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Probing the Critical Nucleus Size in the Metal-Insulator Phase Transition of VO₂

Lei Jin, ^{1,2} Yin Shi, ³ Frances I. Allen, ^{1,4} Long-Qing Chen, ³ and Junqiao Wu[©], ^{1,2,*}

¹Department of Materials Science and Engineering, University of California, Berkeley, California 94720, USA

²Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

³Department of Materials Science and Engineering, Pennsylvania State University, University Park, Pennsylvania 16802, USA

⁴National Center for Electron Microscopy, Molecular Foundry, Lawrence Berkeley National Laboratory,

Berkeley, California 94720, USA

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In a first-order phase transition, critical nucleus size governs nucleation kinetics, but the direct experimental test of the theory and determination of the critical nucleation size have been achieved only recently in the case of ice formation in supercooled water. The widely known metal-insulator phase transition (MIT) in strongly correlated VO_2 is a first-order electronic phase transition coupled with a solid-solid structural transformation. It is unclear whether classical nucleation theory applies in such a complex case. In this Letter, we directly measure the critical nucleus size of the MIT by introducing size-controlled nanoscale nucleation seeds with focused ion irradiation at the surface of a deeply supercooled metal phase of VO_2 . The results compare favorably with classical nucleation theory and are further explained by phase-field modeling. This Letter validates the application of classical nucleation theory as a parametrizable model to describe phase transitions of strongly correlated electron materials.

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Nucleation is the initial step in the formation of a new thermodynamic phase in a supersaturated parent phase [1,2]. For nearly a century, it has been assumed that a successful nucleation event requires the size of the nucleus to exceed a critical nucleation size in order to stabilize the new phase, a hypothesis that constitutes the basis of classical nucleation theory (CNT) [3,4]. Nevertheless, because of the nanoscale size and transient nature of the nucleation process, as well as the difficulty to achieve supercooled matter at ultrahigh purities, direct experimental demonstration of CNT has been challenging. Very recently, Bai et al. [5] have shown that graphene oxide nanosheets serve as seeds to trigger ice nucleation in supercooled water droplets, but they do so only if their diameters are above certain critical sizes. The existence of these critical nucleation sizes is well expected and understood for first-order transitions in conventional condensed matter (e.g., the water-ice transition).

In contrast, the metal-insulator transition (MIT) of vanadium dioxide (VO₂) is fundamentally different. VO₂ switches from an insulating (I) phase at temperatures (T) lower than $T_{\rm MIT}=67\,^{\circ}{\rm C}$ to a metallic (M) phase at temperatures higher than $T_{\rm MIT}$. The MIT is accompanied by a structural transformation from a monoclinic structure at $T < T_{\rm MIT}$ to a tetragonal structure at $T > T_{\rm MIT}$. The nature of the MIT in VO₂ is still under debate and the nucleation process of the MIT is poorly understood [6–10], with unanswered questions such as whether the MIT and the structural transformation are decoupled at the nucleation stage and follow different kinetic pathways afterward. Prior efforts have demonstrated undercooling in VO₂ nanoparticles consistent

with particle size as well as inferred nucleation potency distributions [11,12]. Inspired by the experimental demonstration of the applicability of CNT to water [5], in this Letter we experimentally tested CNT and probed the critical nucleation size of the I-phase nucleus in a supercooled M phase of VO_2 . The experimental results are quantitively explained by phase-field modeling of the energetics of the process.

To probe the critical nucleation size, one must tackle the first challenge that the system needs to be in a deeply supersaturated state to have a sufficiently high thermodynamic driving force for the phase transition. We achieve this in single-crystal VO₂ nanobeams (thickness ~100 nm, width \sim 250 nm, length \sim 50 μ m) by using "irradiation shielding" via helium ion (He⁺) irradiation. As elaborated below, here "shielding" means that VO₂ is locally irradiated in a way that the unirradiated part is geometrically isolated and "shielded" from external influences (e.g., contacts, substrate) that could trigger the nucleation, enabling deep supercooling for that shielded part of VO₂. Achieving supercooled VO₂ has been reported previously by applying substrate strain [13] or using graded tungsten doping [14]. However, the supercooled VO₂ obtained by substate strain or graded W doping exhibits a MIT deviating from the intrinsic behavior of VO₂. In contrast, irradiation shielding can be used to achieve deeply supercooled VO₂ specimens that show a "clean," intrinsic MIT, as demonstrated below. In this approach, we employ the established MIT engineering technique using He^+ irradiation, where the T_{MIT} of VO₂ is controllably reduced by He⁺ irradiation using a Zeiss Orion NanoFab He ion microscope (HIM) [15,16]. In the irradiation process, most of the energetic (\sim 30 keV) He⁺ ions penetrate through the entire thickness of the VO₂ nanobeams, leaving behind uniformly distributed point defects (vacancies and interstitials). The overall effect of these defects is to donate free electrons, which reduces $T_{\rm MIT}$ and increases the electrical conductivity (σ) of I-phase VO₂ [15,17–20]. This effect of reducing $T_{\rm MIT}$ with He⁺ irradiation is shown in Fig. 1(a). The $T_{\rm MIT}$ of the nanobeam can be tuned from the natural $T_{\rm MIT}$ (\sim 341 K) to near room temperature (300 K) by increasing the He⁺ irradiation dose up to 10^{16} ions/cm². The irradiated VO₂ still maintains a high-quality MIT as manifested by the sharp jump in σ , and is fully crystalline in this dose range [15].

Taking advantage of the localized irradiation capability of the HIM, we carry out patterned irradiation along the VO_2 nanobeam. The schematic in Fig. 1(b) depicts the idea

of irradiation shielding. A pristine, unirradiated VO2 segment (gray block) is sandwiched by two He⁺-irradiated segments (green blocks) along a single nanobeam. The platinum (Pt) bonding (gold blocks), deposited using a gallium focused ion beam (FIB) with a platinum-based organometallic precursor, secures electrical and thermal contact between the nanobeam and the underlying electrodes. The nanobeam is suspended from the substrate. In the M to I phase transition during cooling, the He⁺-irradiated segments stay in the M phase until T is lower than their reduced $T_{\rm MIT}$, following Fig. 1(a). Importantly, during this entire process, the central pristine segment also stays in the M phase, because it is shielded by the two neighboring, M-phase segments from sites that could nucleate the I phase in it, such as the substrate and the Pt contacts. It is obvious that the lowest temperature that the shielded VO₂ can be supercooled to $(T_{\rm sc})$ is determined by the reduced

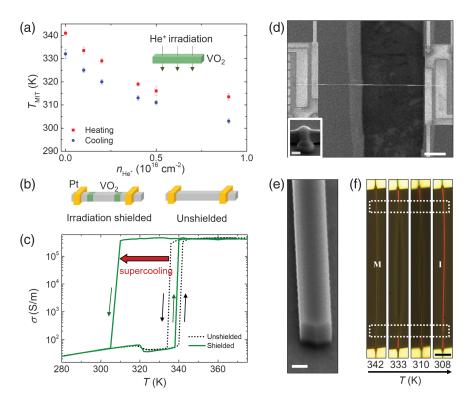


FIG. 1. Creating a deep supercooled state in the MIT of VO₂. (a) The MIT temperature ($T_{\rm MIT}$) of VO₂ nanobeams as a function of the dose of He⁺ ion irradiation. $T_{\rm MIT}$ is determined from electrical transport measurements. The inset shows schematically a VO₂ nanobeam uniformly and globally (i.e., not locally) irradiated with He⁺ ions. (b) Schematics of He⁺ ion irradiation for nucleation shielding in a single VO₂ nanobeam. The middle, pristine VO₂ segment (gray) is shielded against influence from the Pt contacts (yellow blocks) by two end segments of VO₂ that are locally He⁺ irradiated (green). (c) Four-probe measured electrical conductivity of the same VO₂ nanobeams with or without irradiation shielding. Deep supercooling of the *M* phase down to 300 K is observed in the VO₂ end shielded with 5 × 10¹⁵ ions/cm² He⁺ irradiation. (d) Scanning electron microscope (SEM) image of a VO₂ nanobeam supported by two suspended micropads. Scale bar is 10 μm. Inset: SEM image of FIB-deposited Pt bonding of the nanobeam onto the underlying electrode to minimize electrical or thermal contact resistance. Scale bar is 100 nm. (e) SEM image of a VO₂ nanobeam as the temperature drops. A 30-μm-long VO₂ segment is shielded by two 4-μm-long He⁺-irradiated segments at the two ends (indicated by the two white dashed boxes). Despite the fact that the pristine segments between the shields and the electrodes transition to *I* phase at normal $T_{\rm MIT}$ (333 K), the shielded, pristine VO₂ segment stays in the *M* phase (dark) and does not transition to the *I* phase (bright) until 308 K, which is close to the natural $T_{\rm MIT}$ (a) of the irradiated shield segments. Scale bar is 5 μm.

 $T_{\rm MIT}$ of the shields. For comparison, a similar VO₂ nanobeam without irradiation shielding is also shown schematically in Fig. 1(b). Figure 1(c) shows the measured $\sigma(T)$ of two VO₂ nanobeams, with and without irradiation shielding, respectively. The pristine VO₂ nanobeam without irradiation shielding shows a normal MIT at ~341 K with a small hysteresis of ~8 K. The $T_{\rm MIT}=333$ K observed upon cooling the unshielded nanobeam is named $T_{\rm natural}$, as it is the naturally expected $T_{\rm MIT}$ during cooling. In contrast, σ of the shielded VO₂ device stays metallic during cooling until 308 K, showing a deep supercooling of 25 K below the $T_{\rm natural}$.

All VO₂ nanobeams were grown using the vapor transport method published previously [21]. They are all single crystal with rectangular cross section and smooth surface [Fig. 1(e)], maximally eliminating defects such as grain boundaries and domain walls that could nucleate the MIT. Therefore, such a shielded nanobeam, when suspended, provides an ideal supercooled platform for probing the critical nucleation size during the MIT. As shown in Fig. 1(d), two suspended micropads were used to support a nanobeam for electrical and optical measurements at variable temperatures. The micropads were suspended from the substrate with long, flexible arms, fabricated following the method published previously [22], where they were found to allow full axial strain relaxation for the supported nanobeam.

We use the visual color observed under an optical microscope to differentiate the M (dark) and I (bright) phases in the suspended VO₂ nanobeams. Figure 1(f) shows optical images of one such nanobeam during the cooling process. A 30-µm-long pristine segment is shielded by two irradiated segments (indicated by white boxes). The two irradiated segments (shields) switch to I phase once the temperature drops below 308 K, consistent with Fig. 1(a). The two segments between each of the shields and the neighboring Pt contact are pristine, and switch to I phase at 333 K, consistent with the unshielded, pristine MIT as shown in Fig. 1(c). In stark contrast, the VO₂ segment between the shields, although also pristine, remains in M phase when T decreases passing T_{natural} (333 K) until $T_{\rm sc}$ (308 K). Within this temperature window, this shielded VO₂ segment is the ideal supercooled platform where the following experiments are carried out.

The next step is to introduce nucleation seeds with controlled sizes into the supercooled VO₂ segment. As shown in Fig. 2(a), such seeds are created by irradiating the segment with focused Ga⁺ ions (FEI Quanta FIB-SEM instrument). Ga⁺ irradiation is typically used to mill surfaces or cut through samples, as Ga⁺ ions are heavy and when energetic can generate severe lattice damage [23], unlike the much lighter He⁺ ions. In this Letter, the Ga⁺ dose is limited to below 10¹⁷ ions/cm² to avoid significant surface milling (less than a few nanometers as measured by an atomic force microscope and shown in the

Supplemental Material [24]). Special care was taken to focus the Ga^+ beam only into the targeted area. The penetration depth of 30 keV Ga^+ is simulated to be 15 nm in VO_2 using the Stopping and Range of Ions in Matter (SRIM) program [25], much smaller than the thickness of the VO_2 nanobeam. The disk-shaped nucleation seeds as defined by the Ga^+ irradiated zone can be controlled by their diameter (D) and the irradiation dose (n_{Ga+}). The former is varied to probe the critical nucleation sizes, while the latter varies the surface energy to tune the capability of the seed to nucleate the I phase in supercooled, M-phase VO_2 .

As shown in Fig. 2(b), a 50-µm-long single VO₂ nanobeam is patterned with eight periods of segments, where each segment consists of a 3-µm-long pristine zone as the supercooled "test bed," as well as a 3-µm-long, He⁺irradiated zone as the shield. Each shielded test bed is then implanted with one nucleation seed with specific values of D and n_{Ga+} . One test bed is free of seed $(n_{\rm Ga+}=0)$ to allow the measurement of $T_{\rm sc}$, and another small segment outside the shields is also free of seed for measuring T_{natural} , along the same nanobeam. During cooling, the temperature at which each supercooled test bed switches from M to I phase is named the nucleation temperature (T_{nuc}) . By varying D and $n_{\text{Ga+}}$, we measure the corresponding $T_{\rm nuc}$ for each supercooled test bed. If $T_{\rm nuc}$ is between $T_{\rm natural}$ and $T_{\rm sc}$, the *I*-phase nucleation in that test bed is triggered by the nucleation seed. Figure 2(b) shows the color change of each segment when the temperature gradually decreases from $T_{\rm natural}$ (333 K) to $T_{\rm sc}$ (308 K). In this experiment, $n_{\text{Ga}+}$ is fixed at 2.2×10^{16} ions/cm² and D varies from 10 to 180 nm for the eight test beds. For test beds with D of 140 and 180 nm, the I phase was observed immediately at T_{natural} . This indicates that the nucleation seed is large enough to fully suppress the supercooling. At smaller D values, for example, 100 nm, the I phase nucleates at $T_{\rm nuc} = 317$ K, lower than $T_{\rm natural}$ but higher than $T_{\rm sc}$. When D is smaller than ~75 nm, $T_{\rm nuc}$ becomes equal to $T_{\rm sc}$, indicating that such nucleation seeds are too small to trigger the MIT.

For the Ga⁺ doses implemented in this study, we can assume that amorphization of the irradiated disks in the VO₂ has occurred [26,27]. This is supported by our Atomic Force Microscopy (AFM) results, which show sputtering of a few nanometer in depth, indicating that the dose threshold for amorphization has been well surpassed. We note that the actual ion dose profiles and hence the effective seed diameters depend on the Gaussian beam profile of the incident beam, the lateral straggle of the ions interacting with the VO₂, and the scan setting used to pattern each disk. These effects are described in detail in the Supplemental Material [24]. We find that for nominal disk diameters of 25 nm and above, the full width at half maximum of the effective dose profiles are in close agreement with the nominal diameters (within one nanometer).

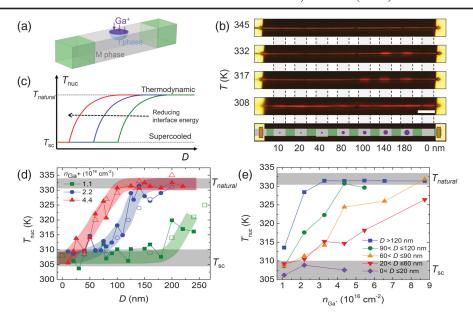


FIG. 2. *I*-phase nucleation in supercooled VO₂. (a) Schematic showing a disk-shaped nucleation seed (purple) created by Ga⁺ ion irradiation on the surface of a shielded pristine VO₂ segment. The Ga⁺ ion penetration depth is ~15 nm, very shallow compared to the diameter of the irradiated area; hence the nucleation can be approximated as a disk shape with zero thickness. (b) Optical image of the *I*-phase nucleation process along a long, suspended VO₂ nanobeam. The nanobeam is divided into eight pristine segments, each shielded by two He⁺-irradiated segments, then a nucleation seed is introduced onto the surface of each of the pristine segments by Ga⁺ irradiation at a dose of 2.2×10^{16} ions/cm², but with different diameters. The lowest panel shows schematically the case where the diameter of the disk-shaped nucleation seed (*D*) increases from 10 to 180 nm. Scale bar is 5 µm. (c) Schematic dependence of *I*-phase nucleation temperature (T_{nuc}) on *D*, as predicted by classical nucleation theory. T_{nuc} is upper bounded by the natural MIT temperature (T_{natural}) and lower bounded by the supercooled temperature (T_{sc}) of the shielded VO₂. With reduced interface energy, stable nuclei with smaller *D* are able to form at a given temperature. (d) Measured (solid symbols) and calculated (open symbols) T_{nuc} as a function of *D* for different Ga⁺ irradiation doses. The thick curved bands are a guide for the eye. The critical sizes of the nucleus can be determined from the onset of the rise above the supercooled limit. (e) Measured dependence of T_{nuc} on Ga⁺ irradiation dose ($n_{\text{Ga}+}$) in the seed at different *D* intervals. A heavier dose in the seed promotes *I*-phase nucleation as shown by the increased T_{nuc} . T_{nuc} for nucleation seeds with large *D* and high doses is found to reach the thermodynamic limit. T_{natural} and T_{sc} are marked by the gray shaded areas in (d) and (e).

We use classical nucleation theory to analyze the critical nucleation diameter $(2r_c)$ as a function of $T_{\rm nuc}$. According to CNT, the change in the Gibbs free energy ΔG for forming a new I phase in a supercooled M phase is expressed as [4]

$$\Delta G = \frac{2}{3}\pi r^3 \cdot \Delta g_v + 2\pi r^2 \cdot \beta + \pi r^2 \cdot \gamma, \tag{1}$$

where r is the radius of the I-phase nucleus, Δg_v is the difference in volumetric Gibbs free energy, β is the interface energy between the I and M phases, and γ is the interface energy between the I phase and the nucleation seed. For simplicity, the I-phase nucleus is assumed to be hemispherical, growing from the disk-shaped nucleation seed as shown in Fig. 2(a). From $\mathrm{d}\Delta G/dr=0$, the critical radius is found to be

$$r_c = \frac{2(\beta + \gamma/2)}{|\Delta g_v|}. (2)$$

In addition [4],

$$\Delta g_v = \frac{\Delta h_f (T_{\text{natural}} - T_{\text{nuc}})}{T_{\text{natural}}},\tag{3}$$

where Δh_f is the volumetric enthalpy of nucleus formation. Subsequently,

$$r_c = \left| \frac{2(\beta + \gamma/2)}{\Delta h_f} \right| \left(\frac{1}{1 - T_{\text{nuc}}/T_{\text{natural}}} \right) \tag{4}$$

and

$$T_{\text{nuc}} = T_{\text{natural}} \cdot \left[1 - \left| \frac{2(\beta + \gamma/2)}{\Delta h_f} \right| \left(\frac{1}{r_c} \right) \right].$$
 (5)

The dependence of $T_{\rm nuc}$ on r_c is shown schematically in Fig. 2(c). $T_{\rm nuc}$ is bounded by the thermodynamic limit $T_{\rm natural}$ and the supercooling limit $T_{\rm sc}$. At these two limits, $T_{\rm nuc}$ no longer depends on D. Between these two limits, the I-phase nucleation is triggered by the seed at $T_{\rm nuc}$ when D is comparable to the critical nucleation size $2r_c$. The I phase is more likely to nucleate at seeds with larger D and higher $n_{\rm Ga+}$. Higher lattice disorder in the nucleation seed arising from higher $n_{\rm Ga+}$ reduces the energy barrier to trigger the

nucleation. This is depicted as a shift of the $T_{\text{nuc}}(D)$ curve toward smaller D with reduced γ , as shown in Fig. 2(c).

 $T_{\rm nuc}$ is experimentally measured by independently varying both $n_{\rm Ga+}$ (from 1.1 to 8.8×10^{16} ions/cm²) and D (from 5 to 260 nm). Figure 2(d) plots $T_{\rm nuc}$ as a function of $n_{\rm Ga+}$ for fixed ranges of D. A monotonic increase in $T_{\rm nuc}$ is observed with n_{Ga+} , suggesting promoted nucleation with higher $n_{\text{Ga+}}$. T_{nuc} is saturated at T_{natural} for D larger than 120 nm and $n_{\rm Ga+}$ higher than $3.3 \times 10^{16}~{\rm ions/cm^2}$. This is the condition of full suppression of the supercooling. If D is smaller than 20 nm or n_{Ga+} is lower than 1.1×10^{16} ions/cm², $T_{\rm nuc}$ saturates at $T_{\rm sc}$, and no I-phase nucleus can be stabilized at the implanted seeds. Figure 2(e) plots T_{nuc} as a function of D for fixed $n_{\text{Ga+}}$, which shows good agreement with the dependence expected from CNT [Fig. 2(c)]. The shift of curves with higher n_{Ga+} to smaller D indicates that the increase in n_{Ga+} reduces the effective interface energy, presumably via a reduction in γ .

To quantitatively analyze the data, we use a previously developed phase-field model of VO₂ [28–31] to calculate the nucleation temperatures of the I phase in freestanding VO₂ nanobeams. This model describes the mesoscopic properties of VO₂ in terms of a structural order parameter field η (characterizing the lattice structural phases), an electronic order parameter field ψ (characterizing the insulating or metallic phases), the free carrier densities, and the elastic strain field. We simplify the model by setting the free carrier densities to be at equilibrium and the elastic energy to zero, because we are only concerned with equilibrium states and the strain in the freestanding nanobeams is fully relaxed. We then set up a VO2 cube with a side length of 300 nm with a stress-free boundary condition imitating part of a freestanding VO₂ nanobeam. The initial state is set to that of a hemispherical *I*-phase nucleus with a given diameter embedded into the surface of the M-phase cube. We then tune the temperature and observe whether the nucleus grows or shrinks to find the nucleation temperature. We take into account the interface energy between the I phase and the Ga^+ -irradiated VO_2 , γ , by renormalizing the gradient energy coefficient (characterizing the domain wall energy) in the phase-field model. γ is related to the renormalized domain wall energy β_t and the true *I-M* domain wall energy β via $\gamma = 2(\beta_t - \beta)$, which simply results from the approximation that the shape of the *I*-phase nucleus is close to a hemisphere. We fit the calculated nucleation temperature as a function of the nucleus diameter to the experimentally measured relation by adjusting the renormalized gradient energy coefficient. The calculated results are shown as open symbols in Fig. 2(d). The yielded γ 's are 1.39, 0.606, and 0.282 J/m² for the Ga⁺ irradiation doses of 1.1, 2.2, and 4.4×10^{16} ions/cm², respectively. The good quantitative agreement between the measured and calculated results strongly supports that the CNT well describes the MIT in VO_2 .

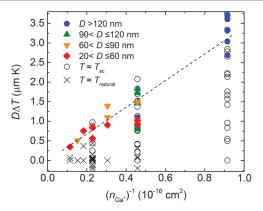


FIG. 3. Measured $D\Delta T$ of all nucleation seeds with different diameters as a function of the reciprocal $\mathrm{Ga^+}$ dose density. The black cross hairs and black open circles are $D\Delta T$ data measured from the nucleation seeds that drive the $\mathrm{VO_2}$ to the thermodynamic or natural and supercooled limit, respectively. Between these two limits (solid and colored symbols), T_{nuc} depends on D and the dose of the nucleation seeds. A linear fit of all colored symbols (black solid line) indicates that the interface energy $(\infty D\Delta T)$ of the nucleation seeds is inversely proportional to their $\mathrm{Ga^+}$ irradiation dose density $n_{\mathrm{Ga+}}$.

Furthermore, based on Eqs. (4) or (5), the total interface energy can be expressed as

$$\beta + \gamma/2 = \frac{|\Delta h_f|}{2T_{\text{natural}}} r_c (T_{\text{natural}} - T_{\text{nuc}}). \tag{6}$$

Between the thermodynamic (natural) and supercooling limits, $D \approx 2r_c$. Given that Δh_f and $T_{\rm natural}$ are constant, it is clear that $D\Delta T \propto \beta + \gamma/2$, where $\Delta T = T_{\rm natural} - T_{\rm nuc}$. In Fig. 3, the measured $D\Delta T$ is plotted as a function of $1/n_{\rm Ga+}$, where colored symbols represent data with I-phase nucleation occurring between (i.e., not reaching) the thermodynamic and supercooling limits. These colored data points can be fitted with a linear dependence on $1/n_{\rm Ga+}$. Such a dependence shows an empirical relationship between the Ga⁺ irradiation dose $(n_{\rm Ga+})$ and the lowering of the interface energy γ . As the interface is one between crystalline VO₂ and the irradiation amorphized VO₂, future work to elucidate the mechanism behind this relationship might provide a useful knob to control the MIT at the level of a single nucleation event.

In conclusion, we show that classical nucleation theory governs the kinetic nucleation process in the coupled structural-electronic phase transition in a strongly correlated electron material. The critical nucleation size in the transition is determined experimentally, and found to be as small as tens of nanometers depending on the interface energy of the heterogeneous nucleus. A deeply supercooled VO₂ test bed is created by shielding it from other nucleation sites using He⁺ ion irradiation. A nucleation seed is introduced to the test bed by surface irradiation with energetic Ga⁺ ions. The achieved deep supercooling state

in the pristine, single-crystal and strain-free VO₂ may also serve as a clean platform for probing the intrinsic properties of its metal-insulator transition.

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- *wuj@berkeley.edu
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