

Fractional Optical Angular Momentum and Multi-Defect-Mediated Mode Renormalization and Orientation Control in Photonic Crystal Microring Resonators

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Whispering gallery modes (WGMs) in circularly symmetric optical microresonators exhibit integer quantized angular momentum numbers due to the boundary condition imposed by the geometry. Here, we show that incorporating a photonic crystal pattern in an integrated microring can result in WGMs with fractional optical angular momentum. By choosing the photonic crystal periodicity to open a photonic band gap with a band-edge momentum lying between that of two WGMs of the unperturbed ring, we observe hybridized WGMs with half-integer quantized angular momentum numbers ($m \in \mathbb{Z} + 1/2$). Moreover, we show that these modes with fractional angular momenta exhibit high optical quality factors with good cavity-waveguide coupling and an order of magnitude reduced group velocity. Additionally, by introducing multiple artificial defects, multiple modes can be localized to small volumes within the ring, while the relative orientation of the delocalized band-edge states can be well controlled. Our Letter unveils the renormalization of WGMs by the photonic crystal, demonstrating novel fractional angular momentum states and nontrivial multimode orientation control arising from continuous rotational symmetry breaking. The findings are expected to be useful for sensing and metrology, nonlinear optics, and cavity quantum electrodynamics.

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Optical microresonators supporting whispering gallery modes (WGMs) have long been studied due to their ability to simultaneously support a long cavity photon lifetime (high quality factor Q) and a strong degree of spatial mode localization, attributes that enhance light-matter interactions in areas such as cavity optomechanics [1], integrated frequency combs [2], and microlasers [3]. WGMs have rotational symmetry in a circular round trip and, within a given wavelength band, have almost constant free spectral range (FSR). The circular geometry dictates the behavior of the electromagnetic field in the azimuthal (ϕ) direction, as matching the field phase across one round trip (i.e., a $\phi = 2\pi$ rotation) results in integer azimuthal mode numbers. Perturbations that disturb the geometry from the circular symmetry change this picture. For example, in deformed microcavities, geometric ray optics predicts more generalized WGMs with fractional angular momentum numbers of m/n [4,5], that is, the optical field exhibits $m \times 2\pi$ phase oscillations across n round trips. Recently, such fractional angular momentum has been demonstrated in a quadrupolar microdisk [6], following the development of numerous free-space approaches because of potential applications in optical trapping, micromanipulation, and providing extra channels for multiplexing [7–14]. However,

the deformed microcavity geometry causes significant Q deterioration, limiting the microcavity's ability to enhance light-matter interactions while exhibiting fractional angular momentum.

Conventional WGMs have also been modified through the introduction of artificial scatterers with designated size and position, in order to efficiently tune intermodal coupling and loss [15,16]. The incorporation of a well-designed photonic crystal can realize coherent scattering between coupled modes and avoid coupling to radiation loss channels [17]. Recently, a “microgear” photonic crystal microring (MPhCR) has been developed [18], in which clockwise (CW) and counterclockwise (CCW) traveling WGMs are hybridized into a set of standing WGMs through backscattering. This geometry enables strong intermodal coupling between WGMs with different angular momentum, resulting in large band gaps (several FSRs) without decreasing the optical Q . As in other prior PhCR work [19–23], however, its photonic crystal structure only hosted conventional WGMs, with integer angular momentum, leaving WGMs with fractional angular momentum unexplored. Moreover, the incorporation of multiple defects and their influence on distributed modes within the ring were not considered. Such understanding of

the multiple defects and mode localization is crucial for utilizing the PhCR platform for cavity quantum electrodynamics.

In this Letter, we show that the MPhCR structure can be modified to realize previously unexplored regimes of modal coupling in WGM resonators. First, we report fractional angular momentum WGMs in a MPhCR. By creating a band gap much larger than the FSR, we successfully hybridize two sets of WGMs with odd and even angular momentum, resulting in WGMs of fractional angular momentum, while preserving the high Q , good coupling, and intuitive design common to conventional WGM resonators. Next, we explore a novel multimode localization process by introducing multiple artificial defects into the MPhCR. Breaking the continuous rotational symmetry of the microring geometry, the orientation and degeneracy of hybridized modes can be well controlled. Our Letter provides understanding of the modes' renormalization and orientation control near the band edge. It also highlights unique features of the PhCR, namely, the coexistence and interplay of the different types of states it supports.

For a microring with continuous rotational symmetry, the dominant traveling electric field of WGMs can be written as $E(r, z, \phi) = E_m(r, z)e^{im\phi}$, where r , z , ϕ are the radial, vertical, and azimuthal coordinate. m represents the azimuthal angular momentum number of the mode, and matching the field phase across one round trip necessitates m being a positive or negative integer, corresponding to the CW and CCW WGMs. Figure 1(a) shows a schematic of the dispersion relation in a conventional microring. Over a certain range of frequencies, the WGMs have nearly equal FSR and symmetric CCW and CW modes due to the rotational symmetry (\mathcal{R}) and mirror symmetry (\mathcal{M}) of the microring.

When N azimuthally periodic modulations are introduced on the microring, it possesses translation symmetry (\mathcal{T}) (in the azimuthal direction) with a period of $2\pi/N$ and becomes a PhCR. The translation symmetry hybridizes four modes that exist near the band gap, with angular momentum $|m_1 + m_2| = N$ [18], generating dielectric and air bands, as shown in Fig. 1(b). To facilitate understanding, we consider that the four constituent traveling wave modes $\{|m_1|, -|m_1|, |m_2|, -|m_2|\}$ prior to mixing via the PhC are in phase:

$$\begin{aligned} E_{m_1}(r, z)(e^{i|m_1|\phi} + e^{-i|m_1|\phi}) + E_{m_2}(r, z)(e^{i|m_2|\phi} + e^{-i|m_2|\phi}) \\ = E_{\tilde{m}}(r, z) \cos(\tilde{m}\phi) \cos(\tilde{m}\phi) \end{aligned} \quad (1)$$

where $E_{m_1}(r, z)$ and $E_{m_2}(r, z)$ are considered to be equal to $E_{\tilde{m}}(r, z)$ for $m \gg 1$. The superposition of the four components gives rise to fractional- m WGM-like modes. The CCW traveling WGMs are generated due to internal modal coupling with the excited CW modes. $\tilde{m} = (|m_1| + |m_2|)/2$ and $\tilde{m} = (|m_2| - |m_1|)/2$ are the numbers describing the angular momentum and envelope modulation,

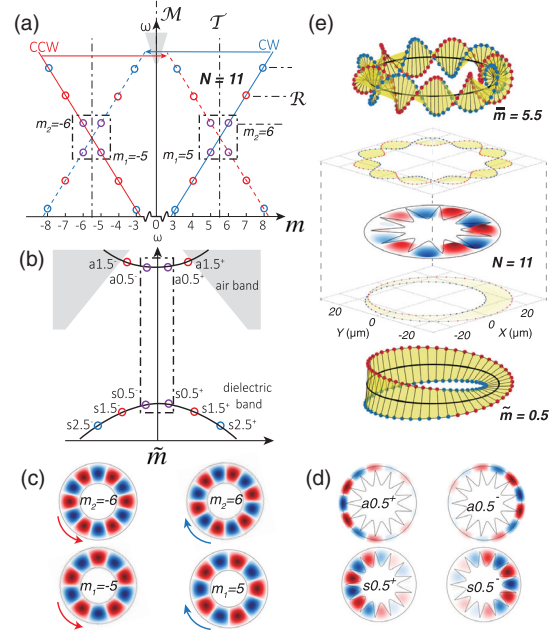


FIG. 1. Fractional angular momentum whispering gallery modes enabled by mode superposition near a band gap. (a) The combination of rotational symmetry (\mathcal{R}) and period translation symmetry (\mathcal{T}) in a photonic crystal ring hybridizes the modes $|m_1 + m_2| = N$, forming WGMs with fractional angular momentum when N is odd. For $N = 11$, the first order mode is formed by $|m_1| = 5$ and $|m_2| = 6$. Counterclockwise modes ($m < 0$) are mirrored (\mathcal{M}) about the y axis of $m = 0$. (b) Band diagram of a photonic crystal ring with an odd number of unit cells. Air (dielectric) band modes denote modes overlapping largely with the air (dielectric) part of the microring. In a real experiment, the degenerate dielectric (air) modes split, showing higher and lower frequencies. We label them with $s(a)\tilde{m}^+$ and $s(a)\tilde{m}^-$, respectively. (c),(d) Qualitative illustrations of the four WGMs before (c) and after (d) hybridization. They correspond to the four purple circles framed by the dashed lines in (a) and (b), respectively. (e) An example of the modes with Möbius-type profile for both angular momentum phase (upper panel) and envelope phase (lower panel). The black circle aligns with the light propagation path.

respectively, of the mixed WGMs. Notably, the complete basis of four mixed modes can be written as $E_{\tilde{m}} \cos(\tilde{m}\phi \pm \pi/4 + \phi_0) \cos(\tilde{m}\phi \pm \pi/4 + \phi_1)$ when the relative phase of the traveling waves are taken into consideration. The first $\pm\pi/4$ corresponds to the dielectric and air band, and second one represents the two orthogonal envelope distributions. ϕ_0 describes the phase offset given by the geometry of the photonic crystal modulation, while ϕ_1 arises from fabrication imperfection. Later, we will show how to fix ϕ_1 beyond this imperfection.

When N is odd, the hybridization of four modes of $|m_1 - m_2| = 1, 3, 5, \dots$ gives mixed modes with fractional $\tilde{m} = N/2$ and $\tilde{m} = 0.5, 1.5, 2.5, \dots$, respectively [i.e., the corresponding modes with the same color in Figs. 1(a) and 1(b)]. For example, the four WGMs with $|m_1| = 5$ and

$|m_2| = 6$ shown in Fig. 1(c) hybridize in a PhCR of $N = 11$, generating four mixed WGMs of $\bar{m} = 5.5$ and $\tilde{m} = 0.5$ shown in Fig. 1(d). Upper (lower) modes correspond to the air (dielectric) band, and left and right are the two orientations of the envelope.

With the fractional angular momentum, phase matching cannot be fulfilled in one round trip of light around the ring. Instead, the mixed WGMs with fractional angular momentum $\bar{m} = N/2$ should undergo two round trips before the phase is matched. Figure 1(e) shows the Möbius-type angular momentum phase and envelope phase topologies of a hybridized mode in the upper and lower panels. For the dots on the edges of the Möbius strip in each panel, their azimuthal angle about the propagation path shows the local phase (i.e., $\bar{m}\phi$ and $\tilde{m}\phi$). After the first round trip (red dots), it has π phase delay. The second round trip (blue dots) fulfills the phase matching with an additional π phase delay, so that the requisite 2π phase delay is reached. Their projection on the x - y plane shows the local amplitude $\propto \cos(\bar{m}\phi)$ and $\propto \cos(\tilde{m}\phi)$, respectively. The mode shape of mixed WGMs with $\{\bar{m}, \tilde{m}\} = \{5.5, 0.5\}$ is shown in the middle panel where $N = 11$.

In a real experiment, \bar{m} is a number much larger than that in the toy schematic shown above. For example, we use a photonic crystal ring to demonstrate fractional m with $\bar{m} = 166.5$. Figure 2(a) shows the scanning electron microscope (SEM) images of the nanofabricated MPhCRs made from a 500-nm-thick stoichiometric silicon nitride layer. The ring has a radius of $\approx 25 \mu\text{m}$ and is spatially modulated with a nominal average width ≈ 1500 nm and modulation amplitude ≈ 1400 nm, as shown in Fig. 2(b). Such a device supports fundamental transverse-electric-like (TE) modes, whose dominant electric field is in the radial direction and can be theoretically modeled by Eq. (1). We carry out finite-element method simulations of these devices over a unit cell. Top and cross-sectional views of the generated field profile are shown in Fig. 2(f), corresponding to $E_{\bar{m}}(r, z_0) \cos(\bar{m}\phi)$ and $E_{\bar{m}}(r, z)$, respectively, in Eq. (1), where $z = z_0$ corresponds to a plane located midway through the thickness of the ring. The dielectric mode is well confined in the silicon nitride core, preserving conventional WGM profiles. We couple light to the device via on-chip waveguides [Fig. 2(a)], and its normalized transmission spectrum is shown in Fig. 2(c). The MPhCR with odd $N = 2 \times 166.5 = 333$ opens a band gap between the $m = 166$ and $m = 167$ WGMs, hybridizing them and their neighbors, and generating the mixed modes $\{s\bar{m}^+, s\bar{m}^-, a\bar{m}^+, a\bar{m}^-\}$ where $\bar{m} = 0.5, 1.5, 2.5, \dots$

The fractional- m WGMs preserve all the main features of conventional WGMs, including high- Q , ease in design and fabrication, and controllable resonator-waveguide coupling. As shown in Fig. 2(d), the total optical Q (Q_t) of the mode $s0.5^+$ is measured to be $Q_t = (5.4 \pm 0.4) \times 10^5$, comparable to conventional WGMs in microrings. The mode spectrum is compressed near the band edge, distinct

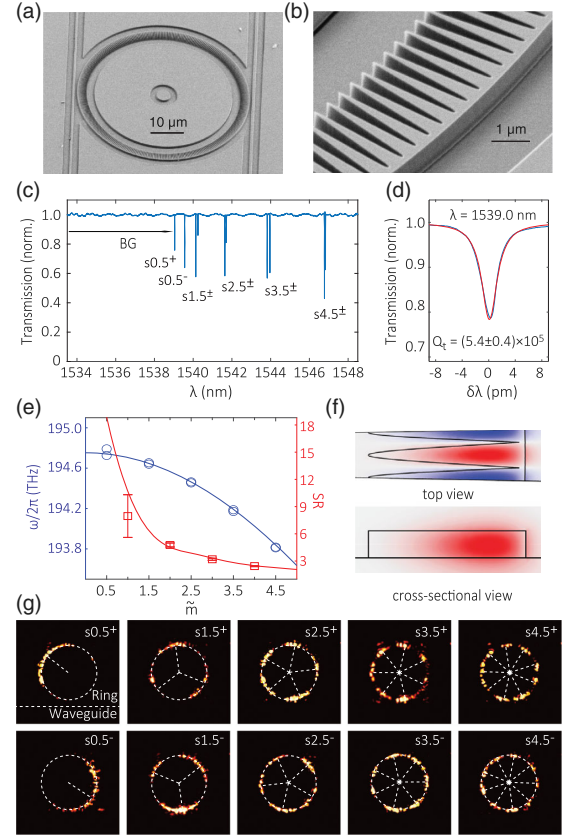


FIG. 2. Fractional angular momentum in a microgear photonic crystal ring (MPhCR). (a),(b) Scanning electron microscope image of the MPhCR device and zoom-in image of the microgear structure. The number of unit cells is $N = 333$, resulting in a fractional angular momentum value $\bar{m} = 166.5$ for the modes discussed below. (c) Linear transmission spectrum of the MPhCR, showing a few pairs of fractional- m slow light modes at the dielectric band edge (labeled $\{s0.5^{\pm}, s1.5^{\pm}, s2.5^{\pm}, s3.5^{\pm}, s4.5^{\pm}\}$). These modes result from the hybridization of $m = \{(\pm 166, \pm 167), (\pm 165, \pm 168), (\pm 164, \pm 169), (\pm 163, \pm 170), (\pm 162, \pm 171)\}$. (d) Enlarged view of the $s0.5^+$ mode (blue), along with a nonlinear least squares fit to resonance (red). (e) The frequencies (blue) and the slow down ratios (red) of each fractional- m mode near the dielectric band edge. (f) Finite-element method simulated mode profile in one unit cell for the $s0.5^+$ mode at 194.25 THz (top and cross-section views). (g) Infrared images of the fractional- m modes from a top view of the MPhCR. The dashed lines mark the antinodes (local maxima) of the light intensity distribution.

from that of an unmodulated ring. In particular, the reduced FSR indicates a decrease in group velocity. The slowdown ratio (SR) defines the factor by which the group velocity in the MPhCR is reduced compared to that in the conventional silicon nitride ring [19]. The FSR between $s0.5$ and $s1.5$ is measured to be (122 ± 33) GHz, corresponding to $SR = 8.0 \pm 2.3$ where the uncertainty arises from the mode splitting of $s0.5$ and $s1.5$. We collect the frequencies of the first five pairs of modes in Fig. 2(c), and calculate their SRs, as shown in Fig. 2(e). The simulated frequencies (blue curve) and SR ratios (red curve) with the MIT Photonic

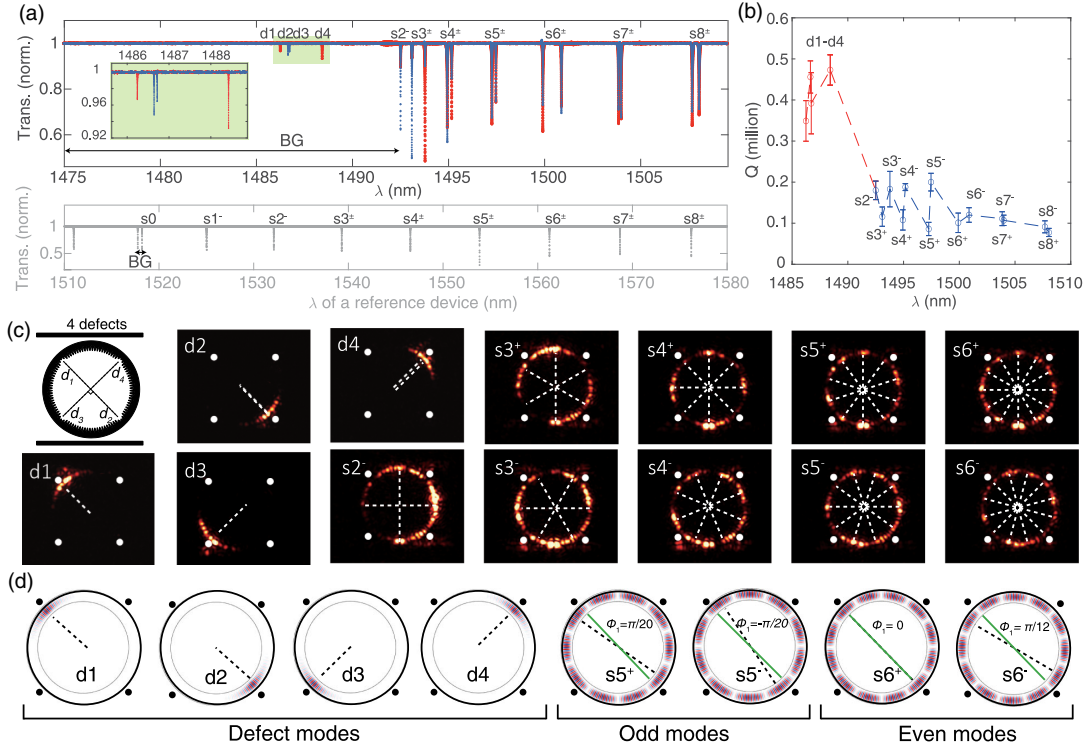


FIG. 3. Photonic crystal ring with multiple defects. (a) The normalized transmission spectrum of a photonic crystal microring with four symmetric defects is shown in the top panel. Red (blue) line corresponds to a measurement taken with the upper (lower) waveguide shown in (c). The spectrum of a reference device with small modulation ($A = 20$ nm) and without defects is illustrated as gray dots in the lower panel. (b) Intrinsic quality factor of the modes. (c) Schematic of the PhCR with four defects and IR image of the modes up to $\tilde{m} = 6$. The white dots label the angular position of the symmetric defects ($\pi/2$ relative phase). The white dashed lines highlight the antinodes of the mode. The four defects modes (labeled d1-d4) are highly localized. The symmetry of the defects decides the orientation of the WGM-like modes (labeled s2 to s6). (d) Schematic of the mode shape. The black dots label the angular location of the defects. The black dashed lines denote the orientation of the modes. For the nonlocalized WGM-like modes, the green lines denote the orientation of the defects. The relative angle between the two lines is ϕ_1 .

Bands method [24] show good agreement with experimental data. Additionally, in the dielectric band, the mode splitting of degenerate modes near the band edge is much larger than for other modes [Fig. 2(c)], indicating stronger coupling near the band edge. This can be attributed to the slow light enhanced coupling, i.e., a higher group index leads to a larger backscattering rate [25], although its interplay with fabrication imperfection requires further investigation. The splitting of modes far from the band edge is attributed to fabrication imperfections.

Infrared images of scattered light from the hybridized dielectric band-edge modes are illustrated in Fig. 2(g). The images in Fig. 2(g) clearly display the number of envelope antinodes to be odd for each displayed mode, and equal to $2\tilde{m}$. Remarkably, different from integer- m WGMs, the fundamental dielectric modes $s_{0.5^+}$ and $s_{0.5^-}$ are localized orthogonally at opposite sides of the ring. This mode localization can be beneficial for increasing coupling to other degrees of freedom. However, the orientation (ϕ_1) of these two modes is currently random, dictated by fabrication imperfections.

The orientation of the WGM-like modes can be controlled by breaking the continuous rotational symmetry via introducing artificial defects. The mode localization produced by the introduction of a single defect in a device with an even number of modulations was studied in Ref. [18]. The Supplemental Material [26] presents a similar study for a single defect in a device with an odd number of modulations, where the localized defect state coexists with band-edge states exhibiting fractional angular momentum. Here, we introduce multiple defects in a device with an even number of modulations. Figure 3 shows a MPhCR with four symmetrical defects. Each defect has $\pi/4$ misalignment with the adjacent waveguide as shown in the schematic of Fig. 3(c). Figure 3(a) shows the normalized transmission spectrum of the MPhCR on the top panel. Four modes are localized to the defects, indicated by their wavelength shift into the band gap (BG). The modes are highly localized, which is evident in transmission spectroscopy because they only appear in the spectrum measured by the adjacent waveguide (red and blue lines, respectively), and more directly through IR imaging of the device as shown in Fig. 3(c). The spectrum of a

reference device with small modulation ($A = 20$ nm) and without defects is illustrated as gray dots on the lower panel in Fig. 3(a). The small modulation only opens a small BG without disturbing other WGMs (whose mode splitting is barely visible). Comparing the multidefect device with the reference, we get the first important conclusion that the mode number is conserved, i.e., four defects localize four modes while other modes remain nearly unperturbed. The second point is that the orientation of the nonlocalized modes is controllable by the introduced defects. The introduced defects break the continuous rotational symmetry while creating a new discrete rotational symmetry with a $\pi/2$ period. The renormalized WGMs have a period of π/\tilde{m} . For the modes with even $\tilde{m} = 2, 4, 6, \dots$, they can match the discrete rotational symmetry of $\pi/2$, so they show orientation $\phi_1 = 0$ and $\phi_1 = \pi/2\tilde{m}$. For the odd modes of $\tilde{m} = 1, 3, 5, \dots$, instead, they show orientation $\phi_1 = \pm\pi/4\tilde{m}$, as shown in Figs. 3(c) and 3(d). Such orientation control based on discrete rotational symmetry also generally applies to devices with other numbers of symmetric defects (see Supplemental Material). Even a single large artificial defect can provide such control. Figure 3(b) shows the measured intrinsic quality factor of the modes. Surprisingly, the defect modes present higher Q than the WGM-like modes. In particular, the defect mode Q s are as high as $\approx 5 \times 10^5$, similar to that for single defect modes in Ref. [18]. However, the WGM-like modes, which exhibit high- Q in cavities without defects (Fig. 2) and cavities with a single defect [18], now show a reduction in Q by a factor of $5 \times$ to $10\times$. The former can potentially be attributed to the high-degree of localization by the defects, which lowers the possibility of the modes to encounter random imperfections on the ring. Conversely, the WGM-like modes interact with all four defects, and though this controls their orientation, it may also increase their radiation and scattering loss. Finally, in Fig. 3(a), we see that as we move away from the band edge, the even modes generally exhibit larger splittings than the odd modes. This is a function of their orientation with respect to the defects [Fig. 3(d)], with the two even modes of a given pair showing complete alignment and misalignment with the defects (nondegenerate), while the two odd modes are in principle equally well misaligned (degenerate).

In summary, we have demonstrated an understanding of renormalization and localization of WGMs near the band edge of a photonic crystal ring. Based on it, we presented optical modes with fractional (half-integer) angular momentum. The photonic crystal patterning enables controlled hybridization of WGMs into half-integer angular momentum states that retain high Q while exhibiting reduced group velocity. Furthermore, we demonstrate a multimode localization process based on the introduction of multiple symmetric artificial defects. These defects strongly localize modes that are closest to the band edge, while impacting the orientation of modes farther away from

the band edge, which largely retain their WGM-like character. These far modes' orientation and splittings are controlled by the defect-induced continuous rotational symmetry breaking. The discrete rotational symmetry of the defects provides an intuitive understanding of how to realize the mode orientation and degeneracy control. Modal-defect interaction without this symmetry is another interesting topic, requiring further investigation. We anticipate that our platform may broaden the usage of fractional- m light in applications including nonlinear photonics [35], quantum photonics [36], and cavity optomechanics [1].

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