Error-Mitigated Quantum Metrology via Virtual Purification

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Quantum metrology with entangled resources aims to achieve sensitivity beyond the standard quantum limit by harnessing quantum effects even in the presence of environmental noise. So far, sensitivity has been mainly discussed from the viewpoint of reducing statistical errors under the assumption of perfect knowledge of a noise model. However, we cannot always obtain complete information about a noise model due to coherence time fluctuations, which are frequently observed in experiments. Such unknown fluctuating noise leads to systematic errors and nullifies the quantum advantages. Here, we propose an error-mitigated quantum metrology that can filter out unknown fluctuating noise with the aid of purification-based quantum error mitigation. We demonstrate that our protocol mitigates systematic errors and recovers superclassical scaling in a practical situation with time-inhomogeneous bias-inducing noise. Our result is the first demonstration to reveal the usefulness of purification-based error mitigation for unknown fluctuating noise, thus paving the way not only for practical quantum metrology but also for quantum computation affected by such noise.

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Introduction.—Quantum metrology with entangled resources has been shown to reach the Heisenberg limit of sensitivity with respect to the number of qubits [1–7]. It may provide significant improvements for versatile applications such as atomic-frequency [8,9] and electron-spin-resonance measurements [10,11], magnetometry [12,13], thermometry [14], and electrometers [15].

The Heisenberg limit is susceptible to decoherence; for example, under the effect of Markovian dephasing, the sensitivity of entangled states scales to the standard quantum limit (SQL), as do separable states [8]. Several theoretical studies predict that scaling beyond the standard quantum limit is possible: they include the superclassical scaling, which is also called Zeno scaling, under the time-inhomogeneous noise model [16,17], quantum scaling by quantum teleportation [18,19], using the collective effect of open quantum systems [20,21], and applying quantum error correction [22,23]. So far, however, sensitivity scaling has been mainly discussed from the viewpoint of statistical errors under the assumption that the noise model can be fully characterized [Fig. 1(a)].

In experiments, we cannot always obtain complete information of a noise model typically due to coherence-time fluctuations [24–27]; accordingly, noise characterization becomes intractable, leading to "systematic errors" [Fig. 1(b)]. Systematic errors usually result from a difference between the actual situation and the theoretical estimator used by experimentalists to estimate the target parameter. Intractable noise characterization leads to a biased estimator and induces systematic errors in estimations. In practice, systematic errors are fatal to quantum metrology because they cannot be reduced even when the number of qubits increases, thus seriously limiting any sensitivity improvement [10,28,29]. Despite systematic errors typically being present in experiments, there is as yet no general approach to dealing with them, although some studies have tackled specific scenarios [30–33].

In the present Letter, we propose a quantum-metrology protocol incorporating quantum error mitigation (QEM) to mitigate systematic errors, thereby improving the scaling of sensitivity even in the presence of unknown fluctuating noise [Fig. 1(c)]. While conventional QEM methods have been designed for suppressing systematic errors in the expectation values produced by near-term quantum algorithms [27,34–43], they are not suitable for suppressing the systematic errors coming from unknown fluctuating noise. For example, probabilistic error cancellation cancels the effect of noise by inverting the noise map based on the characterization of the noise [35,36], while error extrapolation assumes the precise control of the noise model [35,37]; thus, unknown fluctuating noise seriously degrades the performance of QEM. To deal with this problem, we

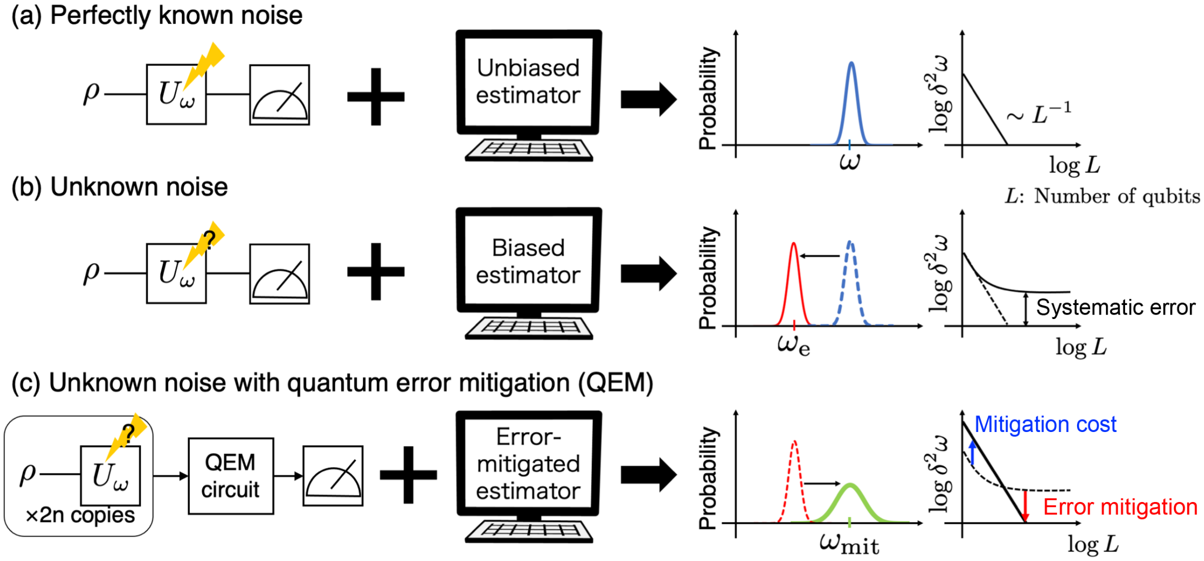

FIG. 1. Schematic illustration of the present Letter. In the standard quantum metrology, we prepare the initial state ρ composed of L qubits, expose it to the target field described by the time-evolution operator U_ω, where some noise process occurs, and measure the parametrized final state. After many iterations, we obtain the average data. Separately, we theoretically calculate the estimator to estimate ω from the average data. (a) Even under noise, perfect knowledge of the noise model provides the unbiased estimator and leads to no systematic errors. (b) Imperfect knowledge of the noise model induced by, e.g., unknown coherence-time fluctuation leads to a biased estimator and involves systematic errors, thus resulting in deterioration of scaling of $\delta^2\omega$, the estimation uncertainty of ω. (c) In the error-mitigated quantum metrology, $2n$ copies of the states after time evolution are put into QEM circuits. Then we calculate the error-mitigated estimator to estimate ω with the average data. Our protocol reduces systematic errors and recovers the scaling of $\delta^2\omega$. Note that the schematic picture of the scaling shows a case of Markovian noise.

construct the protocol of error-mitigated quantum metrology based on purification-based QEM [44,45], so that it can filter out unknown fluctuating noise that differs from one experimental run to another. It is noteworthy that our Letter is the first proposal to use purification-based QEM for overcoming unknown time-fluctuating noise. In numerical simulations, we used it to suppress bias-inducing Markovian and time-inhomogeneous noise and thereby restore the scaling of sensitivity with respect to the number of qubits. In particular, we observed the superclassical scaling with our method.

Quantum metrology with systematic errors.—Here, we describe a general theory for systematic errors in a Ramsey-type measurement of quantum metrology [46–48]. In a typical quantum-metrology setup, we prepare an initial state, expose this state to the target fields characterized by a parameter ω, and obtain a state ρ. Then, we perform a measurement on this state that can be described by a projection operator P producing a binary outcome. The measurement outcome, $m_j \in \{-1, 1\}$, is obtained from the measurement probability,

$$p = \mathrm{Tr}[P\rho] = x + y\omega, \tag{1}$$

where x and y are some scalars. Here, we have assumed that ω is small and have ignored the higher order terms of ω. Repeating the measurement N_{samp} times yields the average value of data, $S_N = \sum_{j=1}^{N_{\mathrm{samp}}} m_j/N_{\mathrm{samp}}$. To estimate the parameter ω, we need to fit this experimental data with a theoretical estimator. To obtain the estimator, we first consider the theoretical density matrix ρ_e and calculate the estimated probability as

$$p_e = \mathrm{Tr}[P\rho_e] = x_e + y_e\omega, \tag{2}$$

where x_e and y_e denote the *estimated* values of x and y, respectively. By reference to Eq. (2), we calculate the estimator of ω as $\omega_e = (S_N - x_e)/y_e$ and estimate ω using the average data S_N with the estimator. If we have imperfect knowledge of noise model, ρ_e would be different from the true one and leads to a biased estimator, thereby leading to systematic errors.

Systematic errors require us to consider the estimation uncertainty of the target quantity [46–48]. The estimation uncertainty of ω is defined as $\delta^2\omega = \langle(\omega - \omega_e)^2\rangle$, with the brackets denoting the ensemble average, and is calculated as

$$\delta^2\omega = \frac{1}{y_e^2}[\mathrm{Var}[p] + (x - x_e)^2], \tag{3}$$

where $\mathrm{Var}[p]$ is the variance of p, which is typically $\mathrm{Var}[p] = p(1-p)/N_{\mathrm{samp}}$, and we have neglected the higher order terms of ω because ω is small. Most of the previous theoretical studies focused on the first term in

Eq. (3) by assuming $\rho = \rho_e$, which comes from the statistical error, as it decreases with increasing $N_{\rm samp}$. The second term in Eq. (3) comes from the systematic error $x - x_e$ induced by the incorrect estimation of the probability $p \neq p_e$. Since $x - x_e$ remains even when $N_{\rm samp}$ increases, it spoils the scaling of $\delta^2\omega$. In the following, we focus on reducing the systematic error $x - x_e$ by using QEM.

Error-mitigated quantum metrology.—Here, we introduce our general framework of error-mitigated quantum metrology, which is inspired by purification-based QEM [44,45]. We assume that we implement $N_{\rm samp}$ experimental runs and the noise fluctuates from one experimental run to another, i.e., we assume a different quantum state for the ith measurement described by ρ_i in the ith experimental run. The key quantity of our framework is the following error-mitigated expectation value of the observable O measured in the quantum circuit in Fig. 2(a):

$$\langle O \rangle_{\rm mit} = \frac{{\rm Tr}[\overline{\rho^n} O]}{{\rm Tr}[\overline{\rho^n}]}, \tag{4}$$

where $\overline{\rho^n} = (1/N_{\rm samp}) \sum_{i=1}^{N_{\rm samp}} (\rho_i)^n$ is the unnormalized purified density matrix made from n copies of noisy density matrices; later, we will discuss its filtering effect on fluctuating noise.

Our protocol is described as follows [Figs. 1(c) and 2]: (1) make $2n$ copies of the initial state $\rho(0)$; (2) expose these $2n$ copies to the target field simultaneously (or almost at the same time) for an interaction time t, so that we can obtain the $2n$ copies of $\rho_i(t)$ even with fluctuating noise; (3) divide these $2n$ copies of $\rho_i(t)$ in half; (4) input n of $2n$ density matrices to the purification circuit and obtain a "single shot" measurement outcome to calculate the numerator in Eq. (4); (5) input the remaining n density matrices in a similar manner to Eq. (4) but with setting $O = I$, to calculate the denominator in Eq. (4) [49]; (6) repeat (1)–(5) and average the obtained data, $S_{\rm num}$ for ${\rm Tr}[\overline{\rho^n} O]$ and $S_{\rm denom}$ for ${\rm Tr}[\overline{\rho^n}]$: then, compute $S_{N_{\rm mit}} \equiv S_{\rm num}/S_{\rm denom}$ as the estimator for $\langle O \rangle_{\rm mit}$; (7) calculate the error-mitigated estimator for ω as $\omega_e = (S_{N_{\rm mit}} - x_{\rm mit})/y_{\rm mit}$ by calculating $\langle O \rangle_{\rm mit} \equiv x_{\rm mit} + y_{\rm mit}\omega$ using estimated ρ_e, and then estimate ω. Note that the density matrix for the ensemble of $N_{\rm samp}$ states input to the purification circuit can be described as $\rho_{\rm in} = (1/N_{\rm samp}) \sum_{i=1}^{N_{\rm samp}} \rho_i^{\otimes n}$; we can then obtain Eq. (4) from a simple calculation [50].

Now, we show that our protocol can filter out the noisy states and extract a dominant pure state even in the presence of fluctuating noise. Denoting the spectral decomposition of ρ_i as $\rho_i = \sum_k p_k^{(i)} |\psi_k^{(i)}\rangle\langle\psi_k^{(i)}|$, we have

$$\overline{\rho^n} = \frac{1}{N_{\rm samp}} (p_{\rm max})^n \sum_{ik} \left(\frac{p_k^{(i)}}{p_{\rm max}}\right)^n |\psi_k^{(i)}\rangle\langle\psi_k^{(i)}|$$
$$= \frac{(p_{\rm max})^n}{N_{\rm samp}} |\psi_{\rm max}\rangle\langle\psi_{\rm max}| \ (n \to \infty), \tag{5}$$

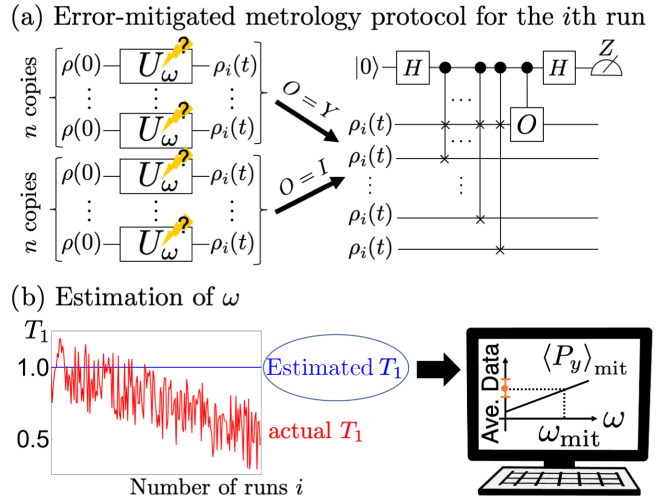

FIG. 2. (a) Error-mitigated metrology protocol for the ith run. $2n$ copies of the initial states are exposed to the magnetic field, and n copies are put to purification circuit [44] with $O = Y$ and the remains are put with $O = I$. (b) Schematic illustration of estimating ω. Using estimated T_1, we calculate the error-mitigated estimator $\langle P_y \rangle_{\rm mit} = {\rm Tr}[\overline{\rho(t)^n} P_y]/{\rm Tr}[\overline{\rho(t)^n}]$ and estimate ω with the data obtained in (a).

where $p_{\rm max} = \max_{i,k} p_k^{(i)}$ and $|\psi_{\rm max}\rangle$ is the corresponding eigenstate. Thus, the contribution of states other than $|\psi_{\rm max}\rangle$ is exponentially suppressed as the number of copies n increases, which means that our method can filter out unknown fluctuating noise. In general, the dominant eigenvector of the mixed state is distorted by noise and differs from the ideal quantum state, which is called coherent mismatch [62]. Nevertheless, our method clearly eliminates the systematic errors, to allow for a dramatic improvement in $\delta^2\omega$ in practical scenarios, as we will see later in the numerical simulations.

Demonstration of error-mitigated quantum metrology.—We demonstrate that our protocol mitigates systematic errors even under unknown fluctuating noise and improves the scaling of $\delta^2\omega$ using entanglement quantum metrology. Here, we consider Markovian and time-inhomogeneous local amplitude damping. We choose the initial probe state $\rho(0) = |{\rm GHZ}\rangle\langle{\rm GHZ}|$ as the L-qubit Greenberger-Horne-Zeilinger (GHZ) state $|{\rm GHZ}\rangle = (|0...0\rangle + |1...1\rangle)/\sqrt{2}$, where $|0\rangle$ and $|1\rangle$ are the eigenstates of the Pauli Z operator σ_z $\sigma_z|1\rangle = |1\rangle$ and $\sigma_z|0\rangle = -|0\rangle$. We consider a uniform magnetic field described by the Zeeman Hamiltonian $H = \sum_{j=1}^{L} \omega\sigma_z^{(j)}/2$ with a parameter ω determined by the target field. Throughout the present Letter, we set $\hbar = 1$ and assume small $L\omega t$. We also assume that the time needed for state preparation, error mitigation, and readout is much shorter than the interaction time with the magnetic fields. We consider that in the ith experimental run, the local amplitude damping with different error rates, $\epsilon_i(t)$, affects each state of the $2n$ copies in the time evolution. The state after the time evolution is described as $\rho_i(t) = \mathcal{E}(\epsilon_i)[e^{-iHt}\rho(0)e^{iHt}]$. Here, $\mathcal{E}(\epsilon_i)$ is the error map denoting the local amplitude damping, in which a single-qubit amplitude damping described by the Kraus operators, $K_1(\epsilon_i) = |0\rangle\langle 0| + \sqrt{1-\epsilon_i(t)}|1\rangle\langle 1|$ and $K_2(\epsilon_i) = \sqrt{\epsilon_i(t)}|0\rangle\langle 1|$, acts on every L qubit. The effect of fluctuating noise is included in $\epsilon_i(t) = 1 - \exp(-t/T_{1,i})$ for Markovian noise and in $\epsilon_i(t) = 1 - \exp[-(t/T_{1,i})^2]$ for time-inhomogeneous noise, where $T_{1,i}$ is the coherence time in the ith experimental run; we consider the actual coherence time drifting from 1.0 to 0.5 as $T_{1,i} = 1.0 - 0.5i/N_{\mathrm{samp}}$ with uniform random fluctuation between $[-0.25, 0.25]$, while the estimated coherence time is assumed to be $T_{1,e} = 1.0$ [Fig. 2(b)]. We also fix the total experimental time as $T = N_{\mathrm{samp}}t = 100$. From the $2n$ copies of $\rho_i(t)$, we obtain $\mathrm{Tr}[\overline{\rho(t)^n}]$ with $O = I$ and $\mathrm{Tr}[\overline{\rho(t)^n}Y]$ with $O = Y$, as shown in Eq. (4), where $Y = 2P_y - I$, $P_y = |\mathrm{GHZ}_y\rangle\langle \mathrm{GHZ}_y|$, and $|\mathrm{GHZ}_y\rangle = (|0\ldots 0\rangle - i|1\ldots 1\rangle)/\sqrt{2}$. These expectation values lead to an error-mitigated probability of $\langle P_y\rangle_{\mathrm{mit}} = \mathrm{Tr}[\overline{\rho(t)^n}P_y]/\mathrm{Tr}[\overline{\rho(t)^n}]$: see the Supplemental Material (SM) [50] for detailed calculations.

We now turn our attention to the numerical results. The performance of our protocol is evaluated by comparing $\delta^2\omega$ with and without QEM; see the numerical details in SM [50]. The ideal case is shown by the black dotted line in Figs. 3(a) and 3(b), when we consider that the actual coherence time is constant and correctly estimated, $T_{1,i} = T_{1,e} = 1.0$: $\delta^2\omega$ follows the conventional SQL scaling for Markovian noise, $\delta^2\omega \sim L^{-1}$, and superclassical scaling for time-inhomogeneous noise, $\delta^2\omega \sim L^{-1.5}$ [16,17]. However, when the actual coherence time is drifted as described above, the systematic error occurs and significantly spoils the scaling of $\delta^2\omega$ as shown by the blue line with points in Figs. 3(a) and 3(b): $\delta^2\omega \sim L^0$ for Markovian noise and $\delta^2\omega \sim L^{-1}$ for time-inhomogeneous noise. The crucial reason for this deterioration is that the systematic error is not reduced by increasing L for both cases as $|x - x_e| \sim L^0$, while the statistical error is reduced. Thus, for large enough L, the second term in Eq. (3) coming from the systematic error is dominant and spoils the scaling.

Now, we show that our protocol mitigates the systematic error and dramatically improves the scaling of $\delta^2\omega$. In Figs. 3(a) and 3(b), the red line with triangles (for $n = 2$) and the green line with squares (for $n = 3$) show $\delta^2\omega$ in our error-mitigated quantum metrology; they demonstrate that our protocol recovers the SQL scaling for Markovian noise and superclassical scaling for time-inhomogeneous noise even when the estimated coherence time is different from the actual one. This demonstrates that our protocol successfully mitigates systematic errors and recovers the scaling; we also demonstrate the usefulness of our protocol for generalized amplitude damping in SM [50]. In the purification-based QEM, as we increase the number of copies n,

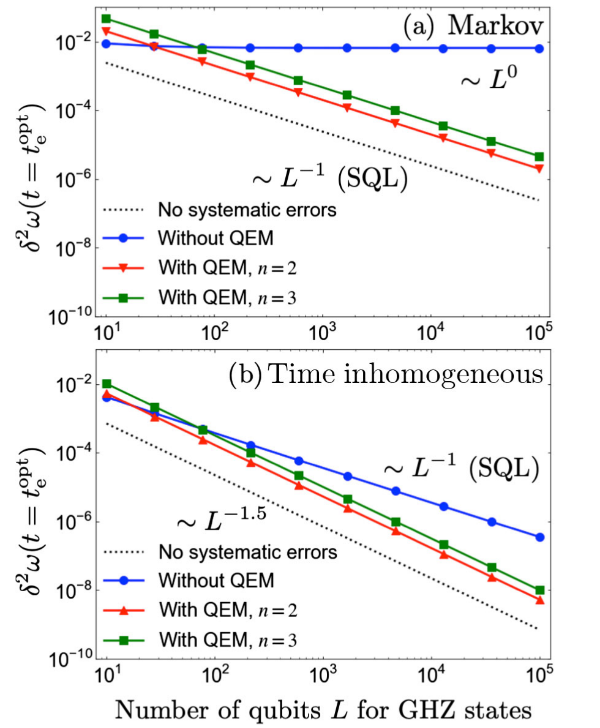

FIG. 3. Estimation uncertainty $\delta^2\omega$ for (a) Markovian and (b) time-inhomogeneous local amplitude damping. Although the ideal scaling is realized by the unbiased estimator with the correct estimation of the noise model (black dotted line), a biased estimator with an incorrect estimation of the noise model leads to systematic errors and spoils the scaling (blue line with circles). Our protocol mitigates systematic errors and recovers the ideal scaling (red and green lines with triangles and squares, respectively).

systematic errors are reduced better with an exponentially higher sampling cost [44]. Therefore, the increase of n improves (reduces) the sensitivity when the systematic (statistical) error is dominant. Accordingly, we see the crossover between $n = 1$ and $n \geq 2$ in Figs. 3(a) and 3(b). We also investigate how the number of copies of the GHZ states affects the uncertainty $\delta^2\omega$ to obtain the quantum enhancement in SM [50].

Considering the trade-off between mitigating systematic error and increasing the statistical error, one may expect that there should be a crossover point between $n = 2$ and $n = 3$; indeed we can see it in the plot for another model in Fig. S4 [50]. In the present example in Fig. 3, however, there is no crossover point between different n for $n \geq 2$, and thus $n = 2$ is always optimal for large enough L. This is because the systematic error decays as fast as or faster than the statistical error by increasing L for $n \geq 2$; the former scales as $L^{-(n-1)}$ (L^{-n}), while the latter scales as L^{-1} $(L^{-3/2})$ for Markovian (time-inhomogeneous) noise. Therefore, the statistical error is always dominant for large enough L for $n \geq 2$, and $n = 2$ is optimal due to its least statistical error among $n \geq 2$. This phenomenon comes from the L dependence of the systematic error for $n \geq 2$ in the present example, $|x_e - x| \sim L^{-(n-1)}$ [50]. We can relate the L dependence of $|x - x_e|$ to the Rényi entropy of the error states [50], which is related to the performance of purification-based QEM [44]. In general, L entangled qubits provide an exponentially increasing state space, and this typically increases the Rényi entropy of error states. Thus, we expect that our protocol will work very well in quantum metrology using entangled states: see also the demonstrations of various models with different Rényi entropy in SM [50].

*Conclusion and outlook.—*We proposed an error-mitigated quantum metrology to reduce systematic errors coming from an incorrect estimation of unknown noise typically induced by coherence-time fluctuations. Our error-mitigated quantum metrology, inspired by purification-based QEM, filters out fluctuating noise that differs from one experimental run to another. We used our method to suppress bias-inducing Markovian and time-inhomogeneous noise, where systematic errors spoil scaling of $\delta^2\omega$, and demonstrated restoration of the scaling. In particular, for the latter case, our method led to super-classical scaling. Here, we should mention that the number of copies of the input density matrix in our method may be reduced by using other methods related to purification-based QEM [63–65] and that coherent error may be further reduced by combining our method with generalized subspace expansion [66]. Our results suggest that our scheme would be useful not only for quantum metrology affected by unknown fluctuating noise but also for quantum computation affected by such noise.

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