Universe and interact with each other following the potential in Eq. (5), corresponding to interactions in the form $(\phi - \phi_{\min})^n$ for $n = 2, 3, \dots$, with ϕ_{\min} being the value of the field at the minimum of the potential. The importance of the higher- n terms depends on the displacement of ϕ from its minimum, which is a measure of the glueball density. If, for example, ϕ is very close to its minimum, only the quadratic term is relevant, which is equivalent to having a massive free field. On the other hand, large amplitudes (i.e., larger densities) for ϕ require increasingly more nonlinear interaction terms (see also Refs. [70,71]).

In Fig. 2 we show the evolution of the glueball field as a function of temperature, starting from different initial conditions set in the deconfined phase. In the very early stage, the field evolution is dominated by the Hubble friction and it remains frozen until H becomes comparable to the temperature-dependent effective glueball mass in the deconfined phase, represented by the gray region labeled as

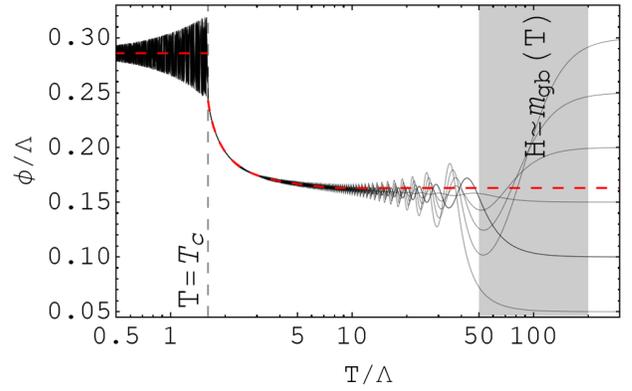


FIG. 2. Evolution of the glueball field for a phase transition scale $\Lambda = 10^{-5} m_P$, $c_1 = 1.225$, and different initial conditions. The gray region indicates the phase in which the glueball mass is comparable with the Hubble parameter, $H \simeq m_{\text{gb}}(T)$. The vertical dashed line marks the phase transition at $T_c = 1.61\Lambda$. The red dashed line shows the evolution of the minimum of the glueball potential.

$H \simeq m_{\text{gb}}(T)$. This happens at a temperature $T_{\text{osc}} \sim \sqrt{M\Lambda}$, when the field starts to oscillate around the minimum of the potential, shown as a dashed red line in Fig. 2, with a damped amplitude. We take $T_{\text{osc}} \gg T_c$, as $M \gg \Lambda$, unless the confinement scale is close to the Planck scale or the dark sector is very cold. Therefore, the oscillations of the glueball field in the deconfined phase have enough time to decay, regardless of initial condition, and ϕ just follows the minimum of the potential (with damped oscillations of small amplitude but with an increasing average speed) until the phase transition occurs at T_c (see Fig. 2). At the critical temperature, the value of the Polyakov loop jumps discontinuously, causing a discontinuous jump in the minimum of the glueball potential, as shown in Fig. 1, generating oscillations with a high initial velocity that wash out any dependence on initial conditions at $T > T_c$.

Glueball relic density.—In the confined phase, ϕ is displaced enough from its minimum to allow for annihilation of n glueballs into $m < n$ glueballs, $n \rightarrow m$, which is possible because of the $(n + m)$ th order interaction term in the Lagrangian. As the glueball number density decreases, all the higher order $n \rightarrow m$ processes become less efficient until the only efficient number-changing process is $3 \rightarrow 2$. Note that the $3 \rightarrow 1$ and $2 \rightarrow 1$ processes are prohibited due to kinematic constraints arising from the energy conservation. The $3 \rightarrow 2$ interactions are precisely the ones determining the relic abundance of glueballs when $\Gamma_{3 \rightarrow 2} < H$. The evolution is that of a simple damped oscillator in a nonlinear potential, and the energy stored in these oscillations around $\phi_{\min} \approx 0.28\Lambda$ corresponds to the relic DM abundance, namely, $\Omega h^2 = \rho / \rho_c$, where the critical density is $\rho_c = 1.05 \times 10^4 \text{ eV cm}^{-3}$, and

$$\rho = \frac{2\pi^3}{45} g_{*,\rho}(T) \frac{T^6}{M^2} \left(\frac{d\phi}{dT} \right)^2 + V[\phi]. \quad (10)$$

This energy density scales as $\sim T^3$, as CDM, when the harmonic approximation is valid, i.e., after the decoupling of $3 \rightarrow 2$ interactions. Numerically solving Eq. (9) down to the temperature T_f , and below this temperature the evolution is simply determined by the cosmological expansion, for $\Lambda \lesssim 0.1M$, the energy density is given by $0.015 \lesssim T_f^{-3} \Lambda^{-1} \rho \lesssim 0.020$ for $1.035 < c_1 < 1.415$. In conclusion, the predicted glueball relic density is

$$0.12 \zeta_T^{-3} \frac{\Lambda}{137.9 \text{ eV}} \lesssim \Omega h^2 \lesssim 0.12 \zeta_T^{-3} \frac{\Lambda}{82.7 \text{ eV}}, \quad (11)$$

and this result should be compared to $\Omega h^2 \sim 0.12 \zeta_T^{-3} \Lambda / 5.45 \text{ eV}$ [1], which overestimates the relic density by one order of magnitude. This difference is due to two main concurrent effects. The first one is an overestimation of the energy stored in the dark gluon field. In the literature, dark gluons are considered as radiation for all temperatures above the phase transition. In our approach, the energy density of dark gluons for temperatures right above the critical one strongly deviates (reduced by a factor ~ 50) from that of an ideal gas, in agreement with lattice results. The second effect is that glueballs do not redshift as CDM immediately after the phase transition, going through a phase in which their equation of state is $-1 \lesssim p/\rho \lesssim 0$, making them dilute slower than dust. The combination of these effects leads to the found discrepancy. We note also that thermal corrections increase the glueball relic density, by displacing the high-temperature minimum of $\sim 10\%$ farther from the low-temperature minimum [48]. We estimated an increase of the relic density up to $\sim 80\%$ due to such thermal corrections, which will be subject of a future investigation.

A possible constraint on the model comes from the contribution of dark gluons to the effective number of relativistic species, constrained to be $\Delta N_{\text{eff}} < 0.35$ at the 95% C.L. (confidence level) [72]. A temperature ratio $\zeta_T \gtrsim 2$ is enough to evade this constraint. Therefore, a dark gauge sector interacting only via gravitational interactions with the SM and a confinement scale at the electron volt scale might explain the DM abundance without spoiling other cosmological observables.

Discussion and conclusions.—In this Letter, we presented a new approach to calculate the glueball CDM relic density which includes the self-interactions in a nonperturbative fashion. We bridge the well-established thermal EFT with the existing lattice results to provide rigorous phenomenological predictions. Because of its generality, it is easy to apply this approach to different gauge groups, but in this Letter we considered only SU(3) for the sake of clarity. Moreover, the method presented in this Letter is suitable for investigations of the glueball formation in modified cosmological histories, requiring only a simple modification of Eq. (9), one of the main results of this Letter. Another interesting question is on the role of thermal

effects in the glueball potential, that we neglected in this preliminary study. We postpone this study to a future work. Our Letter paves the road toward consistent exploration of strongly coupled dark sectors and their cosmological implications.

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- [1] E. D. Carlson, M. E. Machacek, and L. J. Hall, Self-interacting dark matter, *Astrophys. J.* **398**, 43 (1992).
- [2] A. E. Faraggi and M. Pospelov, Selfinteracting dark matter from the hidden heterotic string sector, *Astropart. Phys.* **16**, 451 (2002).
- [3] J. L. Feng and Y. Shadmi, WIMPLess dark matter from non-Abelian hidden sectors with anomaly-mediated supersymmetry breaking, *Phys. Rev. D* **83**, 095011 (2011).
- [4] K. K. Boddy, J. L. Feng, M. Kaplinghat, and T. M. P. Tait, Self-interacting dark matter from a non-Abelian hidden sector, *Phys. Rev. D* **89**, 115017 (2014).
- [5] A. Soni and Y. Zhang, Hidden SU(N) glueball dark matter, *Phys. Rev. D* **93**, 115025 (2016).
- [6] G. D. Kribs and E. T. Neil, Review of strongly-coupled composite dark matter models and lattice simulations, *Int. J. Mod. Phys. A* **31**, 1643004 (2016).
- [7] B. S. Acharya, M. Fairbairn, and E. Hardy, Glueball dark matter in nonstandard cosmologies, *J. High Energy Phys.* **07** (2017) 100.
- [8] K. R. Dienes, F. Huang, S. Su, and B. Thomas, Dynamical dark matter from strongly-coupled dark sectors, *Phys. Rev. D* **95**, 043526 (2017).
- [9] A. Soni and Y. Zhang, Gravitational waves from SU(N) glueball dark matter, *Phys. Lett. B* **771**, 379 (2017).
- [10] A. Soni, H. Xiao, and Y. Zhang, Cosmic selection rule for the glueball dark matter relic density, *Phys. Rev. D* **96**, 083514 (2017).
- [11] P. Draper, J. Kozaczuk, and J.-H. Yu, Theta in new QCD-like sectors, *Phys. Rev. D* **98**, 015028 (2018).
- [12] J. Halverson, B. D. Nelson, F. Ruehle, and G. Salinas, Dark glueballs and their ultralight axions, *Phys. Rev. D* **98**, 043502 (2018).
- [13] L. Forestell, D. E. Morrissey, and K. Sigurdson, Cosmological bounds on non-Abelian dark forces, *Phys. Rev. D* **97**, 075029 (2018).
- [14] L. Forestell, D. E. Morrissey, and K. Sigurdson, Non-Abelian dark forces and the relic densities of dark glueballs, *Phys. Rev. D* **95**, 015032 (2017).

- [15] N. A. Dondi, F. Sannino, and J. Smirnov, Thermal history of composite dark matter, *Phys. Rev. D* **101**, 103010 (2020).
- [16] A. Das, B. Dasgupta, and R. Khatri, Ballistic dark matter oscillates above Λ CDM, *J. Cosmol. Astropart. Phys.* **04** (2019) 018.
- [17] M. Mateo, Dwarf galaxies of the Local Group, *Annu. Rev. Astron. Astrophys.* **36**, 435 (1998).
- [18] D. N. Spergel and P. J. Steinhardt, Observational Evidence for Self-Interacting cold dark matter, *Phys. Rev. Lett.* **84**, 3760 (2000).
- [19] W. J. G. de Blok, The core-cusp problem, *Adv. Astron.* **2010**, 1 (2010).
- [20] D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, The Heterotic String, *Phys. Rev. Lett.* **54**, 502 (1985).
- [21] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Strings on Orbifolds, *Nucl. Phys.* **B261**, 678 (1985).
- [22] L. E. Ibanez, H. P. Nilles, and F. Quevedo, Orbifolds and Wilson Lines, *Phys. Lett. B* **187**, 25 (1987).
- [23] M. Cvetič, T. Li, and T. Liu, Supersymmetric Pati-Salam models from intersecting D6-branes: A road to the standard model, *Nucl. Phys.* **B698**, 163 (2004).
- [24] F. Gmeiner, R. Blumenhagen, G. Honecker, D. Lust, and T. Weigand, One in a billion: MSSM-like D-brane statistics, *J. High Energy Phys.* **01** (2006) 004.
- [25] J. M. Drummond, J. Henn, G. P. Korchemsky, and E. Sokatchev, Hexagon Wilson loop = six-gluon MHV amplitude, *Nucl. Phys.* **B815**, 142 (2009).
- [26] B. S. Acharya, M theory, Joyce orbifolds and super Yang-Mills, *Adv. Theor. Math. Phys.* **3**, 227 (1999).
- [27] A. Grassi, J. Halverson, J. Shaneson, and W. Taylor, Non-Higgsable QCD and the standard model spectrum in F -theory, *J. High Energy Phys.* **01** (2015) 086.
- [28] H. Georgi, G. Kestin, and A. Sajjad, Towards an effective field theory on the light-shell, *J. High Energy Phys.* **03** (2016) 137.
- [29] W. Taylor and Y.-N. Wang, The F -theory geometry with most flux vacua, *J. High Energy Phys.* **12** (2015) 164.
- [30] J. Halverson, Strong coupling in F -theory and geometrically non-Higgsable seven-branes, *Nucl. Phys.* **B919**, 267 (2017).
- [31] M. Panero, Thermodynamics of the QCD Plasma and the Large- N Limit, *Phys. Rev. Lett.* **103**, 232001 (2009).
- [32] J. Halverson, C. Long, A. Maiti, B. Nelson, and G. Salinas, Gravitational waves from dark Yang-Mills sectors, *J. High Energy Phys.* **05** (2021) 154.
- [33] W.-C. Huang, M. Reichert, F. Sannino, and Z.-W. Wang, Testing the dark $SU(N)$ Yang-Mills theory confined landscape: From the lattice to gravitational waves, *Phys. Rev. D* **104**, 035005 (2021).
- [34] M. Reichert, F. Sannino, Z.-W. Wang, and C. Zhang, Dark confinement and chiral phase transitions: Gravitational waves vs matter representations, *J. High Energy Phys.* **01** (2022) 003.
- [35] Z. Kang, J. Zhu, and S. Matsuzaki, Dark confinement-deconfinement phase transition: A roadmap from Polyakov loop models to gravitational waves, *J. High Energy Phys.* **09** (2021) 060.
- [36] J. Halverson, B. D. Nelson, and F. Ruehle, String theory and the dark glueball problem, *Phys. Rev. D* **95**, 043527 (2017).
- [37] F. Sannino, Polyakov loops versus hadronic states, *Phys. Rev. D* **66**, 034013 (2002).
- [38] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier, and B. Petersson, Thermodynamics of $SU(3)$ lattice gauge theory, *Nucl. Phys.* **B469**, 419 (1996).
- [39] M. Okamoto, A. AliKhan, S. Aoki, R. Burkhalter, S. Ejiri *et al.* (CP-PACS Collaboration), Equation of state for pure $SU(3)$ gauge theory with renormalization group improved action, *Phys. Rev. D* **60**, 094510 (1999).
- [40] P. N. Meisinger, T. R. Miller, and M. C. Ogilvie, Phenomenological equations of state for the quark gluon plasma, *Phys. Rev. D* **65**, 034009 (2002).
- [41] P. N. Meisinger and M. C. Ogilvie, Complete high temperature expansions for one loop finite temperature effects, *Phys. Rev. D* **65**, 056013 (2002).
- [42] A. Dumitru and R. D. Pisarski, Event-by-event fluctuations from decay of a Polyakov loop condensate, *Phys. Lett. B* **504**, 282 (2001).
- [43] N. O. Agasian, Dilaton at nonzero temperature and deconfinement in gluodynamics, *JETP Lett.* **57**, 208 (1993), <https://inspirehep.net/literature/362317>.
- [44] B. A. Campbell, J. R. Ellis, and K. A. Olive, QCD phase transitions in an effective field theory, *Nucl. Phys.* **B345**, 57 (1990).
- [45] Y. A. Simonov, Calculating deconfinement temperature through the scale anomaly in gluodynamics, *JETP Lett.* **55**, 627 (1992), <https://inis.iaea.org/search/searchsinglerecord.aspx?recordsFor=SingleRecord&RN=25019795>.
- [46] J. Sollfrank and U. W. Heinz, Chiral and gluon condensates at finite temperature, *Z. Phys. C* **65**, 111 (1995).
- [47] G. W. Carter, O. Scavenius, I. N. Mishustin, and P. J. Ellis, An effective model for hot gluodynamics, *Phys. Rev. C* **61**, 045206 (2000).
- [48] B. J. Schaefer, O. Bohr, and J. Wambach, Finite temperature gluon condensate with renormalization group flow equations, *Phys. Rev. D* **65**, 105008 (2002).
- [49] A. Drago, M. Gibilisco, and C. Ratti, Evaporation of the gluon condensate: A model for pure gauge $SU(3)(c)$ phase transition, *Nucl. Phys.* **A742**, 165 (2004).
- [50] T. Renk, R. Schneider, and W. Weise, Phases of QCD, thermal quasiparticles and dilepton radiation from a fireball, *Phys. Rev. C* **66**, 014902 (2002).
- [51] R. D. Pisarski, Tests of the Polyakov loops model, *Nucl. Phys.* **A702**, 151 (2002).
- [52] C. P. Korthals Altes, R. D. Pisarski, and A. Sinkovics, The potential for the phase of the Wilson line at nonzero quark density, *Phys. Rev. D* **61**, 056007 (2000).
- [53] A. Dumitru and R. D. Pisarski, Degrees of freedom and the deconfining phase transition, *Phys. Lett. B* **525**, 95 (2002).
- [54] J. Wirstam, One loop QCD corrections to the thermal Wilson line model, *Phys. Rev. D* **65**, 014020 (2001).
- [55] M. Laine and O. Philipsen, The Nonperturbative QCD Debye mass from a Wilson line operator, *Phys. Lett. B* **459**, 259 (1999).
- [56] F. Sannino, N. Marchal, and W. Schafer, Partial deconfinement in color superconductivity, *Phys. Rev. D* **66**, 016007 (2002).
- [57] O. Scavenius, A. Dumitru, and A. D. Jackson, Explosive Decomposition in Ultrarelativistic Heavy Ion Collision, *Phys. Rev. Lett.* **87**, 182302 (2001).

- [58] O. Scavenius, A. Dumitru, and J. T. Lenaghan, The K/π ratio from condensed Polyakov loops, *Phys. Rev. C* **66**, 034903 (2002).
- [59] L. Del Debbio, A. Di Giacomo, B. Lucini, and G. Paffuti, Abelian projection in SU(N) gauge theories, [arXiv:hep-lat/0203023](https://arxiv.org/abs/hep-lat/0203023).
- [60] B. Svetitsky and L. G. Yaffe, Critical behavior at finite temperature confinement transitions, *Nucl. Phys.* **B210**, 423 (1982).
- [61] J. Schechter, Effective Lagrangian with two color singlet gluon fields, *Phys. Rev. D* **21**, 3393 (1980).
- [62] J. Schechter, Introduction to effective Lagrangians for QCD, eConf **C010815**, 76 (2002), <https://www.slac.stanford.edu/econf/C010815/docs/schechter.pdf>.
- [63] B. Lucini, A. Rago, and E. Rinaldi, SU(N_c) gauge theories at deconfinement, *Phys. Lett. B* **712**, 279 (2012).
- [64] R. Gomm, P. Jain, R. Johnson, and J. Schechter, Scale anomaly and the scalars, *Phys. Rev. D* **33**, 801 (1986).
- [65] R. Ouyed and F. Sannino, The Glueball sector of two flavor color superconductivity, *Phys. Lett. B* **511**, 66 (2001).
- [66] D. Curtin, C. Gemmell, and C. B. Verhaaren, Simulating glueball production in $N_f = 0$ QCD, *Phys. Rev. D* **106**, 075015 (2022).
- [67] P. Asadi, E. D. Kramer, E. Kuflik, G. W. Ridgway, T. R. Slatyer, and J. Smirnov, Thermal squeezeout of dark matter, *Phys. Rev. D* **104**, 095013 (2021).
- [68] P. Asadi, E. D. Kramer, E. Kuflik, T. R. Slatyer, and J. Smirnov, Glueballs in a thermal squeezeout model, *J. High Energy Phys.* **07** (2022) 006.
- [69] M. D'Elia, A. Di Giacomo, and E. Meggiolaro, Gauge invariant field strength correlators in pure Yang-Mills and full QCD at finite temperature, *Phys. Rev. D* **67**, 114504 (2003).
- [70] N. Yamanaka, H. Iida, A. Nakamura, and M. Wakayama, Dark matter scattering cross section and dynamics in dark Yang-Mills theory, *Phys. Lett. B* **813**, 136056 (2021).
- [71] N. Yamanaka, H. Iida, A. Nakamura, and M. Wakayama, Glueball scattering cross section in lattice SU(2) Yang-Mills theory, *Phys. Rev. D* **102**, 054507 (2020).
- [72] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* **641**, A6 (2020); **652**, C4(E) (2021).