Puviani *et al.* Reply: In a Comment by Benfatto *et al.* [1] two technical points were questioned. Here, we briefly describe the corrective steps taken to avoid the two mistakes of the initial publication.

(1) We firstly admit that only one vertex in Eq. (7) of Ref. [2] needs to be renormalized. We were misled by approximating the Higgs propagator to be frequency independent and momentum independent in order to provide an analytical solution. However, the double vertex renormalization in the Raman susceptibility led to an overcounting of diagrams which we will correct below.

(2) Following Ref. [3] we show in Fig. 1 the lowest order of the particle-particle interaction for (a) the random phase approximation (RPA) and for (b) the vertex function, respectively. In the original Letter [2] we adopted the standard approximation $V_{\mathbf{q}} = V_{\mathbf{k}-\mathbf{k}'} \approx V f_{\mathbf{k}} f_{\mathbf{k}'}$ which yields a **q**-*independent* RPA summation for the Raman response. While this approximation resembles the solution for small **q**, we realized that this consideration is not true for the vertex function $V(f_{[(\mathbf{k}+\mathbf{k}')/2]})^2 \not\approx V f_{\mathbf{k}} f_{\mathbf{k}'}$ in Fig. 1(c). In our original publication, where we assumed a frequency- and momentum-independent Higgs propagator, this approximation seemed to be justified, since $V(f_{[(\mathbf{k}+\mathbf{k}')/2]})^2 \approx V f_{\mathbf{k}} f_{\mathbf{k}'}$ holds for d-wave symmetry and small **q**. Now, we have improved our wrong approximation in Eq. (5) of Ref. [2].

We calculate the vertex correction by using the full Higgs propagator [see Fig. 1(c)], inserting the vertex correction of Fig. 1(b). Note also that the Higgs propagator $H(\mathbf{q}, \omega)$ in Fig. 1(c) contains an RPA-like summation of $V(f_{[(\mathbf{k}+\mathbf{k}')/2]})^2$ with full frequency dependence and momentum dependence. We define the four-momenta $p = (\mathbf{p}, i\Omega)$, $q = (\mathbf{q}, iq_m)$, $k = (\mathbf{k}, ik_n)$, where $i\Omega$ and iq_m are Matsubara bosonic frequencies, while ik_n are fermionic. For the external light we use $\mathbf{p} \to 0$. The lowest order bubble is $\chi_{\Gamma\gamma}$, where $\Gamma = \gamma + \delta\Gamma \equiv \gamma\tau_3 + \Gamma_1\tau_1 + \Gamma_2\tau_2 + \Gamma_3\tau_3$. Then, with some physical cutoff the Higgs propagator (including Coulomb repulsion) is given by

$$\tilde{H}(\mathbf{k}, \mathbf{q}, iq_m) = -f^2_{\mathbf{k}+\mathbf{q}/2}\left[\frac{2}{V} + \sum\nolimits_{\mathbf{k}'} f^2_{\mathbf{k}'+\mathbf{q}/2}\chi_{11}(\mathbf{k}', iq_m) - \frac{(\sum_{\mathbf{k}'} f_{\mathbf{k}'+\mathbf{q}/2}\chi_{13}(\mathbf{k}', iq_m))^2}{\sum_{\mathbf{k}'}\chi_{33}(\mathbf{k}', iq_m)}\right]^{-1}. \quad (1)$$

The advanced solution (involving a complicated Matsubara summation, similar to Ref. [4]) will be presented in a forthcoming publication [5]. Here, we proceed, although knowing the full frequency dependence of the Higgs propagator from Ref. [6], by fairly approximating $\tilde{H}(\mathbf{k}, \mathbf{q}, iq_m) \approx \tilde{H}(\mathbf{k}, \mathbf{q}, 0)$, but still keeping the momentum **q**, where now the Matsubara summation can be solved analytically. Therefore, the contribution for the Raman response coming from the renormalized vertex in each j channel assuming $T = 0$ is

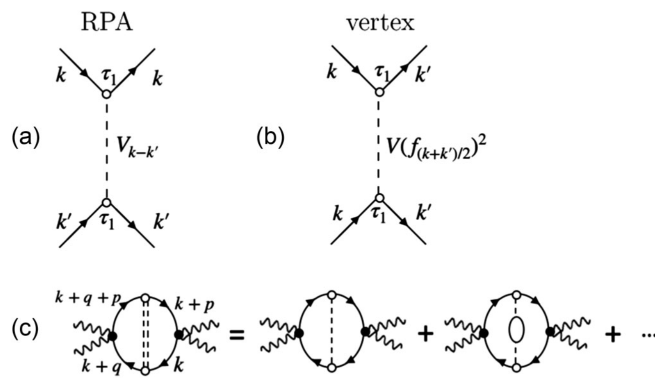

FIG. 1. Lowest order of the pairing interaction used for (a) the RPA series ("Higgs production") and for (b) the vertex correction ("many-body Higgs oscillations") introduced in Fig. 1 of Ref. [2] employing Gorkov notation. (c) Vertex correction due to a single Higgs mode $H(\mathbf{q}, \omega)$ used in our calculation.

$$\chi_{\Gamma_j\gamma}(\Omega) = \frac{1}{2}\sum_{\mathbf{k},\mathbf{q}} \frac{\alpha_j}{E^2_{\mathbf{k}}E^2_{\mathbf{k}+\mathbf{q}}[4E^2_{\mathbf{k}+\mathbf{q}} - (i\Omega)^2][4E^2_{\mathbf{k}} - (i\Omega)^2]} \times \tilde{H}(\mathbf{k}, \mathbf{q}, 0)|_{i\Omega\to\Omega+i\delta}, \quad (2)$$

with $\alpha_1 = \gamma_{\mathbf{k}}\varepsilon_{\mathbf{k}}\Delta_{\mathbf{k}}\gamma_{\mathbf{k}+\mathbf{q}}\varepsilon_{\mathbf{k}+\mathbf{q}}\Delta_{\mathbf{k}+\mathbf{q}}$, $\alpha_2 = -16\gamma_{\mathbf{k}}\gamma_{\mathbf{k}+\mathbf{q}}\Delta_{\mathbf{k}}\Delta_{\mathbf{k}+\mathbf{q}}(i\Omega)^2$, $\alpha_3 = -\Delta^2_{\mathbf{k}}\Delta^2_{\mathbf{k}+\mathbf{q}}\gamma_{\mathbf{k}}\gamma_{\mathbf{k}+\mathbf{q}}$. At this point we can easily add the Coulomb interaction in the Raman response [7], defining $\tilde{\chi}_{\Gamma\gamma}(\Omega) = \chi_{\Gamma\gamma}(\Omega) - \chi_{\Gamma3}(\Omega)\chi_{3\gamma}(\Omega)/\chi_{33}(\Omega)$, leading to a conserving approximation [8]. Finally, we notice that we shall also add the screening coming from the residual interaction for the B_{1g} symmetry as in Refs. [9–11] yielding a smaller B_{1g} Raman response.

In conclusion, while a full calculation with full frequency dependence and momentum dependence in the vertex channel is still in preparation [5], we already find a breaking of particle-hole symmetry due to vertex corrections which reflect many-body interactions on the Higgs mode. This results in a mixing of the τ_1 and τ_3 channel and thus will change the magnitudes of both the A_{1g} and B_{1g} response.

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