Relaxation to a Parity-Time Symmetric Generalized Gibbs Ensemble after a Quantum Quench in a Driven-Dissipative Kitaev Chain

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The construction of the generalized Gibbs ensemble, to which isolated integrable quantum many-body systems relax after a quantum quench, is based upon the principle of maximum entropy. In contrast, there are no universal and model-independent laws that govern the relaxation dynamics and stationary states of open quantum systems, which are subjected to Markovian drive and dissipation. Yet, as we show, relaxation of driven-dissipative systems after a quantum quench can, in fact, be determined by a maximum entropy ensemble, if the Liouvillian that generates the dynamics of the system has parity-time symmetry. Focusing on the specific example of a driven-dissipative Kitaev chain, we show that, similar to isolated integrable systems, the approach to a parity-time symmetric generalized Gibbs ensemble becomes manifest in the relaxation of local observables and the dynamics of subsystem entropies. In contrast, the directional pumping of fermion parity, which is induced by nontrivial non-Hermitian topology of the Kitaev chain, represents a phenomenon that is unique to relaxation dynamics in driven-dissipative systems. Upon increasing the strength of dissipation, parity-time symmetry is broken at a finite critical value, which thus constitutes a sharp dynamical transition that delimits the applicability of the principle of maximum entropy. We show that these results, which we obtain for the specific example of the Kitaev chain, apply to broad classes of noninteracting fermionic models, and we discuss their generalization to a noninteracting bosonic model and an interacting spin chain.

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Introduction.—After a quench, generic isolated quantum many-body systems relax locally to a state that is determined, according to the fundamental postulates of statistical mechanics [1], by maximization of entropy subject to the constraints imposed by integrals of motion [2–4]. For integrable systems, which are characterized by an extensive number of integrals of motion, the resultant equilibrium state is the generalized Gibbs ensemble (GGE) [5–7]. The principle of maximum entropy and, consequently, the structure of the GGE are universal in the sense that model-dependent details affect only the specific form of the integrals of motion and the numerical coefficients that enter the GGE as Lagrange multipliers, and that are determined by the initial state. Likewise, relaxation to the GGE is characterized by a set of universal characteristic traits such as light-cone spreading of correlations [7–9] and linear growth and volume-law saturation of the entropy of a finite subsystem [10–13]. In contrast to this scenario of generalized thermalization of isolated integrable systems, open systems, which are subjected to Markovian drive and dissipation [14], typically evolve toward nonequilibrium steady states that are determined by the interplay of internal Hamiltonian dynamics and the coupling to external reservoirs, and that are, therefore, highly model dependent [15–22]. In particular, the breaking of conservation laws due to the coupling to external reservoirs entails the eventual loss of any memory of the initial state. That is, the constraints that determine the GGE in isolated systems are lifted, and, consequently, the notion of a maximum entropy ensemble appears to be rendered meaningless. Therefore, the existence of model-independent principles that govern the relaxation dynamics and stationary states of open systems is seemingly ruled out.

How are these drastically different paradigms of relaxation in isolated and driven-dissipative systems connected when γ, the strength of the coupling to external reservoirs, is gradually diminished? For any $\gamma > 0$, an open system eventually reaches a stationary state that is vitally determined by the coupling to external reservoirs and takes the form of a GGE only in the limit $\gamma \to 0$ [23–26]. In this Letter, however, we show that the universal principles that govern generalized thermalization after a quantum quench in isolated systems can retain their validity—in suitably generalized form—even for finite values of γ that are comparable to characteristic energy scales of the system Hamiltonian. This robustness is caused by parity-time (PT) symmetry of the Liouvillian [27–39] that generates the dynamics of the system. Focusing on the specific example of a driven-dissipative generalization [40–42] of the Kitaev chain [43], we find that in the PT-symmetric phase, the quadratic eigenmodes of the adjoint Liouvillian oscillate at different frequencies, but crucially, they all decay with the same rate. Consequently, after factoring out exponential decay, dephasing [7,44] leads to local relaxation to a PT-symmetric GGE (PTGGE). In analogy to the GGE for isolated noninteracting fermionic many-body systems and interacting integrable systems that can be mapped to noninteracting fermions [5–7,45–47], we specify the PTGGE in terms of eigenmodes of the generator of the postquench dynamics. However, the PTGGE generalizes the GGE to account for noncanonical statistics of these eigenmodes, and the nonconservation of the associated mode occupation numbers renders the PTGGE intrinsically time dependent. We illustrate relaxation to the PTGGE in terms of the fermion parity and the entropy of a finite subsystem. Thereby, we reveal the directional pumping of fermion parity, which occurs for quenches from the topologically trivial phase of the isolated Kitaev chain to the non-Hermitian topological phase of the driven-dissipative Kitaev chain [42], as a phenomenon that is unique to driven-dissipative systems, and we establish the validity of a dissipative quasiparticle picture [48–50] for values of γ up to the sharply defined boundary of the PT-symmetric phase. Going beyond the example of the Kitaev chain, we show that our results apply to broad classes of noninteracting fermionic models and, in suitably generalized form, also to models of noninteracting bosons and interacting spins.

Model.—We consider a Kitaev chain [43] of length L with hopping matrix element J, pairing amplitude Δ, and chemical potential μ, as described by the Hamiltonian

$$H = \sum_{l=1}^{L} (-J c_l^\dagger c_{l+1} + \Delta c_l c_{l+1} + \text{H.c.}) - \mu \sum_{l=1}^{L} \left(c_l^\dagger c_l - \frac{1}{2} \right). \tag{1}$$

The operators c_l and $c_l^\dagger$ annihilate and create, respectively, a fermion on lattice site l. Unless stated otherwise, we assume periodic boundary conditions with $c_{L+1} = c_1$. The system is prepared in the ground state $|\psi_0\rangle$ for $J = \Delta$ and μ_0. We focus on the limit $\mu_0 \to -\infty$, such that the initial state is the topologically trivial vacuum state $|\psi_0\rangle = |\Omega\rangle$ with $c_l|\Omega\rangle = 0$, but our results are not affected qualitatively by this choice. At $t = 0$, the chemical potential is quenched to a finite value μ, while $J = \Delta$ is kept fixed. At the same time, the system is coupled to Markovian reservoirs. Consequently, the postquench dynamics is described by a quantum master equation for the system density matrix ρ [51,52],

$$i \frac{d}{dt} \rho = \mathcal{L} \rho = [H, \rho] + i \sum_{l=1}^{L} (2 L_l \rho L_l^\dagger - \{ L_l^\dagger L_l, \rho \}), \tag{2}$$

where we choose $L_l = \sqrt{\gamma_l} c_l + \sqrt{\gamma_g} c_l^\dagger$ as a coherent superposition of loss and gain at rates γ_l and γ_g, respectively [40–42]. The mean rate $\gamma = (\gamma_l + \gamma_g)/2$ measures the overall strength of dissipation, whereas the relative rate $\delta = \gamma_l - \gamma_g$ is akin to an inverse temperature: For $\delta = 0$, the system evolves for $t \to \infty$ toward a steady state ρ_{SS} with infinite temperature $\rho_{\text{SS}} = \rho_\infty = 1/2^L$ [42]. In contrast, for $\delta \to \infty$, the steady state is pure, $\rho_{\text{SS}} = |\Omega\rangle\langle\Omega|$.

Since the initial state $\rho_0 = |\psi_0\rangle\langle\psi_0|$ is Gaussian and the Liouvillian $\mathcal{L}$ is quadratic and, therefore, preserves Gaussianity, the time-evolved state $\rho(t) = e^{i\mathcal{L}t}\rho_0$ is fully determined by the covariance matrix

$$g_{l-l'}(t) = \begin{pmatrix} \langle [c_l, c_{l'}^\dagger](t) \rangle & \langle [c_l, c_{l'}](t) \rangle \\ \langle [c_{l'}^\dagger, c_l^\dagger](t) \rangle & \langle [c_{l'}^\dagger, c_l](t) \rangle \end{pmatrix}, \tag{3}$$

where $\langle \cdots (t) \rangle = \text{tr}[\cdots \rho(t)]$. The Fourier transform $g_k = -i \sum_{l=1}^{L} e^{-ikl} g_l$ obeys the equation of motion $dg_k/dt = -i z_k g_k + i g_k z_k^\dagger - s_k$, where z_k and s_k can be expressed in terms of Pauli matrices as $z_k = -i2\gamma\mathbb{1} - 2\sqrt{\gamma_l\gamma_g}\sigma_x + 2\Delta\sin(k)\sigma_y - [2J\cos(k) + \mu]\sigma_z$ and $s_k = -2\delta\sigma_z$ [53]. For $\gamma \to 0$, z_k reduces to the Bogoliubov–de Gennes Hamiltonian of the isolated Kitaev chain [83], and it has inversion symmetry $z_k = \sigma_z z_{-k} \sigma_z$ and time-reversal symmetry $z_k = z_{-k}^*$. These symmetries are broken when $\gamma > 0$. However, the Liouvillian still has PT symmetry in the sense that the traceless part of z_k given by $z_k' = z_k + i2\gamma\mathbb{1}$ is symmetric under the combined operation of inversion and time reversal $z_k' = \sigma_z z_k'^* \sigma_z$. PT symmetry implies that there are two types of eigenvectors and associated eigenvalues $\lambda_{\pm,k}$ of z_k [53]: PT-symmetric eigenvectors, which come in pairs with eigenvalues $\text{Re}(\lambda_{+,k}) = -\text{Re}(\lambda_{-,k})$ and $\text{Im}(\lambda_{\pm,k}) = -2\gamma$, and PT-breaking eigenvectors, for which $\text{Re}(\lambda_{\pm,k}) = 0$ and $\text{Im}(\lambda_{+,k} + i2\gamma) = -\text{Im}(\lambda_{-,k} + i2\gamma)$. The PT-symmetric phase is defined by the eigenvectors of z_k being PT symmetric for all momenta k, which is the case for $2\sqrt{\gamma_l\gamma_g} < |2J - |\mu||$. Then, the eigenvalues of z_k are given by $\lambda_{\pm,k} = -i2\gamma \pm \omega_k$ with $\omega_k^2 = \varepsilon_k^2 - 4\gamma_l\gamma_g$ and $\varepsilon_k^2 = [2J\cos(k) + \mu]^2 + 4\Delta^2\sin(k)^2$. For strong dissipation with $2\sqrt{\gamma_l\gamma_g} > 2J + |\mu|$, all eigenvectors are PT breaking. Finally, in the PT-mixed phase at intermediate dissipation, eigenvectors of both types exist.

PT-symmetric GGE.—We now focus on relaxation dynamics after a quench to the PT-symmetric phase, which is best described in terms of the eigenmodes of the adjoint Liouvillian [53]. With the matrix V_k that diagonalizes z_k, these modes are given by

$$\begin{pmatrix} d_k \\ d_{-k}^\dagger \end{pmatrix} = V_k^\dagger \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}, \quad V_k = \begin{pmatrix} \cos\left(\frac{\theta_k + \phi_k}{2}\right) & i\sin\left(\frac{\theta_k - \phi_k}{2}\right) \\ i\sin\left(\frac{\theta_k + \phi_k}{2}\right) & \cos\left(\frac{\theta_k - \phi_k}{2}\right) \end{pmatrix}, \tag{4}$$

where $c_k = (1/\sqrt{L}) \sum_{l=1}^{L} e^{-ikl} c_l$, $\tan(\theta_k) = -2\Delta\sin(k)/[2J\cos(k) + \mu]$, and $\tan(\phi_k) = 2\sqrt{\gamma_l\gamma_g}/\omega_k$. For $\gamma = 0$, V_k

reduces to the usual unitary Bogoliubov transformation. When $\gamma > 0$, nonunitarity of V_k is reflected in the statistics of the modes d_k as expressed through their anticommutation relations:

$$\begin{pmatrix} \{d_k, d_{k'}^\dagger\} & \{d_k, d_{-k'}\} \\ \{d_{-k}^\dagger, d_{k'}^\dagger\} & \{d_{-k}^\dagger, d_{-k'}\} \end{pmatrix} = f_k \delta_{k,k'}, \tag{5}$$

where $f_k = V_k^\dagger V_k = \mathbb{1} + 2\sqrt{\gamma_l \gamma_g}\sigma_y/\varepsilon_k$. To discuss the dynamics of the modes d_k, we consider their commutators. Expectation values of normal commutators evolve as [53]

$$\langle [d_k, d_k^\dagger](t)\rangle = e^{-4\gamma t}\langle [d_k, d_k^\dagger]\rangle_0 + (1 - e^{-4\gamma t})\langle [d_k, d_k^\dagger]\rangle_{\mathrm{SS}}, \tag{6}$$

where $\langle \cdots \rangle_0 = \mathrm{tr}(\cdots \rho_0)$ and $\langle \cdots \rangle_{\mathrm{SS}} = \mathrm{tr}(\cdots \rho_{\mathrm{SS}})$ denote expectation values in the initial and steady state, respectively. For anomalous commutators, we find

$$\begin{aligned} \langle [d_k, d_{-k}](t)\rangle &= e^{-i2(\omega_k - i2\gamma)t}\langle [d_k, d_{-k}]\rangle_0 \\ &\quad + (1 - e^{-i2(\omega_k - i2\gamma)t})\langle [d_k, d_{-k}]\rangle_{\mathrm{SS}}. \end{aligned} \tag{7}$$

We first consider the case of balanced loss and gain $\delta = 0$. Then, heating to infinite temperature is reflected in the exponential decay and vanishing in the steady state of the expectation values of both normal and anomalous commutators. Crucially, in the PT-symmetric phase, the decay rate is identical for all momentum modes. Thus, after factoring out exponential decay, the system relaxes locally to a maximum entropy ensemble through dephasing of modes with $\omega_k \neq \omega_{k'}$ [7,44]. Since the decay of normal commutators is nonoscillatory, dephasing affects only anomalous commutators. Therefore, we define the PTGGE as the maximum entropy ensemble [84] that is compatible with the statistics given in Eq. (5) and the nondephasing expectation values of normal commutators collected in the diagonal matrix $\zeta_k(t) = e^{-4\gamma t}\mathrm{diag}(\langle [d_k, d_k^\dagger]\rangle_0, \langle [d_{-k}^\dagger, d_{-k}]\rangle_0)$. We find, in terms of spinors $D_k = (d_k, d_{-k}^\dagger)^\top$ [53],

$$\rho_{\mathrm{PTGGE}}(t) = \frac{1}{Z_{\mathrm{PTGGE}}(t)} e^{-2\sum_{k\geq 0} D_k^\dagger f_k^{-1} \mathrm{arctanh}(\zeta_k(t) f_k^{-1}) D_k}, \tag{8}$$

with normalization $Z_{\mathrm{PTGGE}}(t)$ such that $\mathrm{tr}[\rho_{\mathrm{PTGGE}}(t)] = 1$. The PTGGE reduces to the conventional GGE when $\gamma = 0$ such that $f_k = \mathbb{1}$ and $\zeta_k(t)$ becomes time independent. Relaxation to the PTGGE in the PT-symmetric phase stands in stark contrast to the long-time dynamics in the PT-mixed and PT-broken phases, which is determined by the single slowest-decaying mode. Therefore, the boundary of the PT-symmetric phase corresponds to a sharp dynamical transition that delimits the applicability of the principle of maximum entropy.

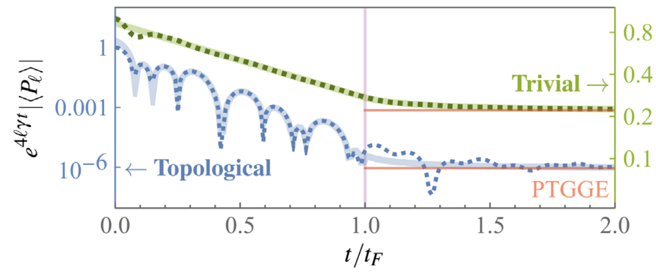

FIG. 1. Subsystem parity after quenches to the trivial (green, $\mu = -4J$) and topological (blue, $\mu = -J$) PT-symmetric phases for $\gamma = 0.3J$, $\delta = 0$, and $\ell = 20$. The solid lines are obtained from Eqs. (9) and (10), where we set $\alpha_+ = 0.08$ and $\alpha_- = 0.11$ to achieve best agreement with the numerical data shown as dashed lines. Straight vertical and horizontal lines indicate $t = t_F$ and the PTGGE predictions for the stationary values, respectively. In all figures, L is chosen large enough to avoid finite-size effects.

When $\delta \neq 0$, the PTGGE captures relaxation dynamics only up to a crossover time scale $t_\times$ that is determined by the equivalence of initial-state and steady-state contributions in Eq. (6), $e^{-4\gamma t_\times}|\langle [d_k, d_k^\dagger]\rangle_0| = (1 - e^{-4\gamma t_\times})|\langle [d_k, d_k^\dagger]\rangle_{\mathrm{SS}}|$. Since $\langle [d_k, d_k^\dagger]\rangle_{\mathrm{SS}}$ is proportional to δ [53], this equation implies $t_\times \sim (1/\gamma)|\ln(c_\times|\delta|)|$ with a constant coefficient $c_\times > 0$ for $\delta \to 0$. Consequently, within the entire PT-symmetric phase, which includes values of γ that are comparable to Hamiltonian energy scales, $t_\times$ can be large enough such that relaxation to the PTGGE can be observed if δ is sufficiently small. The precise condition on the value of δ depends on the observable under consideration. Below, we provide a quantitative discussion for the fermion parity of a finite subsystem.

Relaxation of subsystem parity.—To illustrate relaxation to the PTGGE, we consider the fermion parity of a subsystem that consists of ℓ contiguous lattice sites $P_\ell = e^{i\pi\sum_{l=1}^{\ell} c_l^\dagger c_l}$. The expectation value $\langle P_\ell\rangle = \mathrm{pf}(\Gamma_\ell)$ is given by the Pfaffian of the reduced covariance matrix $\Gamma_\ell = (\Gamma_{l,l'})_{l,l'=1}^{2\ell}$ [85,86], where $\Gamma = iR^\dagger G R$, G is a block Toeplitz matrix built from the 2×2 blocks g_l in Eq. (3), and $R = \oplus_{l=1}^{\ell}(1/\sqrt{2})\left(\begin{smallmatrix} 1 & -i \\ 1 & i \end{smallmatrix}\right)$. For the isolated Kitaev chain, a combined Jordan-Wigner [87] and Kramers-Wannier [88,89] transformation maps the subsystem parity to order parameter correlations in the transverse field Ising model [53]. Based on the analytical results of Calabrese *et al.* [45–47] for the relaxation of order parameter correlations in the space-time scaling limit $\ell, t \to \infty$ with ℓ/t fixed, in Eqs. (9) and (10) below, we formulate analytical conjectures for the time dependence of the subsystem parity in the driven-dissipative Kitaev chain, which we find to be in excellent agreement with numerical results.

First, we consider quenches to the topologically trivial [42] PT-symmetric phase with $|\mu| > 2J$. Then, as shown in Fig. 1, for $\delta = 0$, the behavior of the subsystem parity in the space-time scaling limit is well described by [53]

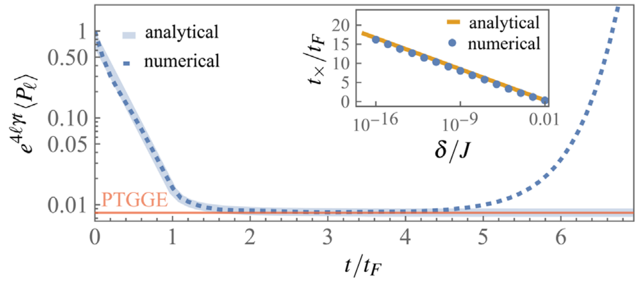

FIG. 2. Deviation from the PTGGE due to $\delta \neq 0$ for $\mu = -2.5J$, $\gamma = 0.1J$, $\delta = 10^{-7}J$, and $\ell = 20$. The rescaled subsystem parity (dashed line) follows Eq. (9) (solid line) up to the crossover timescale $t_\times \approx 5.7 t_F$ defined as $|\langle P_\ell(t_\times)\rangle - \langle P_\ell(t_\times)\rangle_{\mathrm{PTGGE}}| = \langle P_\ell(t_\times)\rangle_{\mathrm{PTGGE}}$. Inset: $t_\times$ diverges logarithmically for $\delta \to 0$. The numerical data are in good agreement with an analytical estimate [53].

$$\langle P_\ell(t)\rangle \sim P_0 e^{-4\ell\gamma t + \int_0^\pi \frac{dk}{2\pi} \min(2|v_k|t,\ell)\mathrm{tr}[\ln(|\zeta_k' f_k^{-1}|)]}, \tag{9}$$

where $\zeta_k' = e^{4\gamma t}\zeta_k(t)$ is time independent. The value $\gamma = 0.3J$ chosen in Fig. 1 leads to sizeable modifications of statistics and dynamics of Liouvillian as compared to Hamiltonian elementary excitations, which are accounted for in Eq. (9) by the appearance of f_k and the definition of the velocity $v_k = d\omega_k/dk$ in terms of ω_k rather than the Hamiltonian dispersion relation ε_k. Relaxation to the PTGGE is best revealed by considering the rescaled subsystem parity $e^{4\ell\gamma t}\langle P_\ell(t)\rangle$, which decays up to the Fermi time [46] $t_F = \ell/(2v_{\max})$ where $v_{\max} = \max_k |v_k|$, before it approaches a stationary value. The prefactor P_0 in Eq. (9) is obtained by fitting the long-time limit of the rescaled subsystem parity to the PTGGE prediction.

For small $\delta \neq 0$, we expect $\langle P_\ell(t)\rangle$ to deviate from Eq. (9) after a crossover time $t_\times \sim (1/\gamma)\ln(c_\times|\delta|)$. This expectation is confirmed in Fig. 2, where we also compare numerical results for $t_\times$ with an analytical estimate [53]. The condition to observe relaxation of the subsystem parity to the PTGGE, therefore, reads $t_F < t_\times$.

Directional parity pumping.—For quenches to the PT-symmetric phase with $|\mu| < 2J$, due to nontrivial non-Hermitian topology of the Liouvillian [42], the rescaled subsystem parity repeatedly crosses zero before it relaxes to a stationary value. Physically, these zero crossings can be interpreted as pumping of parity between the subsystem and its complement. The period of the zero crossings is determined by soft modes of the PTGGE, i.e., momenta $k_{s,\pm}$, for which the exponent in Eq. (8) vanishes. For the isolated Kitaev chain [7,46], the soft modes $k_{s,+} = -k_{s,-}$ are locked onto each other by inversion symmetry [53], and the period of zero crossings is given by $t_s = \pi/(2\varepsilon_{k_{s,+}}) = \pi/(2\varepsilon_{k_{s,-}})$. In contrast, for the PTGGE in Eq. (8), we find that due to the breaking of inversion symmetry when $\gamma > 0$, there are two distinct soft modes with $k_{s,+} \neq -k_{s,-}$ [53], and consequently, two distinct timescales

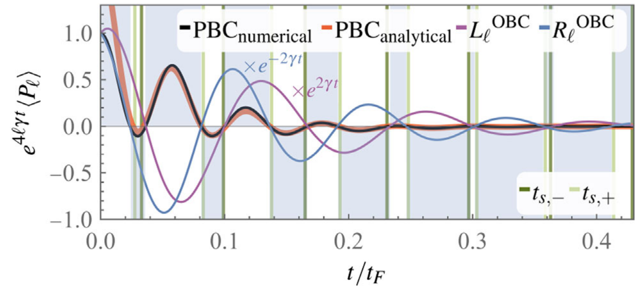

FIG. 3. Directional pumping of subsystem parity for a quench to the topological PT-symmetric phase with $\mu = -0.5J$, $\gamma = 0.3J$, $\delta = 0$, and $\ell = 30$. For periodic boundary conditions (PBC), the subsystem parity [black line, numerics; blue shading, sign of numerical data; red line, Eq. (10) with $\alpha_+ = \alpha_- = 0.09$] crosses zero at multiples of both $t_{s,+}$ and $t_{s,-}$. In contrast, for open boundary conditions (OBC), zero crossings occur only at multiples of $t_{s,-}$ and $t_{s,+}$ for subsystems, respectively, L_ℓ (violet line) and R_ℓ (blue line). Factors $e^{\pm 2\gamma t}$ compensate for additional exponential decay (left end) and growth (right end) due to edge modes [53].

$t_{s,\pm} = \pi/(2\omega_{k_{s,\pm}})$. As shown in Fig. 1, for $t < t_F$, the resulting oscillatory decay of the subsystem parity is captured by the following modified space-time scaling limit [53]:

$$\langle P_\ell(t)\rangle \sim 2\cos(\omega_{k_{s,+}}t + \alpha_+)\cos(\omega_{k_{s,-}}t + \alpha_-)\langle P_\ell(t)\rangle_{\mathrm{nonosc}}, \tag{10}$$

where $\alpha_\pm$ are undetermined phase shifts, and the non-oscillatory part is given by Eq. (9), which also approximately describes the behavior of $\langle P_\ell(t)\rangle$ for $t > t_F$.

The two timescales $t_{s,+}$ and $t_{s,-}$ have a clear physical meaning in terms of the exchange of parity through, respectively, the left and right boundaries of the subsystem. This is confirmed numerically in Fig. 3 by considering a chain with open boundary conditions and subsystems $L_\ell = \{1, \ldots, \ell\}$ and $R_\ell = \{L - \ell + 1, \ldots, L\}$ located at the left and right ends of the chain [53]. Then, zero crossings of $\langle P_\ell(t)\rangle$ occur only with period $t_{s,-}$ and $t_{s,+}$, respectively. In contrast, for a chain with periodic boundary conditions, $\langle P_\ell(t)\rangle$ exhibits zero crossings at multiples of both $t_{s,+}$ and $t_{s,-}$. As we show in the Supplemental Material [53], the occurrence of different periods of parity pumping for subsystems at the left and right ends of the chain requires both mixedness of the time-evolved state and breaking of inversion symmetry and is, therefore, unique to driven-dissipative systems.

Evolution of subsystem entropy.—In isolated systems, a key signature of thermalization is provided by the growth and saturation of the von Neumann entropy of a finite subsystem $S_{\mathrm{vN},\ell} = -\mathrm{tr}[\rho_\ell \ln(\rho_\ell)]$. Here, we consider a subsystem that consists of ℓ contiguous lattice sites, and whose density matrix ρ_ℓ is obtained by taking the trace over

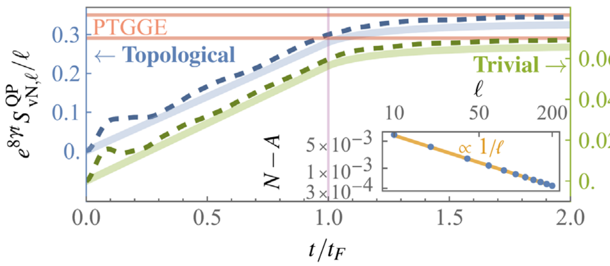

FIG. 4. Quasiparticle-pair contribution to the subsystem entropy after quenches to the trivial (green, $\mu = -4J$) and topological (blue, $\mu = -J$) PT-symmetric phases for $\gamma = 0.3J$, $\delta = 0$, and $\ell = 20$. The numerical data (dashed lines) are close to Eq. (11) (solid lines). Inset: For the trivial quench at $t = 2t_F$, the difference between the numerical data and Eq. (11) (blue dots) vanishes as $1/\ell$ (orange line).

the $L-\ell$ remaining sites $\rho_\ell = \mathrm{tr}_{L-\ell}(\rho)$. Quantitative predictions for the time dependence of $S_{\mathrm{vN},\ell}$ in the space-time scaling limit can be derived from a quasiparticle picture [10–13], according to which the initial state acts as the source of pairs of entangled quasiparticles. The ballistic propagation of quasiparticles leads to growth of the subsystem entropy in proportion to the number of pairs of entangled quasiparticles that are shared between the subsystem and its complement.

In open systems, the subsystem entropy $S_{\mathrm{vN},\ell} = S^{\mathrm{QP}}_{\mathrm{vN},\ell} + (\ell/L)S^{\mathrm{stat}}_{\mathrm{vN}}$ is the sum of two contributions [48–50,90]: $S^{\mathrm{QP}}_{\mathrm{vN},\ell}$ measures correlations due to the propagation of quasiparticle pairs, and $S^{\mathrm{stat}}_{\mathrm{vN}} = S_{\mathrm{vN},L}$ is the statistical entropy due to the mixedness of the time-evolved state. Based on results of Refs. [49,50] for weak dissipation $\gamma \sim 1/\ell$, we conjecture that for quenches to the PT-symmetric phase and $\delta = 0$, the quasiparticle-pair contribution $S^{\mathrm{QP}}_{\mathrm{vN},\ell}$ obeys the following space-time scaling limit [53]:

$$S^{\mathrm{QP}}_{\mathrm{vN},\ell}(t) \sim \int_0^\pi \frac{dk}{2\pi} \min(2|v_k|t, \ell)\mathrm{tr}[S(\zeta_k(t) f_k^{-1}) - S(g_k(t))_d], \tag{11}$$

where $S(\xi) = -(1+\xi/2)\ln(1+\xi/2) - (1-\xi/2)\ln(1-\xi/2)$. The subscript "$d$" in last term indicates that due to dephasing, only the nonoscillatory components of the trace are required to capture the space-time scaling limit. At long times $\gamma t \gg 1$, since $\zeta_k(t), g_k(t) \sim e^{-4\gamma t}$, we can expand $S(\xi) \sim \ln(2) - \xi^2/2$. Then, due to the cancellation of the leading constant term in the difference in Eq. (11), we obtain $S^{\mathrm{QP}}_{\mathrm{vN},\ell}(t) \sim e^{-8\gamma t}$. Therefore, in analogy to the subsystem parity, relaxation to the PTGGE becomes visible by considering the rescaled quasiparticle-pair entropy $e^{8\gamma t} S^{\mathrm{QP}}_{\mathrm{vN},\ell}(t)$. As shown in Fig. 4 the rescaled quasiparticle-pair entropy grows up to the Fermi time t_F before it saturates to a stationary value predicted by the PTGGE.

Discussion.—An important question concerns the validity of the PTGGE beyond the specific example of the Kitaev chain. As we show in the Supplemental Material [53], our results apply directly to symmetry-preserving deformations of the Kitaev chain, and also to a class of fermionic models with a particle-number-conserving Hamiltonian, for which a natural choice of dissipation is provided by incoherent loss and gain. Furthermore, we find that for an interacting spin chain that can be mapped to fermions but with quadratic jump operators, relaxation of a subset of observables is described by the PTGGE. Finally, for a model of noninteracting bosons, we demonstrate relaxation to an ensemble that generalizes the PTGGE for fermions while maintaining the key property of conserving an extensive amount of information about the initial state. It is intriguing to speculate whether PT symmetry can affect also the dynamics of nonintegrable driven-dissipative systems in a similar way so as to induce relaxation to a PT-symmetric Gibbs ensemble.

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