Partition of Two Interacting Electrons by a Potential Barrier

Sungguen Ryu[1,2] and H.-S. Sim[1,*]
[1]Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon 34141, Korea
[2]Institute for Cross-Disciplinary Physics and Complex Systems IFISC (UIB-CSIC), E-07122 Palma de Mallorca, Spain

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Scattering or tunneling of an electron at a potential barrier is a fundamental quantum effect. Electron-electron interactions often affect the scattering, and understanding of the interaction effect is crucial in detection of various phenomena of electron transport and their application to electron quantum optics. We theoretically study the partition and collision of two interacting hot electrons at a potential barrier. We predict their kinetic energy change by their Coulomb interaction during the scattering delay time inside the barrier. The energy change results in characteristic deviation of the partition probabilities from the noninteracting case. The derivation includes nonmonotonic dependence of the probabilities on the barrier height, which qualitatively agrees with recent experiments, and reduction of the fermionic antibunching.

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Interplay of potential scattering of an electron and electron-electron interactions causes nontrivial effects. Generally, the former is used for detecting the latter. For example, the interaction strength of Luttinger liquids [1] and spatial ordering of Wigner crystals [2,3] are read out from anomalous electron tunneling through a potential barrier. And, the latter reduces quantum coherence of the former. It happens in electron interferometers in the quantum Hall regime [4–7], where phase accumulation between scattering events is smeared out by intra-edge [8–10] or interedge-channel interactions [11,12].

The interplay has been investigated in electron quantum optics. Electron scattering at a potential barrier provides a tool not only for studying partition [13,14], antibunching [15], identical particle statistics and anyon braiding [16–20], but also for operating flying qubits [21]. It combines with on-demand generation of wave packets by ac driving [22–37]. An electron packet, generated on a quantum Hall edge at the Fermi level, is partitioned at a barrier. Using partition noise [38], one studies antibunching between the electron and excitations of the Fermi sea [39]. When two packets collide [40,41] at a barrier as in Hong-Ou-Mandel effects, deviation from fermionic antibunching was observed [42,43] and attributed to charge fractionalization [44–46] of Luttinger liquids.

All the above examples involve interaction effects outside a barrier. A recent experiment [47] implies interactions *inside* a barrier. There, two single-electron wave packets are generated far above ($\gtrsim$100 meV) the Fermi level by a quantum-dot pump. These hot electrons copropagate in a depleted region, spatially isolated from other electrons. The observed partition probabilities of the electrons at the barrier [Fig. 1(a)] cannot be decomposed into products of single-electron partition probabilities. The probabilities show nonmonotonic dependence on the barrier height. The results are not described by noninteracting theories [48–50] nor by the charge fractionalization. They remain unexplained, suggesting that the characteristics of the barrier needs to be counted.

In this Letter, we develop a scattering theory of two interacting hot electrons at a potential barrier, and notice a central role of the scattering delay times (sometimes called phase times [51])

$$\tau_{\mathrm{T}}^{D} \equiv \hbar \mathrm{Im} \frac{d \ln \mathbf{t}_{\mathcal{E}}}{d\mathcal{E}}, \qquad \tau_{\mathrm{R}}^{D} \equiv \hbar \mathrm{Im} \frac{d \ln \mathbf{r}_{\mathcal{E}}}{d\mathcal{E}}, \tag{1}$$

of single-electron transmission and reflection at the barrier. $s_{\alpha\beta}(\mathcal{E}) \in \{\mathbf{r}_{\mathcal{E}}, \mathbf{r}'_{\mathcal{E}}, \mathbf{t}_{\mathcal{E}}, \mathbf{t}'_{\mathcal{E}}\}$ is the scattering amplitude of a plane wave of energy $\mathcal{E}$ from an input path β to an output α at the barrier (Fig. 1). We predict the kinetic energy change of the electrons by their Coulomb interaction during the delay times, and compute its effect on their partition at the barrier, considering initially copropagating or counterpropagating electrons. In the copropagating case, our theory

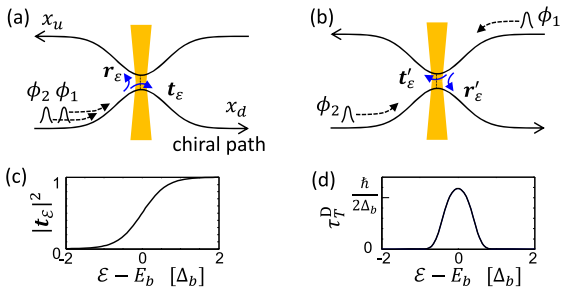

FIG. 1. Potential barrier (shade) on chiral paths u and d. (a) Partition of two copropagating hot electrons $\phi_{i=1,2}$ (peaks) at the barrier. (b) Collision of two counterpropagating electrons at the barrier. (c) Transmission probability $|\mathbf{t}_\mathcal{E}|^2$ and (d) delay time τ_T^D of a plane wave of energy $\mathcal{E}$ at the barrier.

explains the recent experiments [47]. Energy dependence of the delay times causes nonmonotonic dependence of the partition on the barrier height. The scattering probabilities of the two electrons are correlated when the transmission and reflection delay times differ. In the counterpropagating case, we distinguish direct and exchange interaction effects on the partition, especially on the reduction of their antibunching.

Setup.—We consider two hot electrons generated by quantum-dot pumps in a strong magnetic field [33,52]. They approach a potential barrier, propagating along a one-dimensional chiral upper path $\gamma = u$ or a lower path $\gamma = d$ in depleted regions. In Fig. 1(a), they initially copropagate, occupying orthogonal single-electron wave packets $\phi_{m=1,2}$ which usually separate in energy or time in experiments [47]. In Fig. 1(b), they initially counterpropagate, occupying packets $\phi_{m=1,2}$ of the same Gaussian form [31], and arrive at the barrier simultaneously. Each initially has kinetic energy $E_m^{(0)}$ and energy uncertainty σ_E. We assume that their propagation velocity v is energy independent, as the dependence is not strong enough to generate the nonmonotonicity [47].

In the strong magnetic field, the barrier is described by a saddle point constriction [53], and mapped onto a one-dimensional problem [54]. For a plane wave of energy $\mathcal{E}$, the barrier transmission probabilities, $|\mathbf{t}_\mathcal{E}|^2 = |\mathbf{t}'_\mathcal{E}|^2 = 1/[1+\exp(-\pi(\mathcal{E}-E_b)/\Delta_b)]$, change from 0 to 1 over the energy Δ_b around the barrier height E_b where $|\mathbf{t}_{\mathcal{E}=E_b}|^2 = 0.5$ [Fig. 1(c)]. We consider the $\sigma_E < \Delta_b$ regime to predict universal results; here, the wave packet form does not change during its barrier scattering, hence, the results are insensitive to the form. In Ref. [47], $\Delta_b \sim 5\sigma_E$.

The electrons interact through a Coulomb potential [55], $W(x_{\rm rel}) = W_0 e^{-x_{\rm rel}/a_{\rm scr}}/\sqrt{1+(x_{\rm rel}/a_{\rm cut})^2}$. Their separation $x_{\rm rel}$ is simplified as $x_{\rm rel} = x_1 - x_2$ when their coordinates x_m are on the same path, and $x_{\rm rel} = |x_1| + |x_2|$ for them on different paths ($x_m = 0$ at the barrier). $a_{\rm scr}$ is the screening length. The cutoff $a_{\rm cut}$ describes packet broadening to the transverse directions by the magnetic length or the quantum well width confining two-dimensional electrons.

Interaction during delay times.—We compute the partition probabilities P_n that n (= 0, 1, 2) of the two electrons move to the lower path after barrier scattering. They have contributions $P_n = P_n^{(\rm dir)} + P_n^{(\rm ex)}$ from direct and exchange processes, $P_n^{(\rm dir)} = \langle \hat{P}_n \rangle$, $P_n^{(\rm ex)} = \mp \langle \hat{P}_n \mathcal{P}_{\rm ex} \rangle$. $\hat{P}_n$ is the projection operator onto the event of P_n. $\mathcal{P}_{\rm ex}$ is the operator exchanging the two electrons. The sign $-$ $(+)$ is for the electrons in the spin triplet (singlet). We obtain [56] the correction $\delta P_n^{(\rm dir/ex)}$ to the noninteracting probabilities $P_n^{(0)}$ up to the lowest order of the interaction W and σ_E/Δ_b,

$$\delta P_n^{(\rm dir)} = \int_0^\infty dt \left(-\frac{i}{\hbar}\right) \langle [\hat{P}_n, W] \rangle_{\phi_1(t)\otimes\phi_2(t)}$$
$$\delta P_n^{(\rm ex)} = \mp \int_0^\infty dt \left(-\frac{i}{\hbar}\right) \langle [\hat{P}_n \mathcal{P}_{\rm ex}, W] \rangle_{\phi_1(t)\otimes\phi_2(t)} \quad (2)$$

by perturbatively expanding the time evolution operator with respect to W. $[\cdots,\cdots]$ is the commutator.

$\phi_{m=1,2}(t)$ are the packets at time t in the noninteracting case. Their product state is used in computing the expectation values $\langle \cdots \rangle$ in Eq. (2), assuming that the packets are separable at the initial time $t = 0$. Each is decomposed into $\phi_m^{(\rm in)}$ in the input path β, $\phi_m^{(\rm out,\alpha)}$ in an output path α, and $\phi_m^{(\rm bar)}$ in the barrier,

$$|\phi_m(t)\rangle = |\phi_m^{(\rm in)}(t)\rangle + \sum_{\alpha=u,d} |\phi_m^{(\rm out,\alpha)}(t)\rangle + |\phi_m^{(\rm bar)}(t)\rangle. \quad (3)$$

The expression $|\phi_m^{(\rm out,\alpha)}(t)\rangle$ includes the scattering amplitude $s_{\alpha\beta}$. For $\sigma_E \ll \Delta_b$, we derive [56] the probability of electron $m(=1,2)$ being in the barrier,

$$\langle \phi_m^{(\rm bar)}(t) | \phi_m^{(\rm bar)}(t) \rangle = \bar{\tau}_m A_m(t) + \mathcal{O}(\sigma_E^2/\Delta_b^2), \quad (4)$$

in terms of the barrier dwell time [51] (mean delay time) $\bar{\tau}_m \equiv |\mathbf{t}_{E_m^{(0)}}|^2 \tau_{mT}^D + |\mathbf{r}_{E_m^{(0)}}|^2 \tau_{mR}^D$ and the arrival time distribution [57] $A_m(t)$ (the probability per time of arrival at the barrier at t) of electron m.

Inside the barrier, electron m has the kinetic energy $E_m = E_m^{(0)} + \delta E_m^{(\rm dir)} + \delta E_m^{(\rm ex)}$. The change $\delta E_m^{(\rm dir/ex)}$ from the initial value $E_m^{(0)}$ by direct or exchange interactions with the other electron m' occurs in their input paths or the barrier, hence, depending on the trajectory of m'. Using Eqs. (2)–(4), the energy change occurring in the barrier during the dwell time $\bar{\tau}_m$ of m is found [56] as $\bar{\tau}_m \Gamma^{(\rm dir/ex)} + \mathcal{O}(\sigma_E^2/\Delta_b^2) + \mathcal{O}(W^2)$,

$$\bar{\tau}_m \Gamma^{(\rm dir)} = -\bar{\tau}_m \int dt A_m(t) \left\langle v \frac{\partial W}{\partial x_m} \right\rangle_{|0_m\rangle\otimes|\phi_{m'|\alpha}(t)\rangle},$$
$$\bar{\tau}_m \Gamma^{(\rm ex)} = -\bar{\tau}_m {\rm Re} \int dt A_m(t) \left\langle v \frac{\partial W}{\partial x_m} \mathcal{P}_{\rm ex} \right\rangle_{|0_m\rangle\otimes|\phi_{m'|\alpha}(t)\rangle}. \quad (5)$$

The Coulomb power $\Gamma^{(\rm dir/ex)}$ comes from the force $-\partial W/\partial x$ to electron m while m is inside the barrier (described by the state $|0_m\rangle$) and m' moves along a trajectory from its input β to output α without partitioning at the barrier that is described by $|\phi_{m'|\alpha}(t)\rangle \equiv |\phi_{m'}^{(\rm in)}(t)\rangle + |\phi_{m'}^{(\rm barr)}(t)\rangle + s_{\alpha\beta}^{-1}|\phi_{m'}^{(\rm out,\alpha)}(t)\rangle$ [cf. the corresponding state with partitioning in Eq. (3)].

The energy change $\delta E_m^{(\rm dir/ex)}$ modifies the partition probabilities. $P_2 = P_2^{(\rm dir)} + P_2^{(\rm ex)}$ is found as

$$P_2^{(\rm dir)} \simeq \prod_{m=1,2} \int d\mathcal{E} |\tilde{\phi}_m(\mathcal{E})|^2 |s_{d\beta_m}(\mathcal{E} + \delta E_m^{(\rm dir)})|^2,$$
$$P_2^{(\rm ex)} \simeq \mp \Big| \int d\mathcal{E} (\tilde{\phi}_1(\mathcal{E}) s_{d\beta_1}(\mathcal{E} + \delta E_1^{(\rm ex)}))^* \times \tilde{\phi}_2(\mathcal{E}) s_{d\beta_2}(\mathcal{E} + \delta E_2^{(\rm ex)}) \Big|^2 \quad (6)$$

with (i) the amplitude $\tilde{\phi}_m(\mathcal{E})$ of finding the initial packet of electron m in the plane wave having energy $\mathcal{E}$ of its input path β_m and (ii) the scattering amplitude $s_{d\beta_m}$ to the lower output d at the energy shifted by $\delta E_m^{(\mathrm{dir/ex})}$ due to the interaction with the other electron m' moving to the lower output. Equation (6) is valid up to the lowest order of W and σ_E/Δ_b, and gives the noninteracting result at $\delta E_m^{(\mathrm{dir/ex})} = 0$. P_0 is found similarly for two electrons moving to the upper output and $P_1 = 1 - P_0 - P_2$.

Partitioning copropagating electrons.—We consider two copropagating electrons [Fig. 1(a)], the predecessor (labeled by $m = 1$) and successor ($m = 2$) initially separated by distance $\ell > \hbar v/(2\sigma_E)$. Their partition is determined by direct processes. When $\sigma_E \ll \Delta_b$, the partition probabilities in Eq. (6) are written as

$$P_2 \simeq |\mathbf{t}_{\tilde{E}_1+\delta E^{(\mathrm{dir})}_{1|TT}}|^2 |\mathbf{t}_{\tilde{E}_2+\delta E^{(\mathrm{dir})}_{2|TT}}|^2,$$
$$P_0 \simeq |\mathbf{r}_{\tilde{E}_1+\delta E^{(\mathrm{dir})}_{1|RR}}|^2 |\mathbf{r}_{\tilde{E}_2+\delta E^{(\mathrm{dir})}_{2|RR}}|^2. \tag{7}$$

In the noninteracting limit, they are $P_2 = |\mathbf{t}_{E_1^{(0)}}|^2 |\mathbf{t}_{E_2^{(0)}}|^2$ and $P_0 = |\mathbf{r}_{E_1^{(0)}}|^2 |\mathbf{r}_{E_2^{(0)}}|^2$. $\tilde{E}_m - E_m^{(0)}$ is the kinetic energy change of electron m that happens while the electrons copropagate along the input path over distance $L_{m=1,2}$; the predecessor gains energy, $\tilde{E}_1 - E_1^{(0)} = \Gamma_\ell L_1/v$, and the successor losses energy, $\tilde{E}_2 - E_2^{(0)} = -\Gamma_\ell L_2/v$. $\Gamma_\ell \equiv -v(\partial W/\partial x_{\mathrm{rel}})|_{x_{\mathrm{rel}}=\ell}$ (> 0) is the Coulomb power at their separation ℓ. The energy gain or loss is determined by the sign of the force $-\partial W/\partial x_m$ [cf. Eq. (5)]. Electron m has further energy change by $\delta E^{(\mathrm{dir})}_{m|TT}$ ($\delta E^{(\mathrm{dir})}_{m|RR}$) during barrier scattering when they both are transmitted (resp. reflected). We roughly estimate it from Eq. (5),

$$\delta E^{(\mathrm{dir})}_{1|TT} \approx \Gamma_\ell \bar{\tau}_1, \qquad \delta E^{(\mathrm{dir})}_{2|TT} \approx -\Gamma_\ell \bar{\tau}_1 - \Gamma_{\ell - v\tau^D_{1T}} \bar{\tau}_2,$$
$$\delta E^{(\mathrm{dir})}_{1|RR} \approx \Gamma_\ell \bar{\tau}_1, \qquad \delta E^{(\mathrm{dir})}_{2|RR} \approx -\Gamma_\ell \bar{\tau}_1 - \Gamma_{\ell - v\tau^D_{1R}} \bar{\tau}_2. \tag{8}$$

During its dwell time $\bar{\tau}_1$ the predecessor gains energy $\Gamma_\ell \bar{\tau}_1$, while the successor losses $\Gamma_\ell \bar{\tau}_1$. After the predecessor scatters out of the barrier, the successor enters the barrier, as $\ell > \hbar v/(2\sigma_E)$. This moment, their separation is reduced to $\ell - v\tau^D_{1T}$ or $\ell - v\tau^D_{1R}$ by the delay time of the barrier transmission or reflection of the predecessor. Then the successor further losses energy by $\Gamma_{d - v\tau^D_{1T}} \bar{\tau}_2$ or $\Gamma_{d - v\tau^D_{1R}} \bar{\tau}_2$ during its dwell time $\bar{\tau}_2$.

Using Eq. (6), we compute P_n in Fig. 2 for a symmetric saddle point constriction $V_{\mathrm{sym}} = E_b - m^*\omega_0^2(x^2 - y^2)/2$ on the two dimension (x, y). The results qualitatively follow Eqs. (7) and (8). This constriction has $\Delta_b = \hbar\omega_0^2/(2\omega_c)$ [54] and the symmetric delay times, $\tau^D_{mT} = \tau^D_{mR} = \bar{\tau}_m$, hence $\delta E^{(\mathrm{dir})}_{2|TT} = \delta E^{(\mathrm{dir})}_{2|RR}$. ω_c is the cyclotron

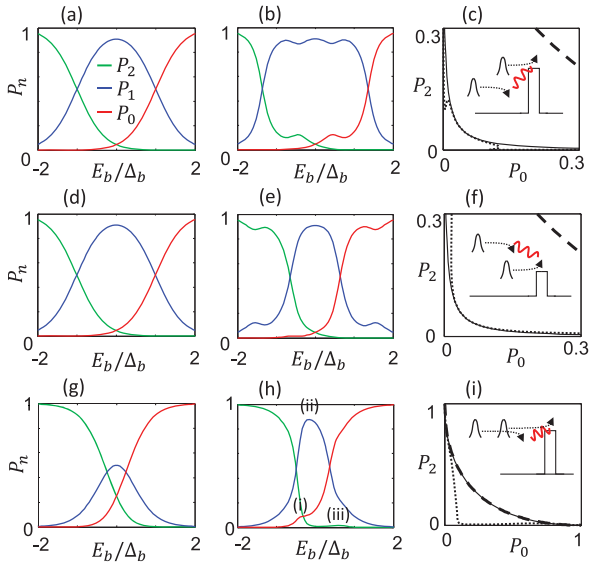

FIG. 2. Partition probabilities $P_{n=0,1,2}$ of two copropagating electrons in Fig. 1(a) by the symmetric saddle point constriction V_{sym}, as a function of the barrier height E_b measured with respect to $(\tilde{E}_1 + \tilde{E}_2)/2$. Left panels: The noninteracing case. Middle: The interacting case. Right: P_0 versus P_2 in the noninteracting (solid curve) and interacting (dotted) cases. The thick dashed curve follows $\sqrt{P_0} + \sqrt{P_2} = 1$. Insets: Schematic kinetic energy change of the electrons during barrier scattering. In (a)–(c), $\tilde{E}_1 = \tilde{E}_2 + 2\Delta_b$. In (d)–(f), $\tilde{E}_1 = \tilde{E}_2 - 2\Delta_b$. In (g)–(i), $\tilde{E}_1 = \tilde{E}_2$. We choose $W_0 = 144$ meV [56], $a_{\mathrm{scr}} = 500$ nm, $a_{\mathrm{cut}} = 10$ nm, $\Delta_b = 5.4$ meV [47], $\sigma_E = 1$ meV [31], $v = 5 \times 10^4$ m/s [52], and $\ell = 3\hbar v/(2\sigma_E)$.

frequency and m^* is the electron effective mass. The partition probabilities exhibit nonmonotonic dependence on E_b in various energy configurations of $\tilde{E}_{m=1,2}$. This originates from the peak structure in the energy dependence of the delay times [Fig. 1]. For instance, the energy exchange $\Gamma_\ell \bar{\tau}_1$ is maximal when the energy $\tilde{E}_1$ of the preceding electron aligns with the barrier height so that $\bar{\tau}_1$ is the largest. The resulting nonmonotonic features of P_n at $\tilde{E}_1, \tilde{E}_2 \sim E_b$, the enhanced P_1 [see (ii) in Fig. 2(h) and Fig. 3(a)] and the reduced P_0 and P_2 accompanied by peaks [(i) and (iii)], agree with the corresponding features of Fig. 3(d) of the experimental report [47].

In an asymmetric saddle point constriction, the transmission and reflection delay times $\tau^{\mathrm{D}}_{1\mathrm{T}}$ and $\tau^{\mathrm{D}}_{1\mathrm{R}}$ differ. Then the partition can violate $\sqrt{P_0} + \sqrt{P_2} \le 1$, a condition [50] for uncorrelated scattering of noninteracting electrons. To see this, we choose an asymmetric constriction $V_{\mathrm{asym}}(x, y) = E_b - m^*(\omega_x^2 x^2 - \omega_y^2 y^2)/2$, where $\omega_x = \omega_{xL}$ and $\omega_y = \omega_{yL}$ for $x < 0$, $\omega_x = \omega_{xR}$ and $\omega_y = \omega_{yR}$ for $x > 0$, and $\omega_{xL}/\omega_{yL} = \omega_{yR}/\omega_{xR} = 1/2$, ω_{yL}/ω_{xR}; the violation does not rely on this specific choice for simplicity of calculation. It has $\Delta_b = \hbar\omega_{xL}\omega_{yL}/(2\omega_c)$ and $\tau^{\mathrm{D}}_{1\mathrm{T}} < \tau^{\mathrm{D}}_{1\mathrm{R}}$ [56]. Then the reflection of the predecessor, in comparison with the transmission, causes larger energy loss of the

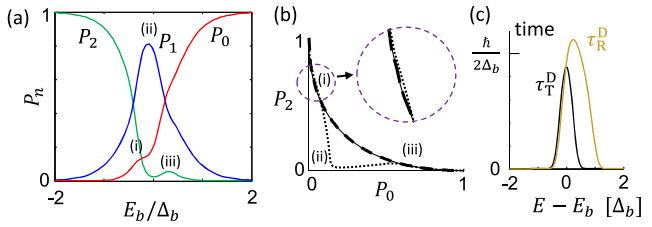

FIG. 3. Partition probabilities P_n of two copropagating electrons, having $\tilde{E}_1 = \tilde{E}_2$, by the asymmetric constriction $V_{\rm asym}$. (a) P_n as a function of E_b in the interacting case [cf. the corresponding symmetric constriction in Figs. 2(g)–2(i)]. (b) P_0 versus P_2 in the noninteracting (solid curve) and interacting (dotted) cases. The interacting case violates $\sqrt{P_0} + \sqrt{P_2} \le 1$ (the dashed curve in the zoom-in plot). (c) Delay times τ_T^D and τ_R^D for the transmission and reflection of a packet of energy E at the asymmetric constriction. The same parameters with Fig. 2 are chosen, except $\omega_{xL}/\omega_{xR} = 1/2$.

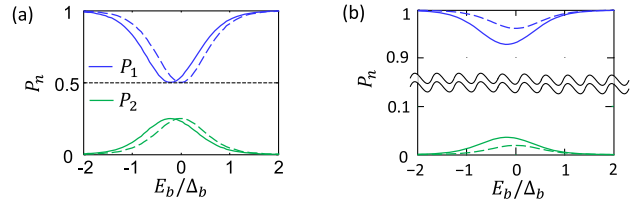

FIG. 4. Partition probabilities P_n by collision of two counterpropagating electrons at the symmetric constriction, as a function of E_b in the noninteracting (dashed curves) and interacting (solid) cases. E_b is measured with respect to $\tilde{E}$. The electrons have (a) opposite spins or (b) same spins. The same parameters with Fig. 2 are chosen.

successor during its dwell time $\bar{\tau}_2$ so that the scattering probabilities of the two electrons are correlated, violating $\sqrt{P_0} + \sqrt{P_2} \le 1$ [Eqs. (7)–(8), Fig. 3]. This may explain the violation observed in Fig. 4 of Ref. [47]. The marks (i), (ii),(iii) in Fig. 3(b) correspond to the nonmonotonic features (i),(ii),(iii) of Fig. 3(a), respectively, in agreement with Ref. [47].

Collision.—We next consider two counterpropagating hot electrons that simultaneously arrive at the symmetric constriction [Fig. 1(b)]. Their wave packets have the same Gaussian form of mean energy $\tilde{E}$ at the barrier entrance. Their spins are in a product state $|\chi_1\rangle \otimes |\chi_2\rangle$, as generated by independent pumps. In this case, the partition probabilities satisfy $P_0 = P_2$ and $P_1 = 1–2P_2$. In Fig. 4, we compute P_n, using Eq. (6). The results qualitatively agree with the relations

$$P_2 \simeq |\mathbf{t}_{\tilde{E}+\delta E^{(\rm dir)}}|^2 |\mathbf{r}'_{\tilde{E}+\delta E^{(\rm dir)}}|^2 - |\langle\chi_1|\chi_2\rangle|^2 |\mathbf{t}_{\tilde{E}+\delta E^{(\rm ex)}}|^2 |\mathbf{r}'_{\tilde{E}+\delta E^{(\rm ex)}}|^2 \quad (9)$$

valid at $\sigma_E \ll \Delta_b$. $\delta E^{(\rm dir/ex)}$ is the kinetic energy change by direct or exchange interactions during the collision.

In Fig. 4(a) we consider electrons having opposite spins, $\langle\chi_1|\chi_2\rangle = 0$. In the noninteracting case, $\delta E^{(\rm dir)} = 0$ and the dependence of P_2 on E_b has a peak of height $P_2 = 1/4$ at $E_b = \tilde{E}$ at which $|\mathbf{t}_{\tilde{E}}|^2 = |\mathbf{r}'_{\tilde{E}}|^2 = 1/2$. In the interacting case, P_n is determined by $\tilde{E} + \delta E^{(\rm dir)}$. $\delta E^{(\rm dir)}$ is negative, as the distance between the electrons decreases in the collision. The peak of P_2 is shifted to lower E_b by $|\delta E^{(\rm dir)}|$, but the peak height is still $1/4$.

In Fig. 4(b) we consider electrons having the same spin, $\langle\chi_1|\chi_2\rangle = 1$. In the noninteracting case, the antibunching of $P_1 = 1$ and $P_2 = P_0 = 0$ happens in the plane wave limit of $\sigma_E = 0$. However, deviation $P_2 \neq 0$ from the antibunching occurs at finite σ_E/Δ_b, where the form of the wave packet changes during barrier scattering [55]. In the interacting case, further deviation happens, since $|\delta E^{(\rm ex)}|$ is smaller than $|\delta E^{(\rm dir)}|$ as usual.

Nonmonotonic behaviors of P_n, similar to those of Fig. 3(a), can happen in the collision, when the electrons arrive at the barrier at different times more than $a_{\rm cut}/v$.

Discussion.—We develop a theory for scattering of two interacting electrons by a potential barrier, and emphasize kinetic energy change by their interaction during scattering delay times. The change depends on whether they copropagate or counterpropagate to approach the barrier, their relative arrival time at the barrier, and the nonmonotonicity and asymmetry in the delay times. For copropagating electrons, the preceding electron gains energy while the succeessor losses energy. For counterprogating cases, they both loss energy. The energy change results in nonmonotic dependence of their partition on the barrier height, correlation of their scattering probabilities, and reduction of fermionic antibunching in the collision. Our finding provides a basic example of multiparticle scattering problems, and will be useful in application of electron quantum optics to flying qubits, as combination of barrier partitioning and Coulomb interactions is essential for coupling multiple qubits.

Our finding does not rely on a specific form of the barrier and Coulomb potentials. Our perturbative treatment of the Coulomb interaction is applicable when $\Gamma_\ell \bar{\tau} \sim \hbar\Gamma_\ell/\Delta_b \lesssim \Delta_b$; this condition is satisfied with usual constrictions [47] where $\bar{\tau} \sim$ subpicoseconds.

We note that in the quantum Hall regime [23,27,41,58] where electron wave packets move along a quantum Hall edge channel, having low energy (≤ 0.1 meV) close to the Fermi level, Γ_ℓ/Δ_b may be so small that our effects are negligible. When the scattering amplitudes of those packets at a quantum point contact are manipulated to be energy dependent (e.g., in nonequilibrium), the scattering delay times will play a role, as in our study. It will be interesting to study interplay between the delay times and electron interactions of chiral Luttinger liquids along the edge.

We considered initial two-electron (anti-symmetrized) product states. This is supported by experiments [59], where the purity of electron states generated by a

quantum-dot pump is very low. Nonetheless, entanglement in initial states can affect P_n, depending on its detailed form. For example, when two electrons are initially in an equal superposition of the initial states of Figs. 2(b) and 2(e), their partition probability P_n equals the average of the results in Figs. 2(b) and 2(e). In this case, P_n does not show the nonmonotonicity. Studies on the effects of general entanglement will be valuable.

When the electrons occupy incoherent wave packets, exchange interactions vanish, so their collision is governed by direct processes. In this case, our theory is applicable [60] also to the regime of $\Delta_b < \sigma_E$ with classical ensemble average, although it is developed for $\Delta_b > \sigma_E$.

It is known that in mesoscopic devices, scattering delay times play a role in nonlinear current response [61–64] at scatterers due to charge screening, although they are short as subpicoseconds ($\sim\hbar/\Delta_b$). Our Letter identifies their new role in multiparticle scattering. This role was unnoticed in theories on two-particle scattering [65] including those for capacitively coupled conductors [66], numerical studies [55] for colliding electrons, and classical descriptions [67]. Note that the delay times differ from the traversal time [31,68,69].

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*hssim@kaist.ac.kr

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