Editors' Suggestion

Noise-Induced Quantum Synchronization

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Synchronization is a widespread phenomenon in science and technology. Here, we study noise-induced synchronization in a quantum spin chain subjected to local Gaussian white noise. We demonstrate stable (anti)synchronization between the endpoint magnetizations of a quantum XY model with transverse field of arbitrary length. Remarkably, we show that noise applied to a single spin suffices to reach stable (anti) synchronization, and find that the two synchronized end spins are entangled. We additionally determine the optimal noise amplitude that leads to the fastest synchronization along the chain, and further compare the optimal synchronization speed to the fundamental Lieb-Robinson bound for information propagation.

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Synchronization is ubiquitous in classical nonlinear systems. Self-sustained periodic oscillators swing in unison, and are thus synchronized, when their phases (frequencies) are locked [1–7]. Synchronous behavior plays a central role in many interconnected systems in fields ranging from biology and chemistry to physics and engineering. Different mechanisms of classical synchronization have been identified [1–7]. For instance, forced synchronization may be generated by an external drive, whereas spontaneous synchronization may be induced by the mutual coupling between subsystems in the absence of external forcing. Another intriguing effect is noise-induced synchronization [8–15], which has recently been observed in sensory neurons [16] and in lasers [17].

In the past years, the study of synchronization has been extended to the quantum domain [18–38]. Quantum synchronization has been examined in systems with a classical analog, such as nonlinear van der Pol oscillators, as well as in systems without a classical counterpart, such as qubits [18–38]. Both forced and spontaneous synchronizations have been investigated in the quantum regime [18–38]. Quantum entrainment may starkly differ from classical entrainment: it has indeed been shown to exhibit counterintuitive nonclassical features, such as enhanced synchronization of far-detuned oscillators and suppressed synchronization of resonant oscillators [30]. Quantum synchronization has recently been experimentally observed in spin-one systems [39,40].

Here, we investigate noise-induced synchronization in an isolated quantum many-body system. By locally applying Gaussian white noise to a quantum spin chain of arbitrary length, we show that stable (anti)synchronization between local spin observables may be achieved when a given condition, on the length of the chain and on the sites at which noise is applied, is satisfied. In that case, local spin observables oscillate with the same frequency, a dynamical criterion for quantum synchronization that has been widely applied [21,24,35,37,38,41]. Remarkably, stable (anti)synchronization can be established between the two ends of the chain, even when noise is applied to only a single site in between. While noise is often assumed to be detrimental for quantum features owing to decoherence, we establish that the synchronized end spins are entangled by evaluating their concurrence [42]. We finally analyze the time needed to fully synchronize the two ends of the chain as a function of the noise strength and of the length of the chain, when noise is added close to one end. We find the existence of an optimal noise amplitude that leads to the shortest synchronization time (or fastest synchronization rate). This optimal time scales like the cube of the chain length, thus stronger than the linear dependence given by the Lieb-Robinson bound, which provides a fundamental upper limit on the speed of information propagation in a quantum system [43].

Synchronization model.—We consider an isolated quantum many-particle system with Hamiltonian H_0 subjected to a stochastic perturbation of the form $\xi(t)V$, where $\xi(t)$ describes classical noise that couples to operator V. For concreteness and simplicity, we take a quantum XY chain of N spins in a transverse field [44]

$$H_0 = \frac{J}{2}\sum_{j=1}^{N-1}(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y) + h\sum_{j=1}^{N}\sigma_j^z, \qquad (1)$$

where $\sigma_j^{x,y,z}$ are the usual Pauli operators, $J > 0$ is the interaction parameter, and $h = 1$ the field strength. We additionally choose delta-correlated (white) Gaussian noise, $\langle\xi(t)\xi(t')\rangle = \Gamma\delta(t-t')$, with zero mean and amplitude Γ. We will in the following consider various operators V, depending on the number and on the position of the sites the noise couples to. This many-particle system may be implemented using trapped ions [45,46], where noise is introduced by locally modulating ac-Stark shifts of the respective spin states [47].

In order to examine the influence of noise on the quantum spin chain, and derive the synchronization condition, it is convenient to describe the time evolution of the system in Liouville space [48]. In this formalism, a density matrix ρ_ξ is mapped onto a vector $|\rho_\xi\rangle\rangle$ (often called supervector) in a higher-dimensional Hilbert space, and the von Neumann equation, $\dot{\rho}_\xi(t) = -i[H_0 + \xi(t)V, \rho_\xi(t)]$, is transformed into the Schrödinger-like equation [48]

$$|\dot{\rho}_\xi(\tau)\rangle\rangle = -i[\mathcal{L}_0 + \xi(\tau)\mathcal{V}]|\rho_\xi(\tau)\rangle\rangle, \qquad (2)$$

which can be analyzed using the usual tools of quantum mechanics. The Liouville superoperator $\mathcal{L}_0$ is given by the supercommutator $\mathcal{L}_0 = [\![H_0, \mathbb{1}]\!]/J = H_0/J \otimes \mathbb{1} - \mathbb{1} \otimes H_0^T/J$ and the perturbation superoperator by $\mathcal{V} = [\![V, \mathbb{1}]\!]$. We have further introduced the normalized time $\tau = Jt$. Equation (2) has the form of a stochastic differential equation with multiplicative noise (which we interpret using the Stratonovich convention) [49]. Averaging over an ensemble of noise realizations, Eq. (2) becomes [50]

$$|\dot{\rho}(\tau)\rangle\rangle = -(i\mathcal{L}_0 + \gamma\mathcal{V}^2/2)|\rho(\tau)\rangle\rangle, \qquad (3)$$

where $\rho(\tau) = \langle\rho_\xi(\tau)\rangle$ is the averaged density operator and $\gamma = \Gamma/J$ is the reduced noise strength.

The stochastic perturbation affects both eigenmodes and eigenfrequencies of the unperturbed quantum system. We start from the spectral decomposition of the free evolution, $|\rho_0(\tau)\rangle\rangle = \exp(-i\mathcal{L}_0\tau)|\rho(0)\rangle\rangle$, given by

$$|\rho_0(\tau)\rangle\rangle = \sum_{k,l} \exp(-i\Lambda_{kl}\tau)|\nu_k, \nu_l\rangle\rangle\langle\langle\nu_k, \nu_l|\rho(0)\rangle\rangle, \quad (4)$$

where eigenfrequencies Λ_{kl} and eigenmodes $|\nu_k, \nu_l\rangle\rangle$ of the Liouvillian $\mathcal{L}_0$ are related to the respective eigenvalues Λ_k and eigenstates $|\nu_k\rangle$ of the Hamiltonian H_0/J via $\Lambda_{kl} = \Lambda_k - \Lambda_l$ and $|\nu_k, \nu_l\rangle\rangle = |\nu_k\rangle \otimes |\nu_l\rangle^*$ [48]. Eigenfrequencies always come in pairs, $\Lambda_{kl} = -\Lambda_{lk}$. For weak noise ($\gamma \ll 1$), eigenmodes and eigenfrequencies of the perturbed system can be determined using perturbation theory in Liouville space [63]. To first order, we obtain

$$\Lambda_{kl}^{\mathrm{p}} \simeq \Lambda_{kl} - i\gamma m_{kl}, \qquad |\nu_k, \nu_l\rangle\rangle^{\mathrm{p}} \simeq |\nu_k, \nu_l\rangle\rangle^{(0)} - \gamma|\nu_k, \nu_l\rangle\rangle^{(1)}, \qquad (5)$$

where we have used the superscript p to label the perturbed quantities [64].

According to Eq. (5), eigenmodes of the quantum many-body system experience a selective exponential decay with rate γm_{kl}. Stable synchronization occurs when all the modes decay to zero except one. This leads to a decoherence-free subspace [65] with only a single eigenmode [66]: after a given time, which we call synchronization time τ_s, the system will be in one eigenstate $|\nu_k, \nu_l\rangle\rangle^s$ in Liouville space and oscillate with the corresponding eigenfrequency Λ_{kl}^s. On the other hand, transient synchronization appears when there is a clear timescale separation between the different decay times [41]. In this situation, a Liouville eigenstate $|\nu_k, \nu_l\rangle\rangle^t$ which oscillates with frequency Λ_{kl}^t can outlive all the others for a very long time—there is no synchronization otherwise [50]. Both types of synchronization occur in the quantum spin chain [Eq. (1)]. The above route to synchronization is reminiscent of the classical synchronization mechanism known as "suppression of natural dynamics" [5–7]: beyond a critical forcing (coupling) amplitude in forced (spontaneous) synchronization, mode locking does not occur, but natural oscillations of the system are suppressed, leaving it synchronized in a new mode [67–72]. However, the present quantum phenomenon is different: (i) it is noise-induced, (ii) it suppresses all the natural modes of the system except one, (iii) it does not require a critical noise amplitude, and (iv) it does not rely on limit cycles.

Stable synchronization condition.—Let us now derive the stable synchronization condition for the quantum spin chain [Eq. (1)]. We concretely focus on the local spin magnetizations, $\langle\sigma_j^z\rangle = \mathrm{Tr}[\sigma_j^z\rho(\tau)]$, at sites j. Our first task is to connect the abstract eigenstates of the Liouvillian (in Liouville space) to the physical eigenstates of the qubit (in Hilbert space). This is achieved by projecting the supervector $|\rho(\tau)\rangle\rangle$ onto the supervector $|\sigma_j^z\rangle\rangle$ [50]:

$$\sigma_j^z(\tau) = \langle\langle\sigma_j^z|\rho(\tau)\rangle\rangle = \sum_{kl} c_{kl} \exp(-i\tilde{\Lambda}_{kl}\tau)\epsilon_{j,kl}. \qquad (6)$$

The magnetization eigenmodes of the jth qubit are given by the projection $\epsilon_{j,kl} = \langle\langle\sigma_j^z|\nu_k, \nu_l\rangle\rangle$, and the coefficients $c_{kl} = \langle\langle\nu_k, \nu_l|\rho(0)\rangle\rangle$ depend on the initial excitations. The magnetization frequencies $\tilde{\Lambda}_{kl}$ are thus a subset of $\{\Lambda_{kl}\}$.

Our next task is to evaluate the decay constants m_{kl}. The quantum XY model [Eq. (1)] is integrable and can be diagonalized exactly with the Jordan-Wigner transformation [44]. The system has a total of $\#\tilde{\Lambda} = \lfloor N^2/4 \rfloor$ magnetization eigenfrequencies of which $\#\tilde{\Lambda}^{\mathrm{non}} = \lfloor N/2 \rfloor$ are nondegenerate and $\#\tilde{\Lambda}^{\mathrm{deg}} = \lfloor N^2/2(N/2 - 1) \rfloor$ are twofold degenerate (here, $\lfloor \cdot \rfloor$ denotes the floor function). The degeneracy strongly affects the decay rates. For nondegenerate eigenstates, the decay constants (in first-order perturbation theory) read as $2m_{kl} = \langle\langle\nu_k, \nu_l|\mathcal{V}^2|\nu_k, \nu_l\rangle\rangle$. On the other hand, since the noise operator V is real and symmetric in the Jordan-Wigner representation, the decay constants for degenerate eigenstates are $2m_{ab}^{\pm} = (\mathcal{V}^2)_{aa} \pm |(\mathcal{V}^2)_{ab}|$, where $\{|a\rangle\rangle, |b\rangle\rangle\}$ denotes the degenerate eigenspace and $(\mathcal{V}^2)_{ab} = \langle\langle a|\mathcal{V}^2|b\rangle\rangle$. The perturbation moreover lifts the degeneracy. The zeroth order eigenmodes are explicitly given by $|a^\pm\rangle\rangle = \{|b\rangle\rangle \pm \mathrm{sgn}[(\mathcal{V}^2)_{ab}]|a\rangle\rangle\}/\sqrt{2}$. In the Hilbert space of H_0, we find that $\langle\langle\sigma_j^z|a^\pm\rangle\rangle = \{1 \pm \mathrm{sgn}[(\mathcal{V}^2)_{ab}]\}\epsilon_{j,a}/\sqrt{2}$.

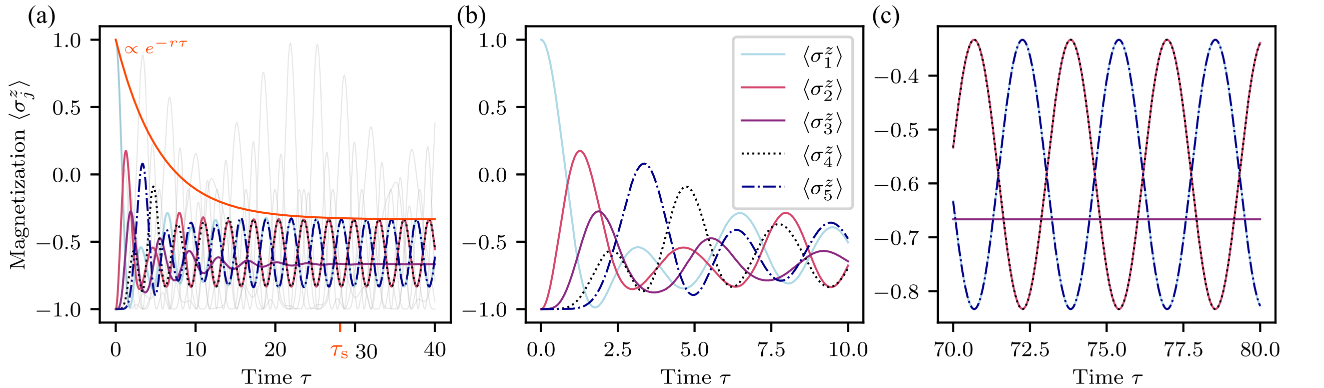

FIG. 1. Stable synchronization. (a) Evolution of the local magnetizations $\langle\sigma_j^z\rangle$ of the quantum XY spin chain [Eq. (1)] of length $N=5$ with white noise amplitude $\gamma=0.2$ applied to site $u=3$ (gray lines in the background show the corresponding noise-free evolutions). The stable synchronization condition [Eq. (8)] is obeyed. The system has $\#\Lambda=\lfloor 5^2/4\rfloor=6$ eigenmodes with respective decay constants $m_{12}=m_{14}=m_{23}=2/3$, $m_{13}=4/9$, $m_{15}=8/9$, and $m_{24}=0$. The smallest nonzero one, $r=\gamma m_{13}$, sets the decay to the synchronized state [Eq. (9)] (orange line). (b) No synchronization occurs for times shorter than the synchronization time $\tau_\mathrm{s}=5/r$. (c) Stable synchronization between the end spins, $\langle\sigma_1^z\rangle$ and $\langle\sigma_5^z\rangle$, as well as between $\langle\sigma_2^z\rangle$ and $\langle\sigma_4^z\rangle$, appears for times larger than τ_s. The initial state is $|\Psi(0)\rangle=|1\rangle_1\otimes\bigotimes_{j=2}^N|0\rangle_j$, where $(|0\rangle_j, |1\rangle_j)$ are the ground and excited states of qubit j.

Depending on the sign of $(\mathcal{V}^2)_{ab}$, decay will therefore be either faster with m_{ab}^+ or slower with m_{ab}^-.

One-site noise: We proceed by applying noise to a single qubit of the chain located at site u and set accordingly $V=\sigma_u^z$ [73]. In this case, $(\mathcal{V}^2)_{ab}<0$, and the rates are thus m_{ab}^-. We concretely obtain [50]

$$m_{kl}^u|_{\mathrm{non}}=\frac{4}{N+1}\left[\sin\left(\frac{uk\pi}{N+1}\right)^2+\sin\left(\frac{ul\pi}{N+1}\right)^2\right]-\frac{16}{(N+1)^2}\left[\sin\left(\frac{uk\pi}{N+1}\right)^2\sin\left(\frac{ul\pi}{N+1}\right)^2\right], \tag{7}$$

for nondegenerate eigenstates, $\tilde{\Lambda}_{kl}\in\tilde{\Lambda}^{\mathrm{non}}$. For degenerate eigenstates, $\tilde{\Lambda}_{kl}\in\tilde{\Lambda}^{\mathrm{deg}}$, the second square bracket in Eq. (7) is multiplied by a factor of 2. The overall decay scales like $1/N$, implying slower decay for longer chains.

Stable synchronization is achieved when all the modes, except one, decay to zero. From Eq. (7), we find that $m_{kl}^u=0$ for a single mode is only possible when [50]

$$\frac{N+1}{3}\in\mathbb{N},\qquad \frac{u}{3}\in\mathbb{N},$$
$$k=\frac{N+1}{3},\qquad l=2k\quad(\text{for } N\geq 5) \tag{8}$$

is fulfilled [74]. The synchronized mode is then [50]

$$|\epsilon_{kl}\rangle^s=\frac{3}{N+1}\begin{cases}(1,-1,0,-1,1,\ldots,1,-1)^T, & N\text{ even}\\ (1,-1,0,-1,1,\ldots,-1,1)^T, & N\text{ odd.}\end{cases} \tag{9}$$

with $|\epsilon_{kl}\rangle=(\epsilon_{1,kl},\epsilon_{2,kl},\ldots,\epsilon_{N,kl})^T$. The corresponding eigenfrequency is $\Lambda_{kl}^s=2$. We see from Eq. (9) that the end magnetizations are synchronized (antisynchronized) for N (odd) even. Remarkably, noise at a single site suffices to (anti)synchronize the endpoints of a chain of arbitrary length. We note that the amplitude of the (anti)synchronized mode scales inversely to the length.

Figure 1(a) shows the time evolution of the magnetization $\langle\sigma_j^z\rangle$ (colored lines) for a chain of length $N=5$ and white noise applied to site $u=3$ (the gray lines in the background indicate the unperturbed evolution in the absence of noise for comparison). This case obeys the stable synchronization condition [Eq. (8)]. Oscillations are out of phase, and no synchronous behavior is seen, for times smaller than the synchronization time τ_s [Fig. 1(b)]. However, for times larger than τ_s, stable synchronization between the endpoints of the chain, $\langle\sigma_1^z\rangle$ and $\langle\sigma_5^z\rangle$, as well as between $\langle\sigma_2^z\rangle$ and $\langle\sigma_3^z\rangle$, appears (the magnetization $\langle\sigma_3^z\rangle$ is independent of time in this regime) [Fig. 1(c)].

Two-site noise: The effect of noise simultaneously applied to several sites may be studied in a similar manner. For two sites, $V=\sigma_u^z+\sigma_v^z$, we find [50]

$$m_{kl}^{u,v}|_{\mathrm{non}}=\frac{4}{N+1}\left[\sin\left(\frac{uk\pi}{N+1}\right)^2+\sin\left(\frac{vk\pi}{N+1}\right)^2+\sin\left(\frac{ul\pi}{N+1}\right)^2+\sin\left(\frac{vl\pi}{N+1}\right)^2\right]-\frac{16}{(N+1)^2}\left[\sin\left(\frac{uk\pi}{N+1}\right)^2+\sin\left(\frac{vk\pi}{N+1}\right)^2\right]\times\left[\sin\left(\frac{ul\pi}{N+1}\right)^2+\sin\left(\frac{vl\pi}{N+1}\right)^2\right], \tag{10}$$

for nondegenerate eigenfrequencies, and

$$m_{kl}^{u,v}|_{\mathrm{deg}}=m_{kl}^{u,v}|_{\mathrm{non}}-\frac{16}{(N+1)^2}\left[\sin\left(\frac{uk\pi}{N+1}\right)\sin\left(\frac{ul\pi}{N+1}\right)+\sin\left(\frac{vk\pi}{N+1}\right)\sin\left(\frac{vl\pi}{N+1}\right)\right]^2, \tag{11}$$

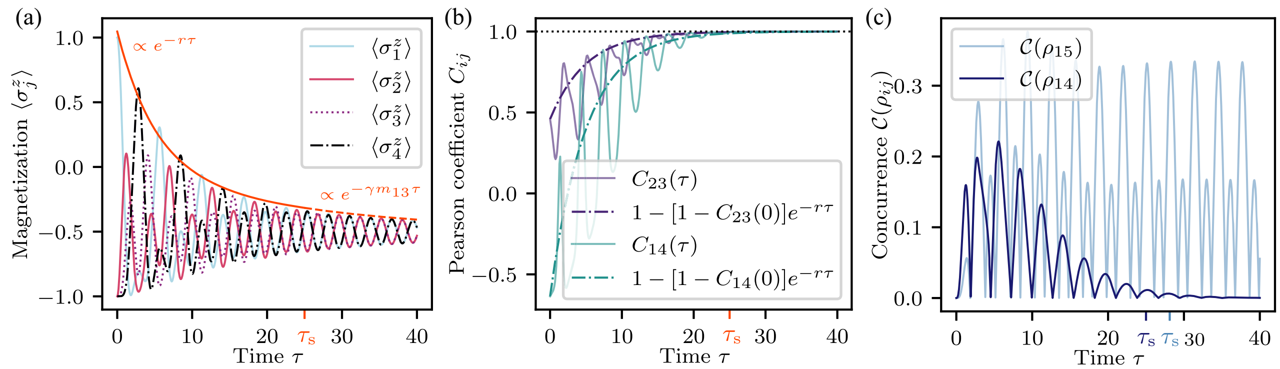

FIG. 2. Transient synchronization and entanglement. (a) Evolution of the local magnetizations $\langle\sigma_j^z\rangle$ of the quantum XY spin chain [Eq. (1)] of length $N=4$ with white noise amplitude $\gamma=0.2$ applied to sites $u=2$ and $v=3$. The stable synchronization condition [Eq. (8)] is not satisfied. The system has $\#\Lambda=\lfloor 4^2/4\rfloor=4$ eigenmodes with respective decay constants $m_{12}=m_{34}=m_{23}=1$ and $m_{13}=1/5$. After $\tau_s=5/(\gamma m_{13})$, transient synchronization (with decay rate $r=\gamma m_{12}$) appears. (b) Pearson correlation coefficients C_{14} and C_{23} showing transient synchronization between the endpoints of the chain, $\langle\sigma_1^z\rangle$ and $\langle\sigma_4^z\rangle$, as well as between $\langle\sigma_2^z\rangle$ and $\langle\sigma_3^z\rangle$. (c) The concurrence $C(\rho_{15})$ for stable synchronization [Fig. 1(a)] displays nonzero steady oscillations after τ_s, indicating entanglement between the edge spins, contrary to transient synchronization $\mathcal{C}(\rho_{14})$ [Fig. 2(a)].

for degenerate eigenfrequencies. The only possible configuration such that $m_{k,l}^{u,v}=0$ for a single mode is [50]

$$\frac{u}{3}\in\mathbb{N},\qquad \frac{v}{3}\in\mathbb{N},\qquad \frac{N+1}{3}\in\mathbb{N},$$
$$k=\frac{N+1}{3},\qquad l=2k. \tag{12}$$

These conditions are equivalent to those indicated in Eq. (8) for a single-site noise. They will thus lead to the same synchronized (antisynchronized) modes. The only difference is that the overall strength of the noise is twice as large here. The two time evolutions will hence be the same with the replacement $\gamma\to\gamma/2$.

Figure 2(a) represents the dynamics of the magnetizations $\langle\sigma_j^z\rangle$ for a chain of length $N=4$ and white noise applied to sites $u=2$ and $v=3$. The stable synchronization condition [Eq. (12)] is not satisfied here. Yet, after the synchronization time τ_s, transient synchronization is observed between the endpoints of the chain, $\langle\sigma_1^z\rangle$ and $\langle\sigma_4^z\rangle$, as well as between $\langle\sigma_2^z\rangle$ and $\langle\sigma_3^z\rangle$. The occurrence of (transient) in-phase oscillation between these qubits is further confirmed by the examination of the corresponding Pearson correlation coefficients, defined as the ratio of the covariance and the respective standard deviations, $C_{ij}=\mathrm{Cov}(\langle\sigma_i^z\rangle,\langle\sigma_j^z\rangle)/\sqrt{\mathrm{Var}(\langle\sigma_i^z\rangle)\mathrm{Var}(\langle\sigma_j^z\rangle)}$ [76]. For $\tau>\tau_s$, both $C_{14}(\tau)$ and $C_{23}(\tau)$ converge to one, implying maximum correlation, and hence synchronous motion, between the local magnetizations [Fig. 2(b)] [77]. We mention that the above quantum synchronization phenomena are robust against weak perturbations, as shown in the Supplemental Material [50].

In order to analyze the entanglement properties of the synchronized edge spins, we plot in Fig. 2(c) the concurrence $\mathcal{C}(\rho_{ij})=\max(0,\sqrt{\kappa_1}-\sqrt{\kappa_2}-\sqrt{\kappa_3}-\sqrt{\kappa_3})$, where $\rho_{ij}=\mathrm{Tr}_{\{1,\dots,n\}\setminus(i,j)}[\rho(t)]$ is the reduced density operator obtained by tracing out the rest of the chain, and κ_n are the ordered eigenvalues of $\rho_{ij}\tilde{\rho}_{ij}$, with $\tilde{\rho}_{ij}$ the spin flipped state [42]. For stable synchronization [Fig. 1(a)], $\mathcal{C}(\rho_{15})$ exhibits steady oscillations with nonzero amplitude after τ_s, revealing that the two end spins are entangled despite the action of the noise (the nondecaying mode is insensitive to the external perturbation). By contrast, for transient synchronization [Fig. 2(a)], $\mathcal{C}(\rho_{14})$ vanishes after τ_s, and the corresponding spins are thus not entangled.

Synchronization time.—The speed of signal propagation in discrete quantum systems with local interactions is upper bounded by the Lieb-Robinson velocity [43], which in the XY model with transverse field is given by $v_{\mathrm{LR}}=2J$. This finite group velocity defines an effective light cone beyond which the amount of transferred information decays exponentially. Consequently, a minimal time is needed for information to travel along a quantum spin chain. Here, we investigate the minimal time it takes to fully (anti) synchronize the two edges of the quantum XY model (1) of arbitrary length N, as a function of the noise strength γ, and compare the result to the Lieb-Robinson bound. To that end, we consider single-site noise applied at site $u=3$, and solve the quantum Liouville equation (3) numerically for varying N and γ. We compute the eigenvalues $-\mu_\alpha(N,\gamma)+i\lambda_\alpha(N,\gamma)$, with real part $\mu_\alpha(N,\gamma)\geq 0$ and imaginary part $\lambda_\alpha(N,\gamma)$. The eigenvalue with the smallest real and nonvanishing imaginary part $\mu^s=\min\{\mu_\beta|\lambda_\beta\neq 0\}$ sets the decay of the synchronized mode. Consequently, $r(N,\gamma)=\min\{\mu_\alpha>\mu^s|\lambda_\alpha\neq 0\}$, sets the relaxation time to the (anti) synchronized state, as seen in Figs. 1 and 2 (orange lines), as well as in Fig. 2(b) for the Pearson coefficients (dashed lines). We thus define the synchronization time as $\tau_s=5/r$.

Figure 3(a) displays the decay rate r as a function of the noise amplitude γ for different chain lengths N. We observe

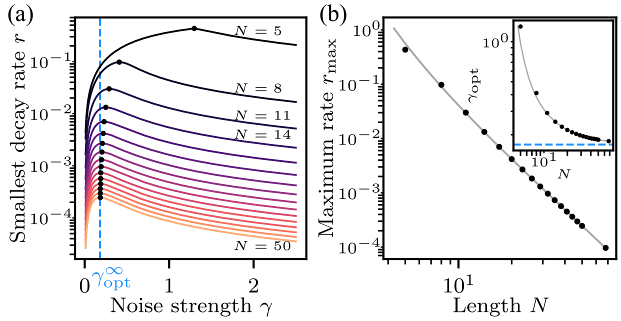

FIG. 3. Optimal synchronization. (a) Decay rate r as a function of the noise strength γ, for various lengths N that fulfill the stable synchronization condition [Eq. (8)]. Dots indicate the optimal strength γ_{opt} leading to fastest synchronization: γ_{opt} tends to the nonzero $\gamma_{\mathrm{opt}}^{\infty}$ for large N (blue dashed line). (b) Maximal rate $r_{\max} = r(\gamma_{\mathrm{opt}})$ as a function of N (inset shows the corresponding strength γ_{opt}). Gray lines are a fit with the function $f(N) = a + b/(N+c)^{\alpha}$ for $N > 5$ (main: $a = 0.00$, $b = 22.65$, $c = -1.75$, $\alpha = 2.94$; inset: $a = 0.17$, $b = 0.78$, $c = -4.84$, $\alpha = 1.01$).

that r first sharply increases with increasing noise strength: intensifying small noise hence significantly speeds up the relaxation, and accordingly reduces the synchronization time. However, beyond an optimal noise amplitude γ_{opt}, the decay rate progressively decreases, and the relaxation is slowed down. This slowing down is related to the phenomenon of noise-induced quantum Zeno effect [78,79]. The maximum decay rate $r_{\max} = r(\gamma_{\mathrm{opt}})$ scales as $1/N^3$, as seen in Fig. 3(b), in agreement with the scaling of the inverse gap of the Liouvillian of the XY model with boundary dissipation [80]. The synchronization time τ_s therefore grows like the third power of the number N of lattice sites, indicating that bigger systems need longer to (anti)synchronize. This dependence is stronger than the $1/N$ scaling of decay rates set by the Lieb-Robinson bound [75,80]. At the same time, the optimal noise strength γ_{opt} decreases as $1/N$, like the related decay rates m_{kl}^{u} in Eq. (7), before saturating at an asymptotic nonzero value $\gamma_{\mathrm{opt}}^{\infty}$, independent of the length N (blue dashed line).

Conclusions.—We have demonstrated the occurrence of stable (anti)synchronization of the endpoints of an isolated quantum spin chain exposed to Gaussian white noise. We have obtained (equivalent) stable synchronization conditions, (8) for one-site noise and (12) for two-site noise, for this noise-induced phenomenon to happen in the quantum domain. We have additionally determined the optimal noise amplitude corresponding to the shortest synchronization time, and shown that the latter grows cubically with the system size, hence stronger than the linear Lieb-Robinson bound. Remarkably, noise applied at a single spin is enough to synchronize a chain of arbitrary length, and synchronized edge spins are nonclassically correlated. This opens up the possibility to employ them for synchronization-based [81,82] quantum communication systems.

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