Erratum: Probing Long-Range Neutrino-Mediated Forces with Atomic and Nuclear Spectroscopy [Phys. Rev. Lett. 120, 223202 (2018)]

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In this Erratum, we point out that the oversimplified approach in the Letter authored by one of us (Y. V. S.) led to limits on a long-range neutrino-mediated force which were overestimated by a factor of 6 in nonhadronic atoms and underestimated by about 5 orders of magnitude in the case of hadronic atoms and the deuteron binding energy. The qualitative conclusion of this Letter, namely that atomic- and sub-atomic-scale experiments provide much better sensitivity to a long-range neutrino-mediated force than macroscopic-scale experiments, remains unchanged.

The form of the potential associated with the long-range neutrino-mediated force, presented in this Letter, reads

$$V_\nu(r) = \frac{G_F^2}{4\pi^3 r^5}\left\{a_1 a_2 - \frac{2}{3} b_1 b_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{5}{6} b_1 b_2 [\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}})]\right\}, \tag{1}$$

where $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the Pauli spin matrix vectors of the two particles, and a_i and b_i represent species-dependent parameters. It is worth noting that the last (tensor) term in Eq. (1) is zero for s orbitals which are the atomic orbitals most strongly affected by a long-range neutrino-mediated force.

A potential scaling as $\propto 1/r^5$ gives divergent integrals ($\int_{r_c} d^3r/r^5 \approx 1/2r_c^2$) in the matrix elements for s-wave atomic states. Using the nuclear radius R as the cutoff length scale, $r_c = R$, would give qualitatively incorrect results in the case of the usual hadronic atoms. A more accurate approach requires first building an effective potential for the electron-quark interactions and then taking into account the nucleon distribution $\rho(r)$ inside the nucleus. To include small distances, we present the neutrino-exchange potential for the finite size R of a nucleus and cutoff for large momenta (small distances r) produced by the Z-boson propagator $1/(q^2 + M_Z^2)$, instead of the contact form $1/M_Z^2$. To start with, we replace $1/r^5$ in the potential (1) above with the following function that takes into account nonzero values of momentum transfer q:

$$F(r) = \frac{8m^4c^4}{3\hbar^4}\frac{I(r)}{r}, \tag{2}$$

where, for $z \equiv M_Z/(2m)$:

$$I(r) = \int_1^\infty e^{-2xmcr/\hbar}\left(x^2 - \frac{1}{4}\right)\frac{\sqrt{x^2-1}\,z^4}{(x^2+z^2)^2}dx. \tag{3}$$

Here, $2m$ is the mass of the fermion-antifermion pair appearing in the loop. The function $F(r) \propto I(r)/r$ gives us the dependence of the interaction between an electron and quark (or an electron and another pointlike fermion) on the distance r between them. For $\hbar/(M_Z c) \ll r \ll \hbar/(mc)$, $F(r) \approx 1/r^5$; at these intermediate distances, the neutrino-mediated potential has the form in Eq. (1). On the long length scales $r \gg \hbar/(mc)$, $F(r) \propto \exp(-2mcr/\hbar)/r^{5/2}$. At the short distances $r \ll r_c = \hbar/(M_Z c)$, $F(r) \propto \ln(r)/r$ and there is no divergence when the function $F(r)$ is integrated over d^3r. Note that the behavior of the neutrino-exchange potential at short distances has been investigated recently in Ref. [1]; however, in that paper, the potential was not studied in the context of the standard model, but instead the authors considered a new scalar particle instead of the Z boson.

Convergence of the integral in the matrix elements of the neutrino-mediated potential, which is proportional to $F(r)$, at the distance $r \sim r_c = \hbar/(M_Z c)$ indicates that this interaction may be treated as a contact interaction in atomic and subatomic systems. In such systems, we can hence replace $F(r)$ by its form in the contact limit, $F(r) \to C\delta(\mathbf{r})$:

$$C = \int_0^\infty F(r)d^3r = \frac{\pi}{3}\frac{M_Z^2c^2}{\hbar^2}, \tag{4}$$

where we have assumed that $z = M_Z/(2m) \gg 1$. Note that if we were to assume the potential form $1/r^5$ with the cutoff $r_c = \hbar/(M_Z c)$, the result would be 6 times larger:

$$C' = \int_{r_c}^{\infty} \frac{1}{r^5} d^3 r = 2\pi \frac{M_Z^2 c^2}{\hbar^2}. \tag{5}$$

Using Eq. (4), the potential in Eq. (1) may be presented as follows in the contact limit:

$$V_\nu^c(r) = \frac{G_F^2 M_Z^2 \delta(\mathbf{r})}{12\pi^2} \left\{ a_1 a_2 - \frac{2}{3} b_1 b_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{5}{6} b_1 b_2 [\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}})] \right\} \equiv g\delta(\mathbf{r}), \tag{6}$$

where we have implemented the natural units $\hbar = c = 1$.

Using the potential in Eq. (6), we find that the limits in our Letter on a long-range neutrino-mediated force were overestimated by a factor of 6 in nonhadronic atoms and underestimated by 5 orders of magnitude in hadronic atoms. The corrected limits based on the $1S-2S$ transition in positronium, ground-state hyperfine splitting (HFS) in positronium, ground-state hyperfine splitting in muonium, as well as the $1S-2S$ isotope shift in hydrogen and deuterium are as follows:

$$G_{\text{eff}}^2 < 1.5 \times 10^9 G_F^2 \qquad \text{positronium } 1S-2S, \tag{7}$$

$$G_{\text{eff}}^2 < 9.0 \times 10^7 G_F^2 \qquad \text{positronium HFS}, \tag{8}$$

$$G_{\text{eff}}^2 < 1.1 \times 10^3 G_F^2 \qquad \text{muonium HFS}, \tag{9}$$

$$G_{\text{eff}}^2 < 1.6 \times 10^6 G_F^2 \qquad \text{hydrogen-deuterium } 1S-2S. \tag{10}$$

In the case of the deuteron nucleus, the wave function may be found by noting the short-range character of the strong interaction between nucleons and the relatively small binding energy of deuteron. In the region outside the interaction range $r_0 \approx 1.2$ fm, we use the solution to the zero-potential Schrödinger equation, whereas within the interaction range, the wave function has a constant value for the s state:

$$\psi(r) = \begin{cases} B\exp(-\kappa r)/r & \text{for } r > r_0, \\ BJ(0)/r_0 & \text{for } r < r_0, \end{cases} \tag{11}$$

where the normalization constant B is given by $4\pi B^2 = 2\kappa$, with $\kappa = \sqrt{2m|E_B|} \approx 46$ MeV, $m \approx m_p/2$ being the reduced system mass and $|E_B| \approx 2.22$ MeV the binding energy. The Jastrow factor, $J(0) \approx 0.4$ [2], is included to account for internucleon repulsion at short distances. Using first-order perturbation theory for the contact potential $g\delta(\mathbf{r})$ in Eq. (6), we obtain the neutrino-exchange-induced shift to the deuteron binding energy:

$$\delta E_B = -\langle\psi|g\delta(\mathbf{r})|\psi\rangle = -\frac{g\kappa[J(0)]^2}{2\pi r_0^2} = -\frac{G_F^2 M_Z^2 \kappa [J(0)]^2}{24\pi^3 r_0^2} \left(a_n a_p - \frac{2}{3} b_n b_p \right) \approx -1.1 \times 10^{-3} \text{ eV}. \tag{12}$$

Following our Letter, and comparing the measured [3] and predicted [4,5] values of the deuteron binding energy, we find the corrected limit:

$$G_{\text{eff}}^2 < 1.2 \times 10^4 G_F^2, \tag{13}$$

which is about 5 orders of magnitude stronger than the limit previously calculated in our Letter. The difference is mainly due to the distance cutoff scale being set by the Z-boson Compton wavelength, instead of the nuclear length scale used in our Letter. Formally, this looks like the second strongest constraint among two-body systems, with the strongest constraint coming from the ground-state hyperfine splitting in muonium. However, deuteron is a system whose structure is predominantly governed by the strong interaction, and so the constraint in Eq. (13) is probably less reliable than the constraints from nonhadronic atoms.

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[1] X.-J. Xu and B. Yu, J. High Energy Phys. 02 (2022) 08.
[2] V. F. Dmitriev, V. V. Flambaum, O. P. Sushkov, and V. B. Telitsin, Phys. Lett. **125B**, 1 (1983).
[3] E. G. Kessler, Jr., M. S. Dewey, R. D. Deslattes, A. Henins, H. G. Boerner, M. Jentschel, C. Doll, and H. Lehmann, Phys. Lett. A **255**, 221 (1999).
[4] D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001(R) (2003).
[5] A. Ekstrom, G. R. Jansen, K. A. Wendt, G. Hagen, T. Papenbrock, B. D. Carlsson, C. Forssen, M. Hjorth-Jensen, P. Navratil, and W. Nazarewicz, Phys. Rev. C **91**, 051301(R) (2015).