Pure Anti–de Sitter Supergravity and the Conformal Bootstrap

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We consider graviton scattering in maximal supergravity on anti–de Sitter space (AdS) in $d+1$ dimensions for $d=3$, 4, and 6 with no extra compact spacetime factor. Holography suggests that this theory is dual to an exotic maximally supersymmetric conformal field theory (CFT) in d dimensions whose only light single trace operator is the stress tensor. This contrasts with more standard cases like type IIB string theory on $\mathrm{AdS}_5\times S^5$ dual to $\mathcal{N}=4$ super-Yang-Mills, where the CFT has light single trace operators for each Kaluza-Klein mode on S^5. We compute the one-loop correction to the pure AdS_{d+1} theory in a small Planck length expansion, which is dual to the large central charge expansion in the CFT. We find that this correction saturates the most general nonperturbative conformal bootstrap bounds on this correlator in the large central charge regime for $d=3$, 4, 6, while the one-loop correction to CFTs with string and M-theory duals all lie inside the allowed region.

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Introduction.—The AdS/CFT duality relates quantum gravity on anti-de Sitter (AdS) space in $d+1$ dimensions times a compact spacetime factor, to certain supersymmetric conformal field theories (CFTs) in d dimensions [1]. In the simplest examples, the compact space is simply a sphere with a similar radius as AdS, and the CFT is maximally supersymmetric. Compactifying the graviton on the sphere generates an infinite tower of Kaluza-Klein (KK) modes in AdS, which are dual to light single trace operators in the CFT. It is an open question if holographic duals exist where the radius of the sphere is parametrically smaller than that of AdS, so that these extra dimensions would be small (see Refs. [2,3] for a recent discussion). In the most extreme case, there would simply be no compact factor at all, and the only single trace operators in the dual CFT would be the stress tensor multiplet. No such pure AdS theory has been constructed, despite much effort [4–13].

We will address this question by studying the stress tensor four-point function, which is dual to scattering of gravitons in the bulk, in maximally supersymmetric CFTs in $d=3,4,6$ dimensions. Consider the large central charge c expansion of this correlator, where c is defined as the coefficient of the stress-tensor two-point function, and is related to the bulk as

$$c\sim(L_{\mathrm{AdS}}/\ell_{\mathrm{Planck}})^{D-2}, \tag{1}$$

where L_{AdS} is the radius of the AdS_{d+1} factor, and ℓ_{Planck} is the Planck length of the full D-dimensional bulk spacetime, including a possible compact factor. We can define the correlator $\mathcal{G}$ in any such theory to any order in $1/c$ as

$$\begin{aligned}\mathcal{G}=\mathcal{G}^{(0)}+c^{-1}\mathcal{G}^{R}+c^{-2}(\mathcal{G}^{R|R}+\kappa\mathcal{G}^{R^4})+\dots\\ \dots+c^{-\frac{D+4}{D-2}}\mathcal{G}^{R^4}+c^{-\frac{D+8}{D-2}}\mathcal{G}^{D^4R^4}+\dots,\end{aligned} \tag{2}$$

where in the first line we wrote the tree level supergravity term $\mathcal{G}^{R}$ and the one-loop term $\mathcal{G}^{R|R}$ with supergravity vertices R, while in the second line we wrote tree level higher derivative corrections that are allowed by supersymmetry [14]. The expansion also includes one-loop terms with such higher derivative vertices, as well as higher loop terms [21]. The $\mathcal{G}^{R|R}$ term has an $\mathcal{G}^{R^4}$ type contact term with coefficient κ as long as the scaling of the R^4 tree level term is smaller than $R|R$, which is the case for string and M theory with $D=10$, 11 [22], respectively, but is not for the pure AdS_{d+1} theory where $D=d+1$ and $d=3,4,6$. All tree and loop supergravity terms $\mathcal{G}^{R|R|\cdots}$ can be computed iteratively using the analytic bootstrap [27,28], but to fix the higher derivative corrections as well as loop contact terms such as $\kappa\mathcal{G}^{R^4}$, we need a UV completion like string/M theory. These terms only affect CFT data with finite spin [29], so at any given order in $1/c$ we can unambiguously determine an infinite set of CFT data for AdS_{d+1} duals with any (or no) compact factor.

Whether or not a pure AdS_{d+1} theory is also defined nonperturbatively in c is a separate question that we will address in the conclusion.

The tree level supergravity correction $\mathcal{G}^R$ at order $1/c$ is unaffected by a compact spacetime factor [27,30–32], but higher loop terms starting with $\mathcal{G}^{R|R}$ at order $1/c^2$ are sensitive to the number of KK modes [28]. We will compute this one-loop term for pure AdS_{d+1} theories in $d = 3, 4, 6$ using the analytic bootstrap, which allows us to extract all CFT data to $O(c^{-2})$. We then can compare this $O(1/c^2)$ data to nonperturbative numerical bootstrap bounds [33–36], which apply to any maximally supersymmetric CFT, and can be computed for any c. We find that for all $d = 3, 4, 6$, the pure AdS_{d+1} one-loop correction precisely saturates the bootstrap bounds in the large c regime.

The one-loop correction has also been computed for maximally supersymmetric CFTs with string/M-theory duals. In 3D, these CFTs are $U(N)_k \times U(N)_{-k}$ ABJM theory with $k = 1,\ 2$, which is dual to M theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ with $c \sim N^{3/2}$ [37,38]. In 4D, they are $\mathcal{N} = 4$ super-Yang-Mills (SYM) with gauge group $SU(N)$ or $SO(N)$ [39], which is dual to type IIB string theory on $\mathrm{AdS}_5 \times S^5$ or $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_2$ with $c \sim N^2$ [1], respectively. In 6D, they are A_{N-1} or D_N (2,0) theories [40,41], which are dual to $\mathrm{AdS}_7 \times S^4$ or $\mathrm{AdS}_7 \times S^4/\mathbb{Z}_2$ with $c \sim N^3$ [42,43], respectively. The one-loop corrections were computed in these various cases in [23–25,44–46]. In all cases, we find that these corrections lie inside the allowed region of the bootstrap bounds for the same regime of large c where the pure AdS_{d+1} theory saturates the bound.

Stress tensor correlator.—We begin by reviewing the constraints of maximal supersymmetry in $d = 3, 4, 6$ on the stress tensor correlator. We consider the superconformal primary $S(x)$, which is a scalar with $\Delta = d - 2$ that transforms in the symmetric traceless representation of the R-symmetry group $SO(8)_R$, $SO(6)_R$, and $SO(5)_R$ for 3D, 4D, and 6D, respectively. Conformal and R symmetry fixes the four-point function to take the form

$$\langle S(x_1, Y_1) S(x_2, Y_2) S(x_3, Y_3) S(x_4, Y_4)\rangle = \frac{(Y_1 \cdot Y_2)^2 (Y_3 \cdot Y_4)^2}{|x_{12}|^{2(d-2)} |x_{34}|^{2(d-2)}} \mathcal{G}(U, V; \sigma, \tau), \tag{3}$$

where we define the cross ratios

$$U \equiv \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \qquad V \equiv \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2},$$
$$\sigma \equiv \frac{(Y_1 \cdot Y_3)(Y_2 \cdot Y_4)}{(Y_1 \cdot Y_2)(Y_3 \cdot Y_4)}, \qquad \tau \equiv \frac{(Y_1 \cdot Y_4)(Y_2 \cdot Y_3)}{(Y_1 \cdot Y_2)(Y_3 \cdot Y_4)}, \tag{4}$$

with $x_{ij} \equiv x_i - x_j$, and Y_i are null polarization vectors that encode the R-symmetry indices. The constraints from supersymmetry are given by the superconformal Ward identities [47], which can be satisfied by expanding $\mathcal{G}$ in superconformal blocks as [48]

$$\mathcal{G}(U, V; \sigma, \tau) = \sum_{\mathcal{M}} \lambda_{\mathcal{M}}^2 \mathfrak{G}_{\mathcal{M}}(U, V; \sigma, \tau), \tag{5}$$

where $\mathcal{M}$ runs over all the supermultiplets appearing in the $S \times S$ operator product expansion (OPE), the $\lambda_{\mathcal{M}}^2$ are the squared OPE coefficients for each such supermultiplet $\mathcal{M}$, and the explicit form of the superblocks can be found for each d in [34,35,47,49]. In the Supplemental Material [50], for each d we summarize the multiplets $\mathcal{M}$ that appear, which we label by the scaling dimension Δ, the spin ℓ, and the R-symmetry representation of the superprimary. We exclude free theory multiplets, which for $d = 4, 6$ restricts us to interacting theories [51]. The $S \times S$ OPE includes long multiplets in the singlet of the R-symmetry group with even spin ℓ and scaling dimension $\Delta > d - 2 + \ell$, as well as protected multiplets such as the stress tensor with fixed Δ. The stress tensor λ^2 is fixed by the conformal Ward identity [52] to be inversely proportional to the central charge coefficient c of the stress tensor two-point function:

$$\lambda_{\text{stress}}^2 \propto 1/c, \tag{6}$$

where the proportionality constant is fixed in 4D so that c is the conformal anomaly [49], in 6D so that a free tensor multiplet has $c = 1$ [34], and in 3D so that the free theory has $c = 16$ [35]. In 4D and 6D, the existence of a protected 2D chiral algebra [53] fixes $\lambda_{\mathcal{M}}^2 \propto 1/c$ for certain protected multiplets, while the remaining protected multiplets $\mathcal{M}_{\text{prot}}$ have λ^2 that remain unconstrained.

An important nonperturbative constraint on the four-point function can be derived by swapping $1 \leftrightarrow 3$ in (3), which yields the crossing equations

$$\mathcal{G}(U, V; \sigma, \tau) = \frac{U^{d-2}}{V^{d-2}} \tau^2 \mathcal{G}(V, U; \sigma/\tau, 1/\tau), \tag{7}$$

which we will now use to constrain the correlator.

One loop from tree level.—We will now restrict to the pure AdS_{d+1} theory, and consider the large c expansion of the correlator $\mathcal{G}$ shown in (2), where we expand long multiplet CFT data as

$$\Delta_{n,\ell} = 2(d-2) + 2n + \ell + \gamma_{n,\ell}^R/c + \gamma_{n,\ell}^{R|R} c^2 + \ldots,$$
$$\lambda_{n,\ell}^2 = (\lambda_{n,\ell}^{(0)})^2 + (\lambda_{n,\ell}^R)^2/c + (\lambda_{n,\ell}^{R|R})^2/c^2 + \ldots. \tag{8}$$

A similar expansion exists for the OPE coefficients of the protected operators, although of course their scaling dimensions are fixed. The long multiplets that appear in (8) are all double trace operators $[SS]_{n,\ell}$ of the schematic form

TABLE I. Fits of the numerical bootstrap bounds at large c, compared to exact $O(1/c^2)$ values for the pure AdS_{d+1} theory for $d = 3$, 4, 6.

3D:	$\Delta_{0,2}$: Exact	$4 - 49.931/c + 2713.6/c^2$
	Fit	$3.99996 - 49.82/c + 2619.4/c^2$
	$\lambda^2_{(A,2)_1}$: Exact	$9.7523 - 98.764/c + 570.43/c^2$
	Fit	$9.7523 - 98.772/c + 580.443/c^2$
	$\lambda^2_{(A,+)_0}$: Exact	$7.1111 + 48.448/c + 97.768/c^2$
	Fit	$7.1111 + 48.445/c + 103.35/c^2$
4D:	$\Delta_{0,2}$: Exact	$6 - 1/c + 0.12976/c^2$
	Fit	$6.0000 - 0.99929/c + 0.14718/c^2$
6D:	$\Delta_{0,2}$: Exact	$10 - 10.909/c - 258.79/c^2$
	Fit	$10.000 - 11.209/c + 270.96/c^2$
	$\Delta_{0,4}$: Exact	$12 - 3.1648/c - 17.157/c^2$
	Fit	$12.000 - 3.1956/c - 17.832/c^2$
	$\lambda^2_{\mathcal{B}[02]_1}$: Exact	$0.75757 - 0.98484/c - 4.2372/c^2$
	Fit	$0.75757 - 0.98009/c - 3.9446/c^2$
	$\lambda^2_{\mathcal{B}[02]_3}$: Exact	$0.43076 - 0.15440/c - 0.15313/c^2$
	Fit	$0.43076 - 0.15432/c - 0.17448/c^2$

$$[SS]_{n,\ell} = S\Box^n \partial_{\mu_1}\ldots\partial_{\mu_\ell} S, \tag{9}$$

with $\Delta^{(0)}_{n,\ell} = 2(d-2) + 2n + \ell$ in the $c \to \infty$ generalized free field theory (GFFT). Note that if the bulk theory had a compact factor, e.g., $\mathrm{AdS}_5 \times S^5$, then we could use the higher KK modes to construct more such long operators, which would be degenerate in the GFFT and thus mix in the $1/c$ expansion. The GFFT and tree correlators, which are insensitive to the bulk factor, were computed in each d in [31,54–56] and used to extract tree level data, which we summarize in Table I. For theories with higher KK modes, we can only extract the average long multiplet anomalous dimensions $\langle \lambda^2_{n,\ell}\gamma^R_{n,\ell}\rangle$, due to the degeneracy at GFFT. For protected multiplets, we can obtain the unique CFT data for all such large c theories.

At one-loop level, we can expand the superblock expansion (5) to get

$$\mathcal{G}^{R|R} = \sum_{n=0}^{\infty}\sum_{\ell\in\text{even}}\left[\frac{1}{8}(\lambda^{(0)}_{n,\ell})^2(\gamma^R_{n,\ell})^2\log^2 U + (\lambda^{(0)}_{n,\ell})^2\gamma^{R|R}_{n,\ell}\frac{\log U}{2} + \ldots\right]\mathfrak{G}_{n,\ell} + \sum_{\mathcal{M}_{\text{prot}}}(\lambda^{R|R}_{\mathcal{M}})^2\mathfrak{G}_{\mathcal{M}}, \tag{10}$$

where the ellipsis refers to other combinations of tree and loop data, and recall that $\mathcal{M}_{\text{prot}}$ denotes protected multiplets whose OPE coefficients are not $1/c$ exact. The significance of the $\log^2 U$ term is that it is the only term at this order that has a double discontinuity (DD) as $U \to 0$ [57]. The Lorentzian inversion formula [58] shows that all CFT data with sufficiently large ℓ can be extracted from the DD as $V \to 0$, so we can obtain this DD from the $\log^2 U$ terms after applying crossing (7). We give the details in the Supplemental Material, and the resulting one-loop corrections for low-lying CFT data are summarized in the exact formulae part of Table I [59]. Note that in the string/M-theory cases, the inversion formula does not converge for low spins, which corresponds to the existence of the contact terms $\kappa\mathcal{G}^{R^4}$ in (2). In the pure AdS_{d+1} case we do not have such contact terms as discussed above, so we can in fact extract all CFT data at one-loop order. One can similarly extract higher spin data from our results, but the one-loop corrections become smaller with spin and so are harder to compare to the numerical bootstrap results in the next section, since the CFT data are then dominated by the tree level correction that is the same for pure AdS and string/M theory.

Numerical conformal bootstrap.—We will now compare these one-loop corrections to the numerical bootstrap bounds on CFT data in the stress tensor correlator for $d = 3$, 4, 6, which were computed for $d = 3$, 4 in [23,25], and which we compute now for 6D following [34]. These bounds come from optimizing the infinite set of constraints imposed by the crossing equations (7) on the superblock expansion in (5), for more details in each case see the original works [33–35], and [60–63] for recent reviews. The convergence of these bounds is monotonic and given by the parameter Λ originally defined in [35], which counts how many derivatives are used in the expansion of conformal blocks around the crossing symmetric point

[64]. These bounds apply to any theory with maximal supersymmetry in the given d and are computed as a function of c, which is related to the stress tensor OPE coefficient as in (6). Since these bounds are nonperturbative in c, we will look at the large c regime where we expect the $1/c$ expansion of the previous section to be good. The large c expansion of CFT data is asymptotic, which means that after a few orders the expansion will actually get worse, unless we look at very large values of c. We observe that the $1/c^2$ corrections get smaller relative to $1/c$ tree corrections as the spin increases, which implies that the asymptotic expansion is getting more accurate at this order. We do not want to look at very high spin data, however, because then the difference between each order will be hard to observe. As a compromise, we will focus on the lowest spin CFT data for which the Lorentzian inversion converges for the string/M-theory CFTs.

We summarize the comparison of the analytic $1/c$ expansion to fits in the large c regime of the bootstrap bounds in Table I [66], which are obtained from Fig. 1, as well as plots of other CFT data given in the Supplemental Material. In all cases, we find that the pure AdS one-loop correction at $1/c^2$ noticeably improves the universal tree correction at $1/c$ and approximately saturates the numerical bounds, unlike the string/M-theory dual corrections that lie inside the allowed region.

In 6D, we also computed an upper bound on c (i.e., a lower bound on the stress tensor OPE coefficient), which applies to any interacting 6D (2,0) CFT, and got

$$c \geq 21.6441, \tag{11}$$

which is weaker than the bound $c \gtrsim 25$ conjectured in [34]. This latter bound was found by extrapolating bounds computed at lower values of Λ to $\Lambda \to \infty$, and was used as evidence that these general bootstrap bounds were saturated by the physical A_1 theory with $c = 25$. We use a different definition of Λ than [34,67], so it is hard to check their conjectured extrapolation against our bound, but since in 3D [68] and 4D [70] we know that the general bounds are not saturated by the string/M-theory theory duals for the smallest such values of c, it seems likely that this general

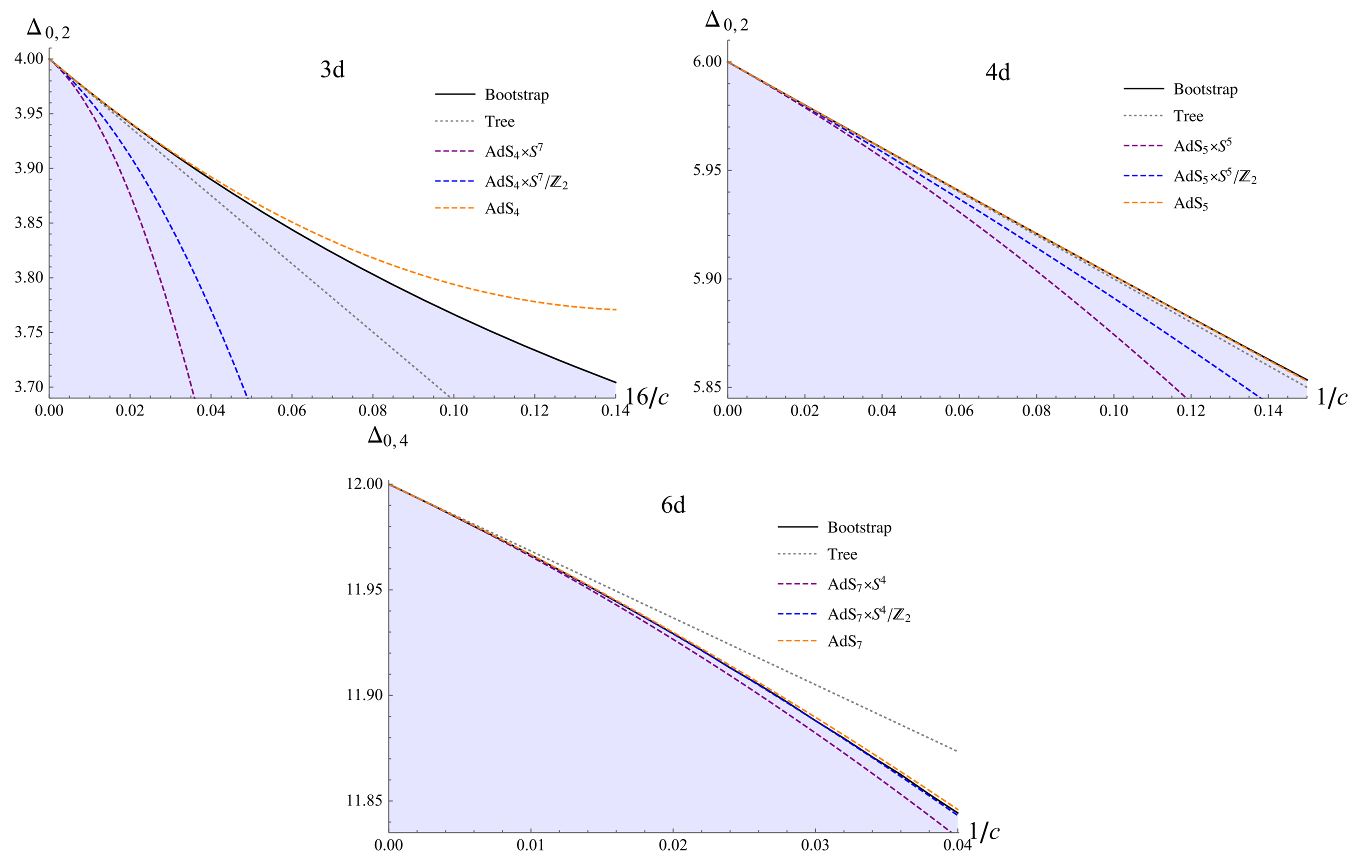

FIG. 1. Numerical bootstrap upper bounds (in black) on the scaling dimension $\Delta_{0,\ell}$ of the lowest dimension spin ℓ long multiplets in various d, made with precision $\Lambda = 83, 123, 91$ for $d = 3, 4, 6$, respectively. These bounds apply to any maximally supersymmetric CFT, and are plotted in terms of the stress-tensor coefficient c in the large c regime (the smooth curve comes from interpolating many raw data points). The gray dotted line denotes the large c expansion to order tree level supergravity $O(c^{-1})$, which does not depend on the compact factor in the bulk. The purple, blue, and orange dashed lines also include the one-loop supergravity correction $O(c^{-2})$ for the pure AdS and string/M-theory dual theories.

6D bound is also not saturated by the A_1 theory even at $\Lambda \to \infty$.

Discussion.—Our results show that pure AdS_{d+1} maximal supergravity saturates the most general nonperturbative bootstrap bounds in the large c regime, while CFTs with string/M-theory duals lie in the allowed region. This suggests that to study the latter theories, one needs to disallow the existence of the pure AdS_{d+1} theory by either looking at a mixed correlator with other single trace operators [69,72], or imposing theory specific constraints like supersymmetric localization [73]. Indeed, in 3D one can strengthen these general bootstrap bounds by inputting the OPE coefficients of the $(B,2)$ and $(B,+)$ multiplets for the $U(N)_k \times U(N)_{-k}$ ABJM theory for $k = 1$, 2, as computed to all orders in $1/N$ using the localization in [74], in which case the one-loop data for the dual $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ theories then saturate the bounds [23,24]. In 4D, one can input the two localization inputs for $SU(N)$ SYM derived in [75,76], which are a function of the complexified coupling τ, in which case the bounds in [71] match four-loop weak coupling results [77] in the appropriate regime, and exclude the general bootstrap bounds shown here for all τ. In 6D there is no localization, but for correlators of single trace operators other than the stress tensor one can input nontrivial OPE coefficients given by the protected 2D chiral algebra [15,78] for the A_{N-1} or D_N theories

We can also use the general bootstrap bounds themselves to further study the pure AdS_{d+1} theory, assuming it continues to saturate the bounds to higher order in $1/c$. In particular, by applying a fit to the large c regime of the numerical bounds, one could read off higher derivative corrections to supergravity such as the $\mathcal{G}^{R^4}$ term discussed in the introduction, to help determine a putative UV completion. Since $\mathcal{G}^{R^4}$ occurs at the same order as higher loop corrections in some cases, e.g., c^{-3} for pure AdS_5 (2), it will be necessary to compute these higher loops, as was recently recently done for the two-loop correction on $\mathrm{AdS}_5 \times S^5$ [79,80]. The pure AdS_{d+1} case should be much easier due to the lack of mixing, and so could even guide the calculation in the more physical cases with compact factors. More ambitiously, we can nonperturbatively define the pure AdS_{d+1} theory as whatever saturates the bootstrap bounds at finite c; it would be fascinating to find independent evidence for or against the existence of such a theory.

Finally, we can ask what theory saturates the stress tensor correlator bootstrap bound with less than maximal supersymmetry. In 3D, the $\mathcal{N} = 6$ bootstrap bounds were found in [81,82] to be saturated by $U(1)_{2N} \times U(1+N)_{-2N}$ ABJ theory [83] for all N, which has a vectorlike large N limit dual to supersymmetric higher spin gravity [84–86]. With no supersymmetry, it was observed in [87–89] that critical $O(N)$ vector models saturate the bound on c [90], so it is likely that the 3D stress tensor correlator bounds in general are saturated by interacting vector model CFTs. In higher dimensions, however, there are no interacting unitary vector models [93], so it is possible that the most general nonsupersymmetric stress tensor bounds could be saturated by pure AdS_{d+1} Einstein gravity with $d > 3$. It would be fascinating to check this by generalizing the nonsupersymmetric stress tensor bootstrap in 3D [99] to higher d. If such nonsupersymmetric pure AdS_{d+1} theories exist for any d, then they suggest that unitary interacting CFTs can be constructed for any d, unlike supersymmetric CFTs which only exist for $d \leq 6$.

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