Pairwise Multiparticle States and the Monopole Unitarity Puzzle

Csaba Csáki,[1,*] Yuri Shirman,[2,†] Ofri Telem,[3,4,‡] and John Terning[5,§]
[1]*Department of Physics, LEPP, Cornell University, Ithaca, New York 14853, USA*
[2]*Department of Physics and Astronomy, University of California, Irvine, California 92697, USA*
[3]*Department of Physics, University of California, Berkeley, California 94720, USA*
[4]*Ernest Orlando Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[5]*QMAP, Department of Physics, University of California, Davis, California 95616, USA*

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We suggest a novel resolution for a decades old mystery—what happens when a positron scatters off a minimal grand-unification-theory monopole in an s wave, a puzzle first discussed by Callan in 1983. Using the language of on shell amplitudes and pairwise helicity we suggest that the final state contains two up quarks and a down quark in an entangled "pairwise" multiparticle state—the only particle final state that satisfies angular momentum and gauge charge conservation. The cross section for this process is as large as in the original Rubakov-Callan effect, only suppressed by the QCD scale. The final state we find cannot be seen in Callan's truncated 2D theory, since our new pairwise state appears only in more than two dimensions.

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Introduction.—The scattering of electrically charged fermions with magnetic monopoles is a very peculiar process [1]. Until recently the theoretical understanding of these processes faced three major difficulties: (i) Weinberg [2] found that the scattering amplitudes are not Lorentz invariant, (ii) multiparticle scattering states with both electric and magnetic charges carry additional angular momentum in the gauge field [3,4] and cannot be written as tensor products of Wigner's one-particle states, and (iii) the analysis of the scattering of grand unified theory (GUT) monopoles seemingly led to the conclusion that one must either give up on conservation of gauge charges or accept the existence of fractional particles [5].

The Lorentz violation problem was resolved by all orders resummation in Ref. [6] and order-by-order for the special case of monopoles bound with antimonopoles in Ref. [7]. The problem with multiparticle states was resolved by the inclusion of an additional quantum number called pairwise helicity [8,9]. In this Letter, we will present a resolution of the final problem.

To understand the essence of the final problem regarding the scattering of GUT monopoles it is helpful to recall a surprising fact about U(1) theories with magnetic monopoles and massless, oppositely charged Weyl fermions. When one of the Weyl fermions scatters with the monopole in such a theory, angular momentum conservation forbids forward scattering in the lowest partial wave. Instead, the massless fermion must flip its chirality [10] by turning into the Charge conjugation $\times$ Parity (CP) conjugate of the other fermion. We can embed this simple theory into a 't Hooft-Polyakov model [11] with an SU(2) gauge group and two Weyl doublets (An even number of doublets is required to avoid the Witten global anomaly.). This theory has an SU(2) flavor symmetry, which is perfectly consistent with the helicity flip process if the flavor flips as well. Things become more subtle in a model with four doublets where the helicity flip process is forbidden by an SU(4) global symmetry. The global symmetry allows processes with one fermion initial state scattering into three fermion final states as well as processes where [5] two incoming fermions scatter to two outgoing fermions [12]. The proton decay catalyzed by the latter type of process produces the leading observational bounds on the relic density of GUT monopoles in the Universe [13].

Callan [5] was the first to study the scattering of a positron off a GUT monopole. The problem reduces to the four flavor 't Hooft-Polyakov model in a limit where some gauge couplings are dropped, implying again that there have to be at least three fermions in the final state. Truncating to 2D and reformulating the problem in terms of solitons, Callan concluded [5,14] that there was no possible three fermion final state that preserved all of the gauge quantum numbers. As noticed by Witten, the truncated theory produced half solitons (also known as "semitons") in the final state [5], which they identified with "half particles" in the full theory. With the fermion masses set to zero these states, if they existed, would have to be true asymptotic final states far from the monopole, where perturbation theory can be reliably applied. Since these states cannot arise in perturbation theory, Callan suggested [5] that there could be some kind of statistical understanding where charge conservation is violated in individual events but is conserved on average. However this explanation was never fully embraced, since gauge invariance of the full 4D theory does not allow for such a probabilistic conservation.

The aim of this Letter is to find the form of the amplitude for this process by making use of helicity amplitudes and pairwise helicity. We will indeed be able to identify the unique form of the amplitude that preserves all gauge quantum numbers, angular momentum, and the approximate global flavor symmetry. As in the helicity-flip process discussed above, forward scattering is forbidden, and surprisingly the unique out-state comprises three fermions in a novel quantum state called a "pairwise" state [8,9]. In this "pairwise" final state, the spins of the fermions combine with the angular momentum in the gauge field into a total $J = 0$ partial wave. This amplitude corresponds to the lowest dimensional operator without derivatives in the low-energy effective theory. Thus we find that monopoles can produce fermions in a "pairwise" quantum state in an overall $J = 0$ partial wave, with a cross section satisfying the s-wave unitarity bound.

Rubakov-Callan interactions.—Consider the minimal SU(5) GUT monopole obtained by embedding the standard 't Hooft-Polyakov monopole [11] into the $\mathrm{SU}(2)_M \subset \mathrm{SU}(5)$ generated by $T^i_M = \mathrm{diag}(0, 0, \tau^i, 0)$. Far away from the monopole, the full SU(5) gauge symmetry is broken by a Higgs in the adjoint of SU(5) to the standard model (SM) gauge group which includes the $\mathrm{U}(1)_M$ generated by T^3_M. The monopole configuration is invariant under the combined rotations generated by $\vec{L} + \vec{T}$, where $\vec{L}$ is the orbital angular momentum operator and $\vec{T}$ is the vector of $\mathrm{SU}(2)_M$ generators T^i_M. In the SU(5) GUT, every generation of SM fermions is embedded in a $\bar{\mathbf{5}}$ and a 10. Under $\mathrm{SU}(2)_M$, these decompose into four doublets:

$$\begin{pmatrix} e \\ -\bar{d}^3 \end{pmatrix}, \begin{pmatrix} \bar{u}^1 \\ u^2 \end{pmatrix}, \begin{pmatrix} -\bar{u}^2 \\ u^1 \end{pmatrix}, \begin{pmatrix} d^3 \\ \bar{e} \end{pmatrix}, \tag{1}$$

where the upper and lower components have charge $e_M = \pm\frac{1}{2}$ under $\mathrm{U}(1)_M$, and all other fermions are $\mathrm{SU}(2)_M$ singlets. We labeled the particles by the corresponding left-handed fields; the right-handed particles correspond to Hermitian conjugates of these fields. Here the 1,2,3 label global color charge, which is broken in the vicinity of the monopole. Furthermore, in this Letter, we take the monopole to have charge $g_M = -1$ for consistency with Rubakov's notation [15]. This means that $q = e_M g_M = -1/2$ for $e, \bar{u}^1, \bar{u}^2, d^3$ while $q = 1/2$ for $\bar{d}^3, u^2, u^1, \bar{e}$. In the early '80s, Rubakov and Callan [12] independently derived a remarkable feature of fermion-monopole scattering. When a pair of $u^1 + u^2$ quarks is incident on the monopole, there are seemingly two possible outgoing states which conserve all SM quantum numbers—the initial state itself (i.e., forward scattering), and the state $d^{3^\dagger} + e^\dagger$. Rubakov and Callan showed that the $J = 0$ partial wave cannot undergo forward scattering. Instead, the incoming u^1, u^2 in this partial wave are converted to $d^{3^\dagger} + e^\dagger$, thus violating the baryon number (B).

To see this effect, both Rubakov and Callan focused on a truncated theory in which one only retains the $J = 0$ partial wave for each fermion. Famously [12], in this truncated theory, fermions with $q = -1/2$ exist only as incoming waves, while those with $q = 1/2$ exist only as outgoing waves. In particular, this implies that forward scattering is always forbidden. Note that the Hermitian conjugates of the fields in Eq. (1) have the opposite charge and helicity so $e^\dagger$ is also an outgoing wave. Consequently, monopoles induce a B-violating process with a cross section saturating the $J = 0$ unitarity bound. When QCD confinement is taken into account, this leads to monopoles catalyzing proton decay with a QCD scale cross section, counter to the naive intuition that the cross section is suppressed by the scattering energy over the GUT scale. In other words, monopole induced B-violation is a *nondecoupling* process. In comparison, the standard B-violating processes in SU(5) GUT are mediated by the X and Y GUT bosons and are suppressed by $(E/M_{\mathrm{GUT}})^n$, $n = 2$–4 and so are negligible compared with monopole catalysis. The leading observational bounds on the relic density of GUT monopoles in the Universe are then derived from proton-neutron decay catalyzed by monopole capture in neutron stars [13].

Monopole catalysis and pairwise helicity.—In Ref. [8], the most general form of the S matrix for the scattering of monopoles and charges was constructed. The main takeaway from this construction is that the multiparticle asymptotic states of the S matrix are *not* tensor products of single particle states. In particular, under Lorentz transformations, they pick up an extra little group phase for every monopole-charge (or dyon-dyon) pair. For example, consider a two-particle state where each particle has electric and magnetic charges (e_i, g_i) and spin s_i. This state transforms as

$$\begin{aligned} &\mathrm{U}(\Lambda)|p_i, p_j; s_i, s_j; q_{ij}\rangle \\ &\quad = e^{iq_{ij}\phi_{ij}} \mathcal{D}_{s'_i, s_i} \mathcal{D}_{s'_j, s_j} |\Lambda p_i, \Lambda p_j; s'_i, s'_j; q_{ij}\rangle, \end{aligned} \tag{2}$$

Here $\mathrm{U}(\Lambda)$ is the unitary representation of the Lorentz transformation Λ, while the $\mathcal{D}_{ab}$ represent single particle little group factors. The extra "pairwise little group" phase $e^{iq_{ij}\phi_{ij}}$ is unique to multiparticle states involving monopoles and charges (or any other mutually nonlocal particles). The pairwise helicity q_{ij} is half integer since it labels charges under the pairwise little group, which is a compact U(1) [8].

It has a natural interpretation as the quantity

$$q_{ij} = e_{Mi}g_{Mj} - e_{Mj}g_{Mi}, \tag{3}$$

which is quantized in half integer units by the Dirac-Zwanziger-Schwinger quantization condition [16].

A more detailed definition of electric magnetic multiparticle states was given in Ref. [9]. The transformation rule [Eq. (2)] implies additional constraints on scattering amplitudes involving monopoles—the functional form of the scattering amplitude has to be such that

$$\begin{aligned}\mathcal{A}(\Lambda p_1, \ldots, \Lambda p_n; \Lambda k_1, \ldots, \Lambda k_m) \\ = e^{-i\sum q_{ij}\phi_{ij}}\tilde{\mathcal{A}}(p_1, \ldots, p_n; k_1, \ldots, k_m),\end{aligned} \tag{4}$$

where $\tilde{\mathcal{A}}$ is the amplitude $\mathcal{A}$ times all of the single particle little group transformations $\mathcal{D}_i$. To construct amplitudes with the required transformation rule Ref. [8] defined new spinor-helicity variables called "pairwise spinors," denoted by $|p_{ij}^{\flat\pm}\rangle$, defined for each pair of particles in the in or out-state. For completeness, we repeat the definition of the standard massless spinor-helicity variables, as well as the pairwise spinor-helicity variables in the Supplemental Material [17]. The spinors have pairwise helicity $\pm$ under the pairwise little group associated with the particles i and j. In other words, they transform as

$$\begin{aligned}\tilde{\Lambda}|p_{ij}^{\flat\pm}\rangle &= e^{\pm\frac{i}{2}\phi(p_i,p_j,\Lambda)}|\Lambda p_{ij}^{\flat\pm}\rangle \\ [p_{ij}^{\flat\pm}|\tilde{\Lambda} &= e^{\mp\frac{i}{2}\phi(p_i,p_j,\Lambda)}[\Lambda p_{ij}^{\flat\pm}|,\end{aligned} \tag{5}$$

where Λ and $\tilde{\Lambda}$ are Lorentz transformations acting in vector and spinor spaces respectively. Finally, the pairwise spinors have the important property that they align with some of the standard spinor helicity variables in the massless limit. In particular,

$$\begin{aligned}\langle i p_{ij}^{\flat+}\rangle &= [i p_{ij}^{\flat+}] = 0 \\ \langle j p_{ij}^{\flat-}\rangle &= [j p_{ij}^{\flat-}] = 0.\end{aligned} \tag{6}$$

The vanishing of these contractions plays a central role in explaining the peculiarities of the Rubakov-Callan effect.

To see the relation between pairwise helicity and the Rubakov-Callan effect, let us consider an incoming state involving the massless fermions u^1, u^2, both with electric charge $e_M = -1/2$ and a scalar monopole M with magnetic charge $g_M = -1$. Let us now focus on the s-wave partial amplitude involving in and out states with total angular momentum $J = 0$. In this case the amplitude splits into an incoming and and outgoing part, each one depending only on the incoming-outgoing momenta and with all spinor indices contracted (since $J = 0$). As $q_{u^1,M} = q_{u^2,M} = -1/2$, the incoming part of the amplitude is

$$[u^1 p_{u^1,M}^{\flat-}][u^2 p_{u^2,M}^{\flat-}], \tag{7}$$

where $|p_{u^i,M}^{\flat-}]$ are pairwise spinors, while $[u^i|$ are the standard massless spinor helicity variables. To see that this in-state transforms correctly, note that the $|p_{u^i,M}^{\flat-}]$ each carry pairwise helicity $-1/2$ under the u^i, M pairwise little group, while the $[u^i|$ transform like a helicity $1/2$ under the single particle little group for u^i, which is suitable since incoming left-handed fermions carry helicity $1/2$ in our *all-outgoing* convention. In contrast, outgoing left-handed fermions carry helicity $-1/2$ in this convention. Note that pairwise helicity is not flipped between incoming and outgoing particles [8].

We can now easily see why there cannot be forward scattering in this process. Let us try to represent the would-be out-state relevant for forward scattering, i.e., involving the same u^1, u^2. The out part of the amplitude has to be

$$\langle u^1 p_{u^1,M}^{\flat+}\rangle\langle u^2 p_{u^2,M}^{\flat+}\rangle. \tag{8}$$

Note that the sign on the pairwise spinors is flipped so as to preserve their pairwise helicity under $|] \to |\rangle$. However, this expression vanishes by [Eq. (6)]. There cannot be forward scattering of fermions on a monopole in the lowest partial wave.

Having established that there is no forward scattering for the Rubakov-Callan in-state, we now turn to write down the only possible final state (There are technically other valid, anomalous, and baryon number violating processes that respect the SU(4) flavor symmetry. These correspond to the two disjoint processes $u^1 + u^2 + M \to e^\dagger + \bar{d}^{3\dagger} + M$ and $M \to M +$ any set of fermions with $\sum$ charges $= 0$. Since the Rubakov-Callan process is already a subprocess of this possibility, we will only address the pure Rubakov-Callan amplitude, as it leads to the inclusive cross section for monopole catalysis.) which respects all SM quantum numbers, as well as the overall SU(4) flavor symmetry. This out-state involves the fermions $e^\dagger, d^{3\dagger}$. The corresponding outgoing part of the amplitude is

$$[e^\dagger p_{e^\dagger,M}^{\flat-}][d^{3\dagger} p_{d^{3\dagger},M}^{\flat-}]. \tag{9}$$

It transforms correctly under the pairwise little group, since $q_{e^\dagger,M} = q_{d^{3\dagger},M} = 1/2$. Since this is the only possible out-state, we have a simple derivation of the Rubakov-Callan amplitude

$$\mathcal{A}_{\text{Rubakov-Callan}} \propto [u^1 p_{u^1,M}^{\flat-}][u^2 p_{u^2,M}^{\flat-}][e^\dagger p_{e^\dagger,M}^{\flat-}][d^{3\dagger} p_{d^{3\dagger},M}^{\flat-}]. \tag{10}$$

The overall cross section for the process satisfies the s-wave unitarity bound, and so should be proportional to $4\pi p_c^{-2}$ where p_c is the center of mass momentum. When taking QCD confinement of the incoming quarks into account, the incoming quarks are confined to within a distance of Λ^{-1} of each other, and the cross section becomes $\mathcal{O}(\Lambda^{-2})$.

Taking aim at a 40 year old mystery.—When a positron $\bar{e}$ scatters off of a GUT monopole, forward scattering is again forbidden by angular momentum conservation, while the flavor symmetry constrains the out-state to have three (mod 4) fermions. The only possible out-state with three fermions which conserves all quantum numbers is

$$\bar{u}^{1\dagger} + \bar{u}^{2\dagger} + \bar{d}^{3\dagger}. \tag{11}$$

Despite this, Callan argued that this final state is impossible, since in the presence of the monopole, the $\bar{d}^{3\dagger}$ cannot exist in a one-particle outgoing partial wave with $J = 0$. Starting with a truncated 2D theory including only fermions in one-particle $J = 0$ waves, Callan then applied 2D bosonization to represent the fermions as solitons. He then found that the 2D final state consists of four semitons, or "half particles." For the initial state of an $\bar{e}$ he found the semitonic final state $1/2(e^{\dagger} + \bar{u}^{1\dagger} + \bar{u}^{2\dagger} + d^{3})$. Since "half particles" do not exist in the 4D theory Callan suggested the interpretation that half the time one would produce a positron, and half the time one would produce a proton. These proposed individual processes do not conserve SM gauge charges, but would do so on average.

The semiton puzzle posed by Callan has since been analyzed by many authors within the framework of a 2D effective theory on the half line $r \geq 0$. Sen [18] claimed that conservation laws ensure that there are no monopole processes allowed with one fermion in the initial state and three fermions in the final state. If this were true then there would either have to be processes with more fermions (3 mod 4) or a mechanism that prevented single fermions from encountering a monopole. Note, however, that the conservation laws that Sen used are only valid in the 2D truncated theory which leaves out the possibility of pairwise multiparticle final states. Within the effective 2D framework, the puzzle has been boiled down to finding the correct boundary condition imposed on the 2D fermions by the monopole collective coordinate. Polchinski [19] showed that the dyon collective coordinate can be effectively integrated out, leaving an effective "dyon" boundary condition which is nonlinear in the 2D fermions [20]. This problem has been systematically analyzed by Affleck and Sagi [21] and more recently by Boyle Smith and Tong [22], by applying Cardy's method relating boundary conditions and boundary states in a 2D conformal field theory with boundary [23]. In the particular case of four fermionic flavors without gauge interactions, Maldacena and Ludwig [24] identified an SO(8) global symmetry and showed that the dyon boundary condition linearizes upon the use of SO(8) triality. The extensive work done within the effective 2D theory effectively solved the unitarity puzzle *within the effective 2D theory*—and showed that the semiton picture is essentially correct. Nevertheless, it is still far from clear how this 2D solution lifts to a physical 4D solution. In particular, it is not clear what happens in the GUT monopole process where the fermions have chiral non-Abelian charges that break the SO(8) symmetry. Moreover, it is not clear that the truncated 2D theory indeed captures all of the relevant degrees of freedom of the dimensionally reduced 4D theory. Indeed, if semitons existed in 4D they would violate Dirac charge quantization.

Kitano and Matsudo [25] suggested that the semitons should be identified in the 4D theory with a "pancake" soliton: a domain wall bounded by a string. These pancakes are supposed to be heretofore unknown asymptotic states of the gauge theory. For this to be a consistent interpretation in the massless fermion limit, the pancake would also have to have arbitrarily small energies since the incoming positron energy can be arbitrarily small.

Using the pairwise helicity formalism, we are able for the first time to propose a simple 4D final state for positron-monopole scattering. This final state does, in fact, consist of the fermions in Eq. (11), which conserve all of the SM quantum numbers and respect the approximate SU(4) flavor symmetry. The novelty here is that the final state fermions are in a "pairwise" quantum state, which is by definition not a tensor product of single particle states. In an entangled pairwise state, the helicity of one fermion can combine with field angular momentum arising from one of the other particles. This allows the multiparticle state to be in an overall $J = 0$ state, even though none of the individual fermions is in a one-particle $J = 0$ state. The amplitude for this process is

$$\mathcal{A}_{\bar{e}} \propto [\bar{e}\, p^{\flat -}_{\bar{e}^{\dagger},M}][\bar{u}^{1\dagger} p^{\flat -}_{\bar{u}^{1\dagger},M}][\bar{u}^{2\dagger} p^{\flat +}_{\bar{d}^{3\dagger},M}][\bar{d}^{3\dagger} p^{\flat -}_{\bar{u}^{2\dagger},M}] - (1 \leftrightarrow 2). \tag{12}$$

Note that we cannot arrange a similar balancing of helicity and field angular momentum when there are only two fermions in the final state. This can be seen by considering the following setup. First, focus on the case where the monopole is heavy and static, and work in monopole rest frame, where the fermion momenta back-to-back along the z-axis (without loss of generality). By definition, the flat momenta are also back to back along the z-axis—and so the contraction of a flat spinor with the regular spinor of the opposite particle gives exactly zero. For finite monopole masses there could be a contribution that is suppressed by the monopole mass, but this cannot saturate the unitarity bound.

Note that truncating to 2D is equivalent to demanding that each one of the outgoing fermions is in an *individual* $J = 0$ partial wave, which in the on shell language means that each single particle spinor $[\bar{e}|, [\bar{u}^{1\dagger}|, [\bar{u}^{2\dagger}|, [\bar{d}^{3\dagger}|$ has to be contracted with its own pairwise spinor. In particular, that means that the correct final state [Eq. (12)] is missed in the 2D truncated theory. Moreover, $[\bar{d}^{3\dagger} p^{\flat +}_{\bar{d}^{3\dagger},M}] = 0$ by Eq. (6), and so the truncated 2D theory seems to not have any allowed final state for this process.

Finally, we can extend our analysis of monopole catalysis processes to an arbitrary gauge group G with Lie algebra $\mathfrak{g}$ and N_F fermions. Note that in an anomaly free theory, our method will always yield a valid out-state for any valid in-state. This is because the cancellation of the gravitational anomaly guarantees that the sum of $\mathrm{U}(1)_j$ charges of all fermions in the theory vanishes for all $\mathrm{U}(1)_j$ in the Cartan subalgebra $\mathfrak{c}$ of $\mathfrak{g}$. Hence for any valid $J_{\mathrm{tot}} = 0$ in-state there will be a valid $J_{\mathrm{tot}} = 0$ out-state which conserves all charges in $\mathfrak{c}$ and leads to an amplitude which respects the global (In that we mean that N_F flavors appear as external legs, as in an 't Hooft vertex.) $\mathrm{SU}(N_F)$. Sometimes there will be more than one such out-state; for example, in the SU(5) GUT setup we find the possibility of the Rubakov-Callan process accompanied by any set of fermions with the sum of charges equal to 0. Conversely, if the theory has a gravitational anomaly, it means that there are some fermions that do not fall into anomaly free multiplets. Take one of these to be the in-state (if there is not one in the presence of the monopole we can always consider scattering with an antimonopole). Then the scattering does not have a valid, charge conserving out-state such that the amplitude respects $\mathrm{SU}(N_F)$.

Challenges and future work.—Our proposed solution [Eq. (12)] to the monopole unitarity puzzle is based entirely on shell reasoning; it is the only 4D final state (up to fermion pair production) which conserves all SM quantum numbers and respects angular momentum selection rules, expressed in terms of pairwise helicity. However, we did not demonstrate that our suggested state is indeed produced by the monopole. In fact, there is a potential difficulty in the dynamical generation of our final state by the monopole since there is an angular momentum barrier for one-particle states. Still, this apparent impediment relies on a non-relativistic quantum mechanical picture of fermions covariantly coupled to the background field of a static monopole. It is not clear to us whether this picture fully captures the dynamics of the soft photon cloud sourced by the monopole and the fermions, along with its associated angular momentum [26]. To check this, one would have to time evolve our proposed pairwise out-state using the full quantum field theory Hamiltonian of monopole QED [27].

Applications.—Since cross sections that saturate partial wave unitarity grow with the inverse of the initial momentum one might naively expect that the positron scattering process we have discussed would lead to an arbitrarily large cross section for B violation in GUT theories. In fact the growth is cut off at energies $E < \Lambda_{\mathrm{QCD}}$, as happens for the Rubakov-Callan processes. There is, however, an important distinction between the two. At energies lower than Λ_{QCD} the cross section for the Rubakov-Callan process remains fixed at $\sigma_{\mathrm{RC}}^{\mathrm{hadronic}} \sim \kappa \Lambda_{\mathrm{QCD}}^{-2}$, where κ is an unknown QCD-dependent $\mathcal{O}(1)$ coefficient. This is because the incoming state for this process involves both a u^1 and a u^2 that arrive to the monopole confined within a distance of $\sim\Lambda_{\mathrm{QCD}}^{-1}$ inside the proton. In contrast, the B-violating cross section for positron-monopole scattering becomes *zero* at low energies. This can be seen as follows. Once the initial energy is below the sum of the monopole and proton masses, the final state of three quarks cannot hadronize into a proton. In the monopole rest frame the three quark state will carry the initial momentum of the positron. Once the separations of the quarks reaches the QCD scale, the quarks will be forced to travel in the same direction, so two of the quarks will have their momentum flipped by QCD interactions. Since QCD also breaks chirality, their chirality can also be flipped, and they can become in-state for a second interaction with the monopole. Two quarks scattering on the monopole produce an antiquark and a lepton. The antiquark can annihilate with the remaining quark to produce two photons or a lepton-antilepton pair. Thus below the proton threshold there is no B violation, as we expect from energy conservation, and the B-violating cross section is cut off at the QCD scale.

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[*]ccsaki@gmail.com
[†]yshirman@uci.edu
[‡]t10ofrit@gmail.com
[§]jterning@gmail.com

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